Lesson 9: Creating scale drawings

Goals

- Compare and contrast (orally) different scale drawings of the same object, and describe (orally) how the scale affects the size of the drawing.
- Create a scale drawing, given the actual dimensions of the object and the scale.
- Determine the scale used to create a scale drawing and generate multiple ways to express it (in writing).

Learning Targets

- I can determine the scale of a scale drawing when I know lengths on the drawing and corresponding actual lengths.
- I know how different scales affect the lengths in the scale drawing.
- When I know the actual measurements, I can create a scale drawing at a given scale.

Lesson Narrative

In previous lessons, students have used scale drawings to calculate actual distances. This is the first lesson where students use the actual distance to calculate the scaled distance and create their own scale drawings. They see how different scale drawings can be created of the same actual thing, using different scales. They also see how the choice of scale influences the drawing. For example, a scale drawing with a scale of 1 cm to 5 m will be smaller than a scale drawing of the same object with a scale of 1 cm to 2 m (since each cm represents a larger distance, it takes fewer cm to represent the object). This prepares them for future lessons where they will recreate a given scale drawing at a different scale.

Noticing *how* scaled drawings change with the choice of scale develops important structural understanding of scale drawings.

Building On

- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- Read, write, and compare decimals to thousandths.

Addressing

- Solve problems involving scale drawings of geometric shapes, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- Clarify, Critique, Correct

- Compare and Connect
- Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Required Preparation

Ensure students have access to geometry toolkits.

Student Learning Goals

Let's create our own scale drawings.

9.1 Number Talk: Which is Greater?

Warm Up: 5 minutes

In this number talk, students compare quantities involving division with whole numbers, decimals, and fractions. In each case, a strategy is available that does not require calculating the quantities. When the quantities are complex, there is motivation for using the structure of the expressions to compare since the actual calculations would be more time-consuming.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Students may misinterpret the last question as $15 \times \frac{1}{3}$ or $15 \times \frac{1}{4}$. Point out that one way to interpret the first expression is “How many one-thirds are there in 15?”

Student Task Statement

Without calculating, decide which quotient is larger.

$11 \div 23$ or $7 \div 13$

$0.63 \div 2$ or $0.55 \div 3$

$15 \div \frac{1}{3}$ or $15 \div \frac{1}{4}$

Student Response

- $7 \div 13$ is greater, because it is greater than $\frac{1}{2}$ while $\frac{11}{23}$ is less than $\frac{1}{2}$.
- $0.63 \div 2$ is greater than $0.55 \div 3$ since $0.63 > 0.55$ and 0.63 is being divided by 2, whereas 0.55 is being divided into more equal parts (3).
- $15 \div \frac{1}{4}$ is greater than $15 \div \frac{1}{3}$ since $\frac{1}{4}$ is less than $\frac{1}{3}$, and dividing by a smaller (unit) fraction gives a larger quotient.

Activity Synthesis

Make sure to bring out different approaches for comparing the quantities, avoiding direct calculation where possible:

- $\frac{7}{13}$ and $\frac{11}{23}$ can both be compared with $\frac{1}{2}$, or students can find a common numerator or denominator, but this requires more calculations.
- 0.63 is greater than 0.55, and 2 is less than 3, or students might notice that $0.63 \div 2$ is greater than 0.3 while $0.55 \div 3$ is less than 0.2.
- Dividing by $\frac{1}{4}$ is equivalent to multiplying by 4 while dividing by $\frac{1}{3}$ is equivalent to multiplying by 3, so $15 \div \frac{1}{4}$ is greater than $15 \div \frac{1}{3}$.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

9.2 Bedroom Floor Plan

10 minutes (there is a digital version of this activity)

In previous lessons in this unit, students have investigated the meaning of scale drawings and have used them to solve problems. The purpose of this activity is to prepare students for creating their own scale drawing. The discussion highlights that a scale can be expressed in different ways, or that different pairs of numbers may be used to show the same relationship. For example, a scale of 4 cm to 1 m is equivalent to a scale of 1 cm to 0.25 m.

As students work, notice those who use the language of scale appropriately, e.g., by saying that “every 4 cm on the drawing represents 1 m,” or “every 0.25 m shows up as 1 cm on the drawing.” Also notice those who do and do not attend to the relationship between actual and scaled lengths in finding missing measurements.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Tell students that a floor plan is a top-view drawing that shows a layout of a room or a building. Floor plans are usually scale drawings. Explain that sometimes the scale of a drawing is not specified, but we can still tell the scale if we know both the scaled and actual lengths.

Arrange students in groups of 2. Give students 4–5 minutes of quiet work time and partner discussion.

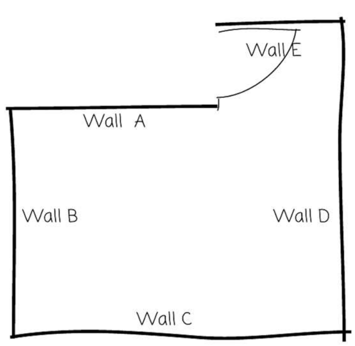
For students using the Digital Activity, teachers can allow exploration with the applet during think time.

Anticipated Misconceptions

Students may see that one value is 4 times the other and write the scale backwards, as “1 cm to 4 m.” Prompt students to pay attention to the units and the meaning of each number.

Student Task Statement

Here is a rough sketch of Noah’s bedroom (not a scale drawing).



Noah wants to create a floor plan that is a scale drawing.

1. The actual length of wall C is 4 m. To represent wall C, Noah draws a segment 16 cm long. What scale is he using? Explain or show your reasoning.

2. Find another way to express the scale.

3. Discuss your thinking with your partner. How do your scales compare?

4. The actual lengths of wall A and wall D are 2.5 m and 3.75 m. Determine how long these walls will be on Noah's scale floor plan. Explain or show your reasoning.

Student Response

1. 4 cm to 1 m; 1 cm to 0.25 m; or 16 cm to 4 m. Sample explanation: Since 16 cm represents an actual length of 4 m, then 1 cm must represent $\frac{1}{16}$ of 4 m, which is 0.25 m.

2. Answers vary depending on response to the first question.

3. Answers vary.

4. Wall A: 10 cm. Wall D: 15 cm. Sample explanations:

   – Since every 1 m is shown as 4 cm on the drawing, I multiