Lesson 4: Dividing powers of 10

Goals

- Generalise a process for dividing powers of 10, and justify (orally and in writing) that $\frac{10^n}{10^m} = 10^{n-m}$.
- Use exponent rules to multiply and divide with 10^0, and justify (orally) that 10^0 is 1.

Learning Targets

- I can evaluate 10^0 and explain why it makes sense.
- I can explain and use a rule for dividing powers of 10.

Lesson Narrative

Students continue to use repeated reasoning to discover the exponent rule $\frac{10^n}{10^m} = 10^{n-m}$. For now, students work with expressions where n and m are positive integers and $n > m$. In the last activity, students extend to the case where $n = m$ to make sense of why 10^0 is defined to be equal to 1 and critique a faulty argument that it should be defined to be equal to 0. Students make sense of this rule when they recognise that separating the same number of factors from the numerator and denominator, then dividing has the effect of multiplying by 1. For example $\frac{10^3}{10^2} = \frac{10\times10}{10\times10} \times 10$ which is the same as $1 \times 10 = 10$ or the same as $10^{3-2} = 10^1$. In a subsequent lesson, students will extend this rule to include situations where $n < m$.

Building On

- Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognising multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n\times a}{n\times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Addressing

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

Building Towards

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

Instructional Routines

- Stronger and Clearer Each Time

- Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for the rule $\frac{10^n}{10^m} = 10^{n-m}$. For a guiding example, consider $\frac{10^5}{10^2} = \frac{10\times10\times10\times10\times10}{10\times10} = \frac{10\times10}{10\times10} \times 10 \times 10 \times 10 = 1 \times 10^3 = 10^3$.

Create a visual display for the rule $10^0 = 1$. For an example, you can show $\frac{10^6}{1} = \frac{10^6}{10^0} = 10^{6-0} = 10^6$. Another possibility is to write $10^5 \times 1 = 10^5 \times 10^0 = 10^{5+0} = 10^5$ and use visual aids to highlight that each of these examples implies $10^0 = 1$.

Student Learning Goals

Let's explore patterns with exponents when we divide powers of 10.

4.1 A Surprising One

Warm Up: 10 minutes

In this activity, students investigate fractions that are equal to 1. This is an important concept that helps students make sense of the exponent division rule explored later in this lesson. It is expected that students will try to compute the numerator and denominator of the fraction directly in the first problem. Notice any students who instead make use of structure to show multiplication by 1.

Launch

Give students 5 minutes of quiet work time. Expect students to attempt to work out all of the multiplication without using exponent rules. Follow with a brief whole-class discussion.

Student Task Statement

What is the value of the expression?

$$\frac{2^5 \times 3^4 \times 3^2}{2 \times 3^6 \times 2^4}$$

Student Response

$\frac{2^5 \times 3^4 \times 3^2}{2 \times 3^6 \times 2^4}$ is equal to 1. Strategies vary. Sample strategies:

- Compute the numerator and denominator and then realise that they are equal $(\frac{23\,328}{23\,328})$, giving an overall value of 1.
- Notice that there are 5 factors that are 2 and 6 factors that are 3 in both numerator and denominator, thus making the entire fraction equal to 1.

Activity Synthesis

The key takeaway is that a fraction is often easier to analyse when dividing matching factors from the numerator and denominator to show multiplication by 1. Select any students who made use of structure in this way. If no students did this, provide the following example: $\frac{2 \times 3 \times 7 \times 11}{5 \times 3 \times 7 \times 11} = \frac{3 \times 7 \times 11}{3 \times 7 \times 11} \times \frac{2}{5} = 1 \times \frac{2}{5} = \frac{2}{5}$.

If time allows, consider the following questions for discussion:

- "What has to be true about a fraction for it to equal 1?" (The numerator and denominator must be the same value and something other than 0.)
- "Create your own fraction that is equivalent to 1 that has several bases and several exponents."

