Lesson 16: Representing contexts with equations

Goals

- Coordinate (orally and in writing) verbal descriptions, equations, and diagrams that represent the same situation involving an unknown amount in the context of temperature or height above sea level.
- Write equations of the form $x + p = q$ or $px = q$ to represent and solve a problem in an unfamiliar context, and present the solution method (using words and other representations).

Learning Targets

- I can explain what the solution to an equation means for the situation.
- I can write and solve equations to represent situations that involve rational numbers.

Lesson Narrative

In the previous lesson students looked at methods for solving equations with rational numbers. In this lesson students choose equations that represent a context, and write their own equations given a context. Students are also encouraged to look at the structure of an equation and decide if its solution is positive or negative, without solving it.

Building On

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Addressing

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
- Solve real-world and mathematical problems involving the four operations with rational numbers. Calculations with rational numbers extend the rules for manipulating fractions to complex fractions.

Building Towards

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Collect and Display
- Compare and Connect
- Discussion Supports

Student Learning Goals

Let’s write equations that represent situations.

16.1 Don't Solve It

Warm Up: 5 minutes

For this warm-up students use what they have learned about arithmetic with negative and positive numbers to determine the sign of solutions to equations. Students who focus on the signs of the numbers and the relative magnitudes without actually calculating are at an advantage.

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer. Follow with a whole-class discussion.

Student Task Statement

Is the solution positive or negative?

$(-8.7)(1.4) = a$

$-8.7b = 1.4$

$-8.7 + c = -1.4$

$-8.7 - d = -1.4$

Student Response

a is negative

b is negative

c is positive

d is negative

Activity Synthesis

For each question, ask at least one student to explain their reasoning. Make sure there is agreement for each question on whether the solution is positive or negative. Ask if anyone used the third question to help them answer the fourth.

16.2 Warmer or Colder than Before?

10 minutes (there is a digital version of this activity)

In this activity, students work with changing temperatures to build understanding of equations that represent situations with negative coefficients, variables, and solutions. Students choose from a bank of equations to find two equations, one that represents the situation using a variable and the other representing the path to solve for the variable. They interpret the meaning of the variable in the context of each situation, solve for the value of the variable that makes the equations true, and explain how the equations and their solutions describe the situation. Students contextualise and decontextualise between the contexts of changing temperatures and the equations that represent them.

Note that the last question involves some ambiguity. In order to select the anticipated response, students need to use the assumption that the temperature is less at midnight than it is at 9. Since it's the last question, they could also use some process of elimination to help lead them to making the assumption.

Instructional Routines

- Discussion Supports

Launch

Give students 5 minutes of quiet work time followed by whole-class discussion.

Students using the digital activity have a thermometer applet to help visualise the changes in temperature.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, to get students started, provide a smaller bank of equations and only the first two situations. Once students have successfully completed the three steps for each, introduce the remaining equations and situations.
Supports accessibility for: Conceptual processing; Organisation Representing, Listening: Discussion Supports. Demonstrate the first situation to clarify what students need to do by thinking aloud using mathematical language and reasoning. This will help invite more student participation, conversations, and meta-awareness of language representing situations with negative coefficients, defining variables, and determining values that make

equations true.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

For students who struggle to understand how equations represent these situations, suggest that they draw a number line in the form of a thermometer and show the changes along the number line while reasoning about the rising and falling temperatures.

Student Task Statement

For each situation,

- Find *two* equations that could represent the situation from the bank of equations. (Some equations will not be used.)
- Explain what the variable v represents in the situation.
- Determine the value of the variable that makes the equation true, and explain your reasoning.

Bank of equations:

$-3v = 9$

$-4 \times 3 = v$

$-4 \times -3 = v$

$v = -\frac{1}{3} \times 9$

$v = -16 + 6$

$v = 4 + (-12)$

$-3v = -6$

$v = -\frac{1}{3} \times (-6)$

$v = \frac{1}{3} \times (-6)$

$v = -16 - (6)$

$-6 + v = -16$

$v = 4 + 12$

$v + 12 = 4$

$v = 9 + 3$

$-4 = \frac{1}{3}v$

1. Between 6 a.m. and noon, the temperature rose 12 degrees Fahrenheit to 4 degrees Fahrenheit.
2. At midnight the temperature was -6 degrees. By 4 a.m. the temperature had fallen to -16 degrees.
3. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is -6 degrees at a certain time.
4. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is 9 degrees at a certain time.
5. The temperature at 9 p.m. is one third the temperature at midnight.

