Chapter 1 Applications of Matrices and Determinants

1.If $|adj\ (adjA)| = |A|^9$, then the order of the square matrix A is

(1)3 (2)4 (3)2 (4)5

Sol: $|adj(adjA)| = |A|^{(n-1)^2}$

$|adj\ (adjA)| = |A|^9$

$(n-1)^2 = 9 = 3^2$

n-1=3⇒n=4 (Option:2)

2.If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(1)A (2)B (3)I_3 (4)B^T

Sol: $BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$

$= (A^{-1}A^T)\ ((A^T)^T(A^{-1})^T$

$= (A^{-1}A^T)(A(A^{-1})^T)$

$= (A^{-1}\ A)(A^T(A^T)^{-1}) = I$

(Option:3)

3.If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adjA and C = 3A , then $\frac{|adj\ B|}{|C|} =$

(1)$\frac{1}{3}$ (2)$\frac{1}{9}$ (3)$\frac{1}{4}$ (4)1

Sol: $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, |A| = 6 - 5 = 1$

$C = 3A \Rightarrow |C| = |3A| = 3^2|A| = 9 \times 1 = 9$

$$\frac{|adj\ B|}{|C|} = \frac{|adj\ (adjA)|}{|C|} = \frac{|A|^{(2-1)^2}}{9} = \frac{1}{9}$$

(Option:2)

4.If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

(1)$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (2)$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

(3)$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (4)$\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

Sol: $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 6I$

$A = 6I\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}^{-1}$

$= 6I\frac{1}{(4+2)}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

(Option:3)

5.If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

(1)A^{-1} (2)$\frac{A^{-1}}{2}$ (3)$3A^{-1}$ (4)$2A^{-1}$

Sol:

$9I_2 - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$A^{-1} = \frac{1}{(14-12)}\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = 2\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\frac{A^{-1}}{2} = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$9I_2 - A = \frac{A^{-1}}{2}$ (Option:2)

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adjAB| =$

(1)-40 (2)-80 (3)-60 (4)-20

Sol:$|adjAB| = |adjB|\ |adjA|$

$= 10 \times (-8) = -80$ (Option:2)

7.If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1)15 (2)12 (3)14 (4)11

Sol: $|adjA| = |A|^{n-1}$

$\begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{vmatrix} = 4^{(3-1)}$

$-2(3-x) = 16$

$-6 + 2x = 16$

$x = 11$ (Option:4)

8. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(1)0 (2)-2 (3)-3 (4)-1

Sol: $|A| = 2$

$$a_{23} = \frac{(co\ factor\ of\ 2)}{|A|} = \frac{-\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}}{2} = \frac{-2}{2} = -1$$

(Option:4)

9. If A, B and C are invertible matrices of same order, then which one of the following is not true?

(1) $adj\ A = |A|A^{-1}$

(2) $adj\ (AB) = (adjA)(adjB)$

(3) $\det A^{-1} = (detA)^{-1}$

(4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(Option:2)

10.If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

(1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$

(3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

Sol:

$(AB)^{-1} = B^{-1}A^{-1}$

$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix} = B^{-1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

$\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}^{-1} = B^{-1}$

$$B^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}\frac{1}{(3-2)}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 36-34 & 12-17 \\ -57+54 & -19+27 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$$

(Option:1)

11.If A^TA^{-1} is symmetric then $A^2 =$

(1) A^{-1} (2) $(A^T)^2$

(3) A^T (4) $(A^{-1})^2$

Sol: $A^TA^{-1} = (A^TA^{-1})^T = (A^{-1})^T(A^T)^T$

$A^TA^{-1} = (A^{-1})^TA$

$A^TA^{-1} = (A^T)^{-1}A \Rightarrow (A^T)^2 = A^2$

(Option:2)

12.If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$

(3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

Sol: $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}^T$

$$= \begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$$

(Option:4)

13. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

Sol: $A^T = A^{-1} \Rightarrow AA^T = I$

$$\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}.x + \frac{4}{5} \times \frac{3}{5} = 0$$

$5(3x) + 12 = 0 \Rightarrow x = -\frac{4}{5}$ (Option:1)

14. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then B =

(1) $\left(cos^2\frac{\theta}{2}\right)A$ (2) $\left(cos^2\frac{\theta}{2}\right)A^T$