4.2 Dividing Powers of Ten

10 minutes

Explore division of powers of 10 to derive the rule $\frac{10^n}{10^m} = 10^{n-m}$. At this point, students will only be working in cases where $n > m$ and will later extend the rule to include $n = m$. The case of $n < m$ is left for the next lesson. The rule arises from the fact that $\frac{10^n}{10^m} = \frac{10^m}{10^m} \times 10^{n-m}$ which is equivalent to 10^{n-m}. These problems are also meant to underscore the connection between fractions and division, and students should be expected to use both fractions and division interchangeably as they work.

Instructional Routines

- Stronger and Clearer Each Time
- Notice and Wonder

Launch

Before students begin working, ask students to notice and wonder about the "expanded" column for the expression $10^4 \div 10^2$.

It is important for students to understand that the "expanded" column shows each power of 10 expanded into factors, the division written as a fraction, and a certain number of factors in the numerator and denominator being grouped because their quotient is 1.

Give students 5–7 minutes quiet work time followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support use of structure. For example, check in with students within the first 2-3 minutes of work time. Ask students to share how they made sense of the first row of the table. Use colour or annotation to highlight connections between the "expanded" column and the single power.
Supports accessibility for: Memory; Organisation

Student Task Statement

a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the "expanded" column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10\times10\times10\times10}{10\times10} = \frac{10\times10}{10\times10} \times 10 \times 10 = 1 \times 10 \times 10$	10^2
	$\frac{10\times10\times10\times10\times10}{10\times10} = \frac{10\times10}{10\times10} \times 10 \times 10 \times 10 =$ $1 \times 10 \times 10 \times 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

1. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression of the form $10^{\square}$.
2. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Student Response

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10\times10\times10\times10}{10\times10} = \frac{10\times10}{10\times10} \times 10 \times 10 = 1 \times 10 \times 10$	10^2
$10^5 \div 10^2$	$\frac{10\times10\times10\times10\times10}{10\times10} = \frac{10\times10}{10\times10} \times 10 \times 10 \times 10 = 1 \times 10 \times 10 \times 10$	10^3
$10^6 \div 10^3$	$\frac{10\times10\times10\times10\times10\times10}{10\times10\times10} = \frac{10\times10\times10}{10\times10\times10} \times 10 \times 10 \times 10 = 1 \times 10 \times 10 \times$	10^3

	10	
10^{43} $\div 10^{17}$	skip	10^{26}

b. I chose to skip the expanded column of $10^{43} \div 10^{17}$ because there is not enough space in the table for all of the factors.

1. $\frac{10^n}{10^m} = 10^{n-m}$ because m factors in the numerator and denominator are divided to make 1, leaving $n - m$ factors remaining.

2. There are roughly 100 trees per person because 10^{12} trees divided equally among 10^{10} people is 10^{12-} $= 10^2$ trees per person.

Are You Ready for More?

expression	expanded	single power
$10^4 \div 10^6$		

Student Response

expression	expanded	single power
$10^4 \div 10^6$	$\frac{10\times10\times10\times10}{10\times10\times10\times10\times10\times10} = \frac{10\times10\times10\times10}{10\times10\times10\times10} \times \frac{1}{10} \times \frac{1}{10} = 1 \times \frac{1}{10} \times \frac{1}{10}$	$\frac{1}{10^2}$

Activity Synthesis

The key idea for writing $\frac{10^n}{10^m}$ with one exponent is that m factors in the numerator and denominator can be divided to make 1, leaving $n - m$ factors in the numerator. Here are some sample questions to guide students to this idea:

- "Say we want to write $10^6 \div 10^3$ with a single power of 10. What happens to the 3 factors that are 10 in the denominator? How many factors that are 10 are left in the numerator?" (The 3 factors that are 10 in the denominator are matched with 3 of the factors that are 10 in the numerator, then divided to make 1. Since 3 factors that are 10 from the numerator are used to make 1, there are still 3 left.)