Student Response

Explanations vary. Sample responses:

1. $v = 4 + (-12)$, $v + 12 = 4$, v represents the temperature at 6 a.m., $v = -8$. Adding 12 to -8 will bring the temperature to 4.
2. $-6 + v = -16$, $v = -16 + 6$, v represents the change in temperature between midnight and 4 a.m., $v = -10$. The temperature had to drop 10 degrees to go from -6 to -16.
3. $-3v = -6$, $v = -\frac{1}{3} \times (-6)$, v represents the number of hours after midnight that it took to reach -6 degrees, $v = 2$. Dropping at a rate of 3 degrees per hour it will take 2 hours to go from 0 degrees to -6 degrees, so the time is 2 a.m.
4. $-3v = 9$, $v = -\frac{1}{3} \times 9$, v represents the change in time from midnight when the temperature was 9 degrees, $v = -3$. If the temperature has been dropping at 3 degrees per hour, and is at 0 degrees at midnight, then the temperature was 9 degrees sometime before midnight, so the change in time is negative. It took 3 hours for the temperature to fall from 9 degrees to 0 degrees, so the time was 3 hours before midnight, or 9 p.m.
5. $-4 = \frac{1}{3}v$, $-4 \times 3 = v$, v represents the temperature at midnight, $v = -12$. We expect that the temperature at midnight will be lower than the temperature at 9 p.m., so $-4 = \frac{1}{3}v$ matches this situation. (Note that $v = \frac{1}{3} \times -6$ could also work, if v represents the temperature at 9 p.m., but there is no equivalent equation to partner it with.)

Activity Synthesis

This discussion has two goals: First, for students to connect the equation that represents a situation to the equation that represents the solution strategy, and second, to ensure that students understand how to represent negative quantities in an equation. Ask students to share examples of how they chose:

- the equation that represents the situation
- the equation that represents the solution strategy

and in what order they chose them. Also ask for examples of how they knew that a situation required an equation with negative values.

16.3 Animals Changing Altitudes

Optional: 15 minutes

In this optional activity, students use expressions and number line diagrams to represent situations involving the changing height and depth of sea animals. They discuss how there is more than one correct way to write an equation that represents each situation. As students work, identify students who are writing their equations differently.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

Launch

Reading, Representing, Conversing: Collect and Display. While students work, circulate and listen to students talk about the similarities and differences between the changing height and depth of sea animals. Write down common or important phrases you hear students say about there being more than one correct way to write an equation that represents each representation. Write the students' words on a visual display of the expressions and number line diagrams. This will help students read and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Some students may struggle to match the verbal descriptions with the number line diagrams. Prompt them to examine whether each situation is looking for the animal's final altitude or change in altitude. If it is asking for a change in altitude, the number line will have one arrow and one point, instead of two arrows.

Student Task Statement

1. Match each situation with a diagram.
    a. A penguin is standing 3 feet above sea level and then dives down 10 feet. What is its depth?

b. A dolphin is swimming 3 feet below sea level and then jumps up 10 feet. What is its height at the top of the jump?

c. A sea turtle is swimming 3 feet below sea level and then dives down 10 feet. What is its depth?