(3) $(cos^2\theta)I$ (4) $\left(sin^2\frac{\theta}{2}\right)A$

Sol: $AB = I_2 \Rightarrow B = A^{-1}$

$$B = \frac{1}{1+tan^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$B = \frac{1}{sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} = (cos^2\frac{\theta}{2})A^T$$

(Option:2)

15.If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adjA) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

(1) 0 (2) $\sin\theta$

(3) $\cos\theta$ (4) 1

Sol: $A(adjA) = (adjA)A = |A|I$

$$A(adjA) = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = kI$$

$$|A|I = 1I$$

$k = 1$ (Option:1)

16.If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(1) 17 (2) 14

(3) 19 (4) 21

Sol: $\lambda A^{-1} = A \Rightarrow \lambda\, I = A^2$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 19 & 19 \\ 19 & 19 \end{bmatrix}$$

$\lambda = 19$ (Option:3)

17.If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adj\, B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then $adj\,(AB)$ is

(1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$

(3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

Sol: $adj\,(AB) = adj\,(B)adj\,(A)$

$$= \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 2-8 & 3+2 \\ -6+4 & -9-1 \end{bmatrix}$$

$adj\,(AB) = \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (Option:2)

18.The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1)1 (2)2 (3)4 (4)3

Sol: $= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$(R_2 \to R_2 - 2R_1, R_3 \to R_3 + R_1)$

$\rho(A) = 1$ (Option:1)

19.If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively

(1)$e^{\left(\frac{\Delta_2}{\Delta_1}\right)}, e^{\left(\frac{\Delta_3}{\Delta_1}\right)}$ (2)$\log\left(\frac{\Delta_1}{\Delta_3}\right), \log\left(\frac{\Delta_2}{\Delta_3}\right)$

(3)$\log\left(\frac{\Delta_2}{\Delta_1}\right), \log\left(\frac{\Delta_3}{\Delta_1}\right)$ (4)$e^{\left(\frac{\Delta_1}{\Delta_3}\right)}, e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$

Sol:$a\,logx + b\,logy = m$

$c\,logx + d\,logy = n$

Applying Cramer's rule,

$$logx = \frac{\begin{vmatrix} m & b \\ n & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{\Delta_1}{\Delta_3} \Rightarrow x = e^{\left(\frac{\Delta_1}{\Delta_3}\right)}$$

Similarly we get,$y = e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$ (Option:4)

20.Which of the following is/are correct

(i)Adjoint of a symmetric matrix is also symmetric matrix.

(ii)Adjoint of a diagonal matrix is also a diagonal matrix.

(iii)If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$

(iv)$A(adjA) = (adjA)A = |A|I$

(1)only (i) (2)(ii) and (iii)

(3)(iii) and (iv) (4)(i),(ii) and (iv)

(Option:4)

21 If $\rho(A) = \rho(A|B)$, then the system AX=B of linear equations is

(1) consistent and has a unique solution

(2) consistent

(3) consistent and has infinitely many solutions

(4) inconsistent (Option:2)

22.If $0 \le \theta \le \pi$ and the system of Equations

$$x + (sin\theta)y - (cos\theta)z = 0,$$
$$(cos\theta)x - y + z = 0,$$
$$(sin\theta)x + y - z = 0$$

has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4)$\frac{\pi}{4}$

Sol:$\begin{vmatrix} 1 & sin\theta & -cos\theta \\ cos\theta & -1 & 1 \\ sin\theta & 1 & -1 \end{vmatrix} = 0$

$sin^2\theta = cos^2\theta \Rightarrow \theta = \frac{\pi}{4}$ (Option:4)

23.The augmented matrix of the system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7, \mu \ne -5$ (2) $\lambda = -7, \mu = 5$

(3) $\lambda \ne 7, \mu \ne -5$ (4) $\lambda = 7, \mu = -5$

Sol:

The system has infinitely many solutions,

$$\rho(A) = \rho(A, B) \ne 3$$

$\lambda = 7, \mu = -5$ (Option:4)

24.Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(1)2 (2)4 (3)3 (4)1

Sol:$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} . \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix} \Rightarrow$

$a_{13} = 0$

$\frac{-2}{4} - \frac{x}{4} + \frac{3}{4} = 0 \Rightarrow x = 1$ (Option:4)

25.If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adjA)$ is

(1)$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (2)$\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$

(3)$\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (4)$\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

$adj(adjA) = |A|^{n-2}A = |A|^{3-2}A$

$adj(adjA) = |A|A = 1A = A$

(Option:1)

YOUTUBE:MATHSTIMES_THIRUMURUGAN