Grades 9-12 (A)

Duration: 15 min

Tools: one Logifaces Set / class

Individual work

Keywords: Square root, Heron's formula, Measurements

413 - Area with Heron's Formula

MATHS / 2D GEOMETRY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

2019-1-HU01-KA201-0612722019-1

DESCRIPTION

Students use Heron's formula to calculate the area of the top and the base triangles of the blocks.

LEVEL 1 Students measure the lengths of the edges, aiming for high accuracy. The class can discuss the accuracy of the results and the size of the error depending on the measurements.

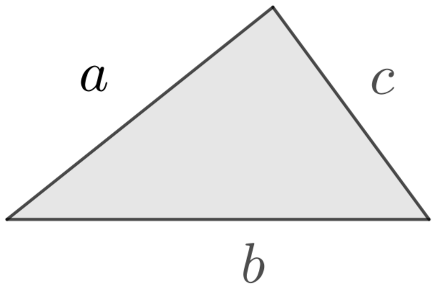
LEVEL 2 Students use the standard units to calculate the exact areas.

SOLUTIONS / EXAMPLES

Heron's formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \frac{a+b+c}{2}$



LEVEL 1 Area of the equilateral triangle:

$\sqrt{7.5 \times 2.5 \times 2.5 \times 2.5} \approx 10,825\ cm^2$

Area of the top triangle of blocks 112, 122, 223, 233:

$\sqrt{7.65 \times (7.65 - 5) \times (7.65 - 5.15) \times (7.65 - 5.15)} \approx 11.26\ cm^2$

Area of the top triangle of blocks 113, 133:

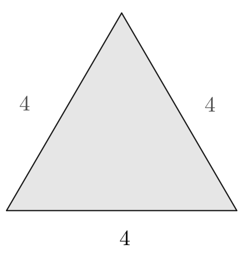
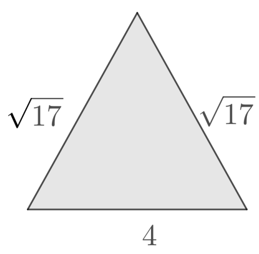
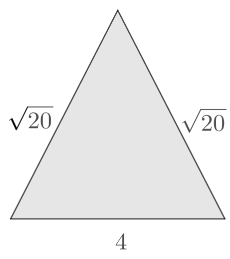
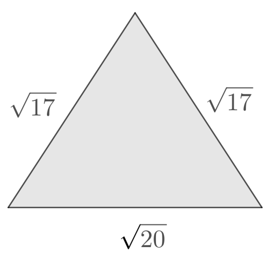
$\sqrt{8.09 \times (8.09 - 5) \times (8.09 - 5.59) \times (8.09 - 5.59)} \approx 12.5\ cm^2$

Area of the top triangle of blocks 123, 132:

$\sqrt{7.95 \times (7.95 - 5.15) \times (7.95 - 5.15) \times (7.95 - 5.59)} \approx 12.1\ cm^2$

LEVEL 2 See exercise [404 - Top Edges] for calculations of the lengths of the top edges.

There are 4 different types of triangles. The base faces are all equilateral triangles, the top faces are either equilateral triangles or isosceles triangles (3 types).

block 111, 222, 333	block 112, 122, 223, 233	block 113, 133	block 123, 132
			
$s = \frac{4+4+4}{2} = 6$	$s = \frac{4+\sqrt{17}+\sqrt{17}}{2} = 2 + \sqrt{17}$	$s = \frac{4+\sqrt{20}+\sqrt{20}}{2} = 2 + \sqrt{20}$	$s = \frac{\sqrt{20}+\sqrt{17}+\sqrt{17}}{2} = \sqrt{5} + \sqrt{17}$
$A = 4\sqrt{3}$	$A = 2\sqrt{13}$	$A = 8$	$A = 2\sqrt{15}$

CALCULATIONS

area of equilateral triangle:

$$A = \sqrt{6(6-4)(6-4)(6-4)} = \sqrt{6 \times 2 \times 2 \times 2} = 4\sqrt{3}$$

area of isosceles triangle with edges

$$4, \sqrt{17}, \sqrt{17}: A = \sqrt{(2+\sqrt{17})(2+\sqrt{17}-4)(2+\sqrt{17}-\sqrt{17})(2+\sqrt{17}-\sqrt{17})} =$$
$$= \sqrt{(2+\sqrt{17})(-2+\sqrt{17}) \times 2 \times 2} = \sqrt{52} = 2\sqrt{13}$$

$$4, \sqrt{20}, \sqrt{20}: A = \sqrt{(2+\sqrt{20})(2+\sqrt{20}-4)(2+\sqrt{20}-\sqrt{20})(2+\sqrt{20}-\sqrt{20})} =$$
$$= \sqrt{(2+\sqrt{20})(-2+\sqrt{20}) \times 2 \times 2} = \sqrt{64} = 8$$

$$\sqrt{20}, \sqrt{17}, \sqrt{17}: A = \sqrt{(\sqrt{5}+\sqrt{17})(\sqrt{5}+\sqrt{17}-\sqrt{20})(\sqrt{5}+\sqrt{17}-\sqrt{17})(\sqrt{5}+\sqrt{17}-\sqrt{17})} =$$
$$= \sqrt{(\sqrt{5}+\sqrt{17})(\sqrt{5}+\sqrt{17}-\sqrt{20}) \times \sqrt{5} \times \sqrt{5}} = \sqrt{60} = 2\sqrt{15}$$

PRIOR KNOWLEDGE

Square root, Triangle, Edge

RECOMMENDATIONS / COMMENTS

The area of a triangle can be calculated with the Heron's formula if all side lengths are known. There is no need to calculate the altitude of the triangle.

In exercise [411 - Area of Triangles] the same areas are calculated using the altitude of the triangle.

Level 1 is recommended when the goal is to calculate the area of the real object and discuss accuracy.

Level 2 is recommended when the goal is to practise calculations including square roots of integers.