- "If you wanted to write $10^{80} \div 10^{20}$, what would happen with the 20 factors that are 10 in the denominator? How many factors that are 10 would still be left in the numerator?" (To make this connection explicit, you might show the calculation $\frac{10^{80}}{10^{20}} = \frac{10^{20}\times10^{60}}{10^{20}} = \frac{10^{20}}{10^{20}} \times 10^{60}$. Compare this to $10^{80} \div 10^{20} = 10^{80-20} = 10^{60}$.)

Make another "Exponent Rule" poster for the rule $\frac{10^n}{10^m} = 10^{n-m}$. An example to illustrate the rule could be $\frac{10^5}{10^3} = \frac{10\times10\times10\times10\times10}{10\times10\times10} = \frac{10\times10\times10}{10\times10\times10} \times 10 \times 10 = 1 \times 10 \times 10 = 10^2$. Use colours and other visual aids to highlight the fact that the exponents are subtracted because m factors that are 10 in the numerator and denominator are used to make a factor of 1, leaving $n - m$ factors that are 10.

Writing, Speaking: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their explanation of the patterns they noticed in the task. Ask each student to meet with 2–3 other partners in a row for feedback. Provide student with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “Can you give an example?”, “Can you say that another way?”, “How do you know…?”, etc.). Students can borrow ideas and language from each partner to strengthen their final explanation.
Design Principle(s): Optimise output (for explanation)

4.3 Zero Exponent

15 minutes

Students extend exponent rules to discover why it makes sense to define 10^0 as 1. Students create viable arguments and critique the reasoning of others when discussing Noah’s argument that 10^0 should equal 0.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes to answer the first 3 problems and a few minutes to discuss the last problem with a partner before a whole-class discussion.

Student Task Statement

So far we have looked at powers of 10 with exponents greater than 0. What would happen to our patterns if we included 0 as a possible exponent?

1. Write $10^{12} \times 10^0$ with a power of 10 with a single exponent using the appropriate exponent rule. Explain or show your reasoning.
2. What number could you multiply 10^{12} by to get this same answer?
3. Write $\frac{10^8}{10^0}$ with a single power of 10 using the appropriate exponent rule. Explain or show your reasoning.
4. What number could you divide 10^8 by to get this same answer?

5. If we want the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?

6. Noah says, "If I try to write 10^0 expanded, it should have zero factors that are 10, so it must be equal to 0." Do you agree? Discuss with your partner.

Student Response

1. 10^{12} because $10^{12} \times 10^0 = 10^{12+0} = 10^{12}$.
2. 1 because $10^{12} \times 1 = 10^{12}$.
3. 10^8 because $\frac{10^8}{10^0} = 10^{8-0} = 10^8$.
4. 1 because $\frac{10^8}{1} = 10^8$.
5. The value of 10^0 has to be 1 in order for the exponent rules to work when the exponent is 0.
6. Answers vary. Sample response: Noah's answer appears to make sense with how we first defined exponents. It is true that there will be no factors of 10 but there will be a factor of 1 since the base is not equal to 0.

Activity Synthesis

The important concept is that $10^0 = 1$ is a convenient definition that extends the usefulness of the exponent rules to a wider range of numbers. This idea is developed in the next lesson to further extend exponent rules to include negative exponents.

Ask students to share their thinking about what 10^0 means. Noah's argument raises a valid concern that 10^0 doesn't fit the definition that we use when the exponent is positive. One way to address this concern is to allow alternate definitions and see what happens to the exponent rules. For example, if we want to define 10^0 as 0, then we can choose to do that. However, when we look at an example like $10^3 \times 10^0$, the result would be zero. This just means 10^0 does not match the pattern of the exponent rule, which says the result should be 10^{3+0} which is equal to 10^3.