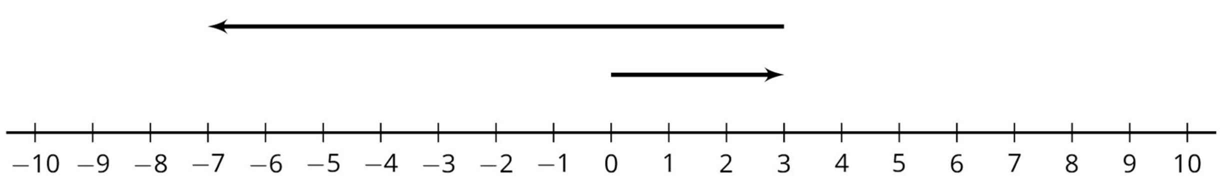
d. An eagle is flying 10 feet above sea level and then dives down to 3 feet above sea level. What was its change in altitude?

e. A pelican is flying 10 feet above sea level and then dives down reaching 3 feet below sea level. What was its change in altitude?

f. A shark is swimming 10 feet below sea level and then swims up reaching 3 feet below sea level. What was its change in depth?

Diagrams

A



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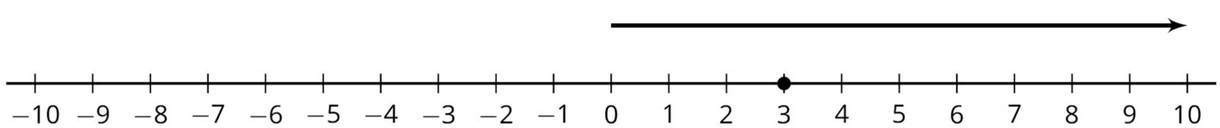

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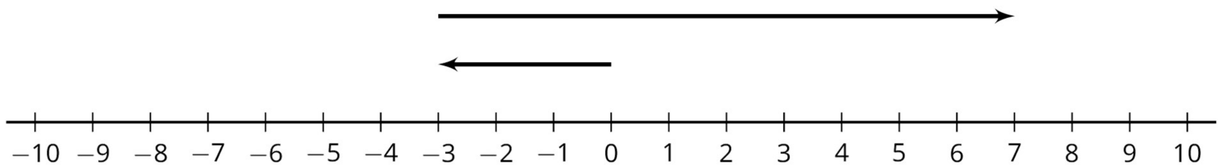
D



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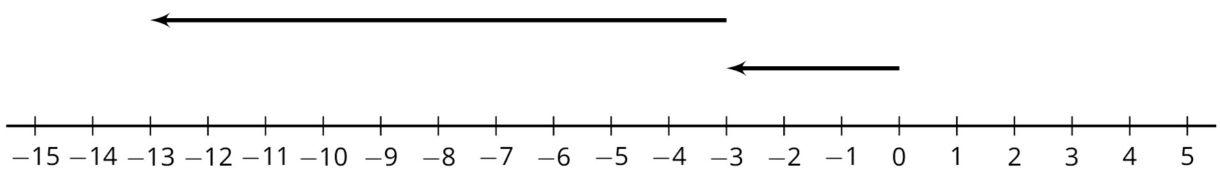
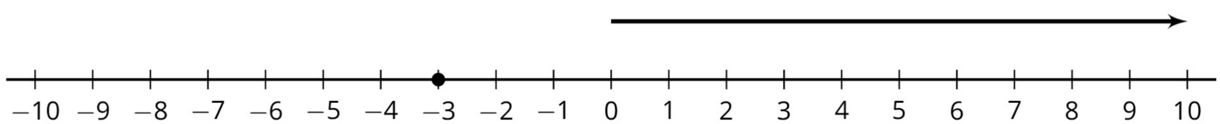


F



2. Next, write an equation to represent each animal's situation and answer the question. Be prepared to explain your reasoning.

Student Response

1.
    a. Diagram A
    b. Diagram D
    c. Diagram E
    d. Diagram B
    e. Diagram F
    f. Diagram C
2. Answers vary. Sample responses:
    a. $3 - 10 = a; a =$ -7

b. $-3 + 10 = b; b = 7$

c. $-3 - 10 = c; c = -13$

d. $10 - d = 3; d = 7$

e. $10 - e = -3; e = 13$

f. $-10 + f = -3; f = 7$

Activity Synthesis

Poll the class on which diagram matched which situation. The majority of the discussion should focus on how the students wrote the equations to represent each situation. Have previously identified students share their equations with the class and have the class discuss whether their equations correctly represent the situation. Try to get at least two different equations for every situation.

