

DESCRIPTION

LEVEL 1 Students form a hexagon using 6 blocks of their own choice such that the hexagon has a continuous surface. They calculate and compare the sums of the volumes of the blocks marked with the same colour in the figure.

LEVEL 2 Students prove that this is always true: the two sums are equal in any possible configuration of a continuous surface hexagon.

SOLUTIONS / EXAMPLES

LEVEL 1 See the solution of exercise <u>517 - Heights and Volumes</u> or exercises <u>515 - Simple Volumes</u> and <u>516 - Truncated Volumes</u> for the volumes of the blocks.

After the calculation, observe that the two sums are equal.

LEVEL 2 Use the following formula to calculate the sum of the volumes: $\frac{1}{3}A(h_1 + h_2 + h_3)$

The proof is based on the fact that when two blocks meet, they have two heights in common.

Let the middle height be h_0 and the outer heights be h_1 , h_2 , ..., h_6 respectively, as shown in the figure.



Sum of the volumes of the blocks marked in light colour:

$$\frac{1}{3}A(h_0 + h_1 + h_2) + \frac{1}{3}A(h_0 + h_3 + h_4) + \frac{1}{3}A(h_0 + h_5 + h_6) = \frac{1}{3}A(3h_0 + h_1 + h_2 + h_3 + h_4 + h_5 + h_6)$$

Sum of the volumes of the blocks marked in dark colour:

$$\frac{1}{3}A(h_0 + h_2 + h_3) + \frac{1}{3}A(h_0 + h_4 + h_5) + \frac{1}{3}A(h_0 + h_6 + h_1) = \frac{1}{3}A(3h_0 + h_1 + h_2 + h_3 + h_4 + h_5 + h_6)$$

The two sums are equal, hence the statement is proven.

PRIOR KNOWLEDGE

Volume formula for the truncated prism

RECOMMENDATIONS / COMMENTS

The calculations of the Level 1 exercise for the different blocks or colours can be given to different students.

After the calculations in Level 1, the whole group or the whole class can discuss the question of Level 2.

We recommend exercise <u>517 - Heights and Volumes</u> as a previous exercise in order to be familiar with the formula used in this exercise.

Exercise <u>518 - Proof of the Volume Formula</u> is the proof of the formula used in this exercise.