Introduce the visual display for the rule $10^0 = 1$. Display for all to see throughout the unit. To illustrate the rule, consider displaying the example $\frac{10^6}{10^0} = 10^{6-0} = 10^6$. For this to be true, the denominator 10^0 must be equal to 1. Another possibility is to write $10^5 \times 10^0 = 10^{5+0} = 10^5$ which can only be true if $10^0 = 1$.

Speaking: Discussion Supports. To scaffold students in explaining the patterns they noticed when 10^0, provide sentence frames for students to use when they are working with their partner. For example, "I think ____ because _____." or "I (agree/disagree) because ______."
Design Principle(s): Support sense-making; Optimise output for (explanation)

4.4 Making Millions

Optional: 10 minutes

This activity expands on a previous cool-down as students generate different representations of the same number to solidify what they have learned about exponent sum, product, and subtraction rules. Notice students who use the exponent rules they have learned in different ways to achieve an exponent of 6, especially those who combine different exponent rules together. It is not expected that students will make an exponent of 6 using negative exponents, but do not discourage it if they do. Explain to these students that, while the rules still work when using negative exponents, it is not yet clear what the value of, say, 10^{-2} is.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Give 5–7 minutes of quiet work time before asking students to share their response with their partner. Follow with a whole-class discussion.

Student Task Statement

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents, multiplication, and division. What patterns do you notice with the exponents?

Student Response

Answers vary. Sample responses:

- 1 000 000
- A million
- $10^4 \times 10^2$
- $(10^2)^3$
- $10^8 \div 10^2$
- $\frac{10^{80}}{10^{74}}$ For multiplying powers of 10, the exponents must always have a sum of 6. For dividing powers of 10, the exponents must always have a difference of 6. When raising powers of 10 to another power, the exponents must have a product that is 6.

Activity Synthesis

Tell students to compare their response with their partner's to discuss what differences there might be. In a whole-class discussion, select students to share a variety of responses. Look especially for examples that show creativity or that combine multiple rules together.

The main idea is for students to show flexibility with the exponent rules they have learned so far. To include more students in the discussion, consider asking: “How can you build on __’s method to come up with another expression that makes a million?” If time allows, ask whether it’s possible to make an exponent of 6 using negative numbers and indicate that a future lesson will explore negative exponents in detail.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making

Lesson Synthesis

The goal of the discussion is to check that students can explain why the exponents are subtracted when rewriting a quotient of powers of 10 with a single exponent, and why it makes sense to define 10^0 as equal to 1. Consider recording student responses and displaying them for all to see.

Here are possible discussion questions:

- “How can you write $\frac{10^{36}}{10^{12}}$ using a single exponent?” (10^{24})
- “It’s a common mistake for students to try to divide the exponents. Why do we subtract the exponents instead of divide them?” (The exponents subtract because we are counting the number of factors that are 10 that survive division. In this case, we have $\frac{10^{36}}{10^{12}} = \frac{10^{12} \times 10^{24}}{10^{12}} = 10^{24}$.)
- “Why did we define 10^0 to be equal to 1?” (10^0 is defined to be equal to 1 so that it fits with the exponent rules we have discovered for positive exponents. It is a logical extension of the rules. For example, the rules indicate $10^4 \times 10^0$ should be equal to 10^{4+0}, which is just 10^4. So 10^0 is a number that doesn’t change the value of other numbers when it is multiplied. The only number with this property is 1, so it only makes sense to define 10^0 as 1.)

4.5 Why Subtract?

Cool Down: 5 minutes

Student Task Statement

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11}? Explain or show your thinking.

Student Response

Answers vary. Sample response: $\frac{10^{15}}{10^4} = 10^{11}$ because 4 factors that are 10 in the numerator and denominator are used to make 1, leaving 11 remaining factors that are 10. In other words, $\frac{10^{15}}{10^4} = \frac{10^4 \times 10^{11}}{10^4} = 10^{11}$.