16.4 Equations Tell a Story

10 minutes

Unlike the previous activities, this activity gives students a chance to generate an equation by themselves, in preparation for the work in the upcoming lessons.

Instructional Routines

- Group Presentations
- Compare and Connect

Launch

Arrange students in groups of 2–3 and provide tools for making a visual display. Assign one situation to each group. Note that the level of difficulty increases for the situations, so this is an opportunity to differentiate by assigning more or less challenging situations to different groups.

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, create an exemplar display including all required components, highlighting the explanation for how inverses were used in problem solving.
Supports accessibility for: Attention; Social-emotional skills Speaking, Listening, Representing: Compare and Connect. As students prepare the visual display of their given situation, look for students with different strategies for writing and solving their equations. As students analyse each other's work, ask students to share what worked well in a particular approach when representing the situations with equations. Then encourage students to make connections between the use of multiplicative and arithmetic inverses when solving for the unknown quantities. During this discussion, listen for and amplify language students use to make sense of how the operations in the equation represent the

relationships in the situation. This will foster students' meta-awareness and support constructive conversations as they compare strategies for writing and solving variable equations, and make connections between the situations and visual representations of equations and solutions on the visual display.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Student Task Statement

Your teacher will assign your group *one* of these situations. Create a visual display about your situation that includes:

- An equation that represents your situation
- What your variable and each term in the equation represent
- How the operations in the equation represent the relationships in the story
- How you use inverses to solve for the unknown quantity
- The solution to your equation

1. As a $7\frac{1}{4}$ inch candle burns down, its height decreases $\frac{3}{4}$ inch each hour. How many hours does it take for the candle to burn completely?
2. On Monday $\frac{1}{9}$ of the enrolled students in a school were absent. There were 4 512 students present. How many students are enrolled at the school?
3. A hiker begins at sea level and descends 25 feet every minute. How long will it take to get to a height above sea level of -750 feet?
4. Jada practises the violin for the same amount of time every day. On Tuesday she practises for 35 minutes. How much does Jada practise in a week?
5. The temperature has been dropping $2\frac{1}{2}$ degrees every hour and the current temperature is -15°F. How many hours ago was the temperature 0°F?
6. The population of a school increased by 12%, and now the population is 476. What was the population before the increase?
7. During a 5% off sale, Diego pays £74.10 for a new hockey stick. What was the original price?
8. A store buys sweaters for £8 and sells them for £26. How many sweaters does the store need to sell to make a profit of £990?

Student Response

Answers vary. Sample responses:

1. $-\frac{3}{4}h = -7\frac{1}{4}$ where h represents how many hours the candle has been burning, $-\frac{3}{4}h$ represents how much the height of the candle has changed, and $-7\frac{1}{4}$ represents the entire candle burning down. The situation involves multiplication by a negative to show how much of the candle has burned away after each hour. The equation can be solved by multiplying by $-\frac{4}{3}$. The solution is $h = 9\frac{1}{2}$, which means that at this rate it will take $9\frac{1}{2}$ hours for the candle to burn down completely.

2. $(s - \frac{1}{9}s) = 4\,512$ where s represents the total number of students enrolled at the school, $\frac{1}{9}s$ represents the students that were absent, and 4 512 represents the students that were at school on Monday. Students may not know how to solve this because they haven't practiced any equations with more than one variable yet. Students can reason that if $\frac{1}{9}$ of the students are absent, then $\frac{8}{9}$ of the students are present. They can write $\frac{8}{9}s = 4\,512$ and solve $s = \frac{9}{8} \times 4\,512 = 5\,076$. This means that there are 5 076 students enrolled at the school.

3. $-25m = -750$ where m represents how many minutes they have been hiking, $-25m$ represents the height above sea level they have hiked to after m minutes, and -750 represents the goal height above sea level they are trying to get to. The situation involves multiplication by a negative to show how far the hiker has descended after the number of minutes. The equation can be solved by multiplying by $-\frac{1}{25}$. The solution is $m = 30$, which means that at this rate it will take the hiker 30 minutes to reach a height above sea level of -750 feet.

4. $35 \times 7 = p$ (or $\frac{1}{7}p = 35$) where p represents the total amount of time Jada practises in a week, $\frac{1}{7}p$ represents the amount of time Jada practises on one day, and 35 is the number of minutes that she practised on Tuesday. The situation involves multiplication by a fraction to show that the time she practises on Tuesday is $\frac{1}{7}$ of the total time she practises during the week. The equation can be solved by multiplying by 7. The solution is $p = 245$, which means that Jada practises a total of 245 minutes, or 4 hours and 5 minutes, during the week.

5. $-2\frac{1}{2}h = -15$ where h represents the number of hours since the temperature was 0 degrees Fahrenheit, $-2\frac{1}{2}h$ represents how much the temperature has dropped after h hours, and -15 represents the current temperature. The situation involves multiplication by a negative to show how far the temperature has dropped after the number of hours. The equation can be solved by multiplying by $-\frac{2}{5}$ because that is the reciprocal of $-\frac{5}{2}$ which is equivalent to $-2\frac{1}{2}$. The solution is $h = 6$, which means that the temperature was 0 degrees Fahrenheit 6 hours ago.

6. $p + 0.12p = 476$ where p represents the school's population before the increase, $0.12p$ represents 12% of the original population that the school increased by, and 476 represents the new population of the school after the increase. The equation can be solved by reasoning that 112% of the original population is 476. Then $1.12p = 476$ and $p = 476 \div 1.12 = 425$. This means that the school population before the increase was 425 students.

7. $p - 0.05p = 74.10$, where p represents the original price of the hockey stick, $0.05p$ represents the discount of 5% of the original price, and 74.10 represents the price Diego pays for the stick, assuming there is no sales tax. The equation can be solved by reasoning that 95% of the original price is 74.10. Then $0.95p = 74.10$ and $p = 74.10 \div 0.95 = 78$. This means that the original price of the hockey stick was £78.

8. $(26 - 8)s = 990$, where s represents the number of sweaters sold, $(26 - 8)$ represents the profit the store makes on each sweater, and 990 represents the total profit they want to make. The equation can be solved by dividing by $26 - 8 = 18$. The solution is $s = 55$, which means the store has to sell 55 sweaters to make the desired profit.

Are You Ready for More?

Diego and Elena are 2 miles apart and begin walking towards each other. Diego walks at a rate of 3.7 miles per hour and Elena walks 4.3 miles per hour. While they are walking, Elena's dog runs back and forth between the two of them, at a rate of 6 miles per hour. Assuming the dog does not lose any time in turning around, how far has the dog run by the time Diego and Elena reach each other?

Student Response

Diego and Elena are approaching each other at a rate of $3.7 + 4.3$ or 8 miles per hour. We can write the equation $2 = 8t$ to find the time, t, it takes them to cover 2 miles together. Solving the equation, we find $t = \frac{1}{4}$. This means they walk, and the dog runs, for a quarter of an hour. In that time, the dog covers $\left(\frac{1}{4}\right)(6)$, or 1.5 miles.

Activity Synthesis

Invite groups to present their solutions or to view all the solutions on display. The discussion should focus on

- how students decided what their variable would represent
- how to write the equation
- how to solve the equation
- how to interpret the solution in terms of the context.

Students should also address any equations with negative quantities and discuss how they represent the situations.

Lesson Synthesis

- When writing an equation to represent a situation, how do you decide what your variable represents?
- How do you solve the equation?

16.5 Floating Above a Sunken Canoe

Cool Down: 5 minutes

Student Task Statement

A balloon is floating above a lake and a sunken canoe is below the surface of the lake. The balloon's vertical position is 12 metres and the canoe's is -4.8 metres. The equation $12 + d = -4.8$ represents this situation.

1. What does the variable d represent?
2. What value of d makes the equation true? Explain your reasoning.

Student Response

1. The difference in height above sea level. ("Change" in height above sea level should also be accepted.)
2. -16.8. The equation $12 + d = -4.8$ can be rewritten $d = -4.8 - 12$, and $-4.8 - 12 = -16.8$.