Student Lesson Summary

In an earlier lesson, we learned that when multiplying powers of 10, the exponents add together. For example, $10^6 \times 10^3 = 10^9$ because 6 factors that are 10 multiplied by 3 factors that are 10 makes 9 factors that are 10 all together. We can also think of this multiplication equation as division: $10^6 = \frac{10^9}{10^3}$. So, when dividing powers of 10, the exponent in the denominator is subtracted from the exponent in the numerator. This makes sense because $\frac{10^9}{10^3} = \frac{10^3 \times 10^6}{10^3} = \frac{10^3}{10^3} \times 10^6 = 1 \times 10^6 = 10^6$. This rule works for other powers of 10 too. For example, $\frac{10^{56}}{10^{23}} = 10^{33}$ because 23 factors that are 10 in the numerator and in the denominator are used to make 1, leaving 33 factors remaining.

This gives us a new exponent rule: $\frac{10^n}{10^m} = 10^{n-m}$.So far, this only makes sense when n and m are positive exponents and $n > m$, but we can extend this rule to include a new power of 10, 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means that if we want the rule to work when the exponent is 0, then it must be that $10^0 = 1$

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   Evaluate:

   a. 10^0

   b. $\frac{10^3}{10^3}$

   c. $10^2 + 10^1 + 10^0$

   Solution

   a. 1

   b. 1

   c. 111

2. **Problem 2 Statement**

Write each expression as a single power of 10.

a. $\frac{10^3\times10^4}{10^5}$

b. $(10^4)\times\frac{10^{12}}{10^7}$

c. $\left(\frac{10^5}{10^3}\right)^4$

d. $\frac{10^4\times10^5\times10^6}{10^3\times10^7}$

e. $\frac{(10^5)^2}{(10^2)^3}$

Solution

a. 10^2

b. 10^9

c. 10^8

d. 10^5

e. 10^4

3. **Problem 3 Statement**

The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

Solution

10^3 (or 1 000). This can be determined by calculating $\frac{10^5}{10^2}$, since both the Sun and KG Sagittarii's widths can be compared to the width of Earth.

4. **Problem 4 Statement**

Bananas cost £1.50 per pound, and guavas cost £3.00 per pound. Kiran spends £12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

a. Write an equation relating the two variables.

b. Rearrange the equation so b is the independent variable.

c. Rearrange the equation so g is the independent variable.

Solution

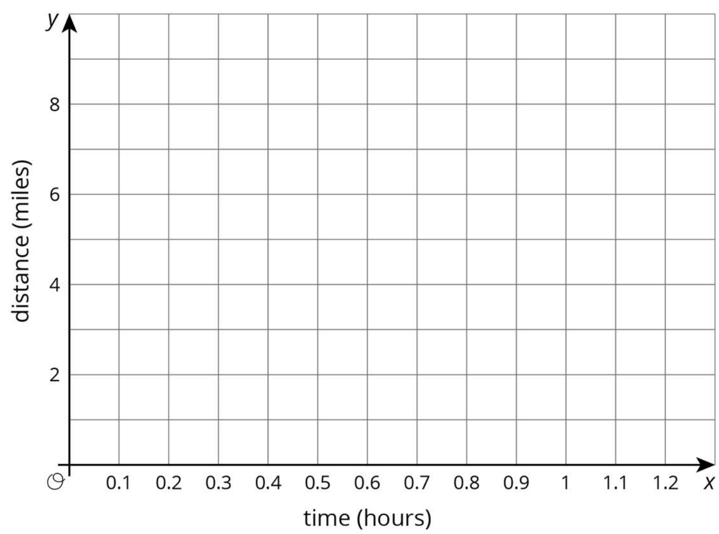
a. $1.5b + 3g = 12$

b. $g = 4 - \frac{1}{2}b$

c. $b = 8 - 2g$

5. **Problem 5 Statement**

Lin's mum bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mum bikes. Sketch a graph of both of these relationships.



Solution