Student Lesson Summary

We can use variables and equations involving directed numbers to represent a story or answer questions about a situation.

For example, if the temperature is -3°C and then falls to -17°C, we can let x represent the temperature change and write the equation:

$-3 + x = -17$

We can solve the equation by adding 3 to each side. Since $-17 + 3 = -14$, the change is -14°C.

Here is another example: if a starfish is descending by $\frac{3}{2}$ feet every hour then we can solve $-\frac{3}{2}h = -6$ to find out how many hours h it takes the starfish to go down 6 feet.

We can solve this equation by multiplying each side by $-\frac{2}{3}$. Since $-\frac{2}{3} \times -6 = 4$, we know it will take the starfish 4 hours to descend 6 feet.

Lesson 16 Practice Problems

1. **Problem 1 Statement**

   Match each situation to one of the equations.

   a. A whale was diving at a rate of 2 metres per second. How long will it take for the whale to get from the surface of the ocean to a height above sea level of -12 metres at that rate?

   b. A swimmer dived below the surface of the ocean. After 2 minutes, she was 12 metres below the surface. At what rate was she diving?

   c. The temperature was -12 degrees Celsius and rose to 2 degrees Celsius. What was the change in temperature?

   d. The temperature was 2 degrees Celsius and fell to -12 degrees Celsius. What was the change in temperature?

   1. $-12 + x = 2$
   2. $2 + x = -12$
   3. $-2x = -12$
   4. $2x = -12$

   Solution

   – A: 3

   – B: 4

   – C: 1

   – D: 2

2. **Problem 2 Statement**

   Starting at noon, the temperature dropped steadily at a rate of 0.8 degrees Celsius every hour.

   For each of these situations, write and solve an equation and describe what your variable represents.

   a. How many hours did it take for the temperature to decrease by 4.4 degrees Celsius?

   b. If the temperature after the 4.4 degree drop was -2.5 degrees Celsius, what was the temperature at noon?

Solution

a. $-0.8h = -4.4$, where h is the number of hours it took for the temperature to decrease. Solve the equation by multiplying each side by $\frac{-1}{0.8}$. It took $5\frac{1}{2}$ hours for the temperature to drop 4.4 degrees Celsius.

b. $T - 4.4 = -2.5$, where T is the temperature at noon. The temperature at noon was 1.9 degrees Celsius.

3. **Problem 3 Statement**

Kiran mixes $\frac{3}{4}$ cups of raisins, 1 cup peanuts, and $\frac{1}{2}$ cups of chocolate chips to make trail mix. How much of each ingredient would he need to make 10 cups of trail mix? Explain your reasoning.

Solution

The ingredients listed will make $2\frac{1}{4}$ cups of trail mix. So to get 10 cups of trail mix, multiply each amount by $10 \div 2\frac{1}{4} = \frac{40}{9}$. Kiran will need $\frac{30}{9}$ cups of raisins, $\frac{40}{9}$ cups of peanuts, and $\frac{20}{9}$ cups of chocolate chips.

4. **Problem 4 Statement**

Find the value of each expression.

a. $12 + -10$

b. $-5 - 6$

c. $-42 + 17$

d. $35 - -8$

e. $-4\frac{1}{2} + 3$

Solution

a. 2

b. -11

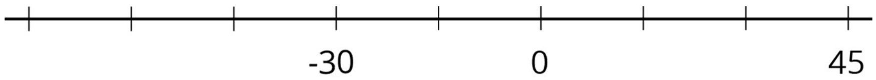
c. -25

d. 43

e. $-1\frac{1}{2}$

5. **Problem 5 Statement**

The markings on the number line are evenly spaced. Label the other markings on the number line.



Solution

-75, -60, -45, -30, -15, 0, 15, 30, 45

6. **Problem 6 Statement**

Kiran drinks 6.4 oz of milk each morning. How many days does it take him to finish a 32 oz container of milk?

a. Write and solve an equation for the situation.

b. What does the variable represent?

Solution

a. $6.4n = 32$. The solution is $n = 5$.

b. The variable n represents the number of days it takes Kiran to finish the container of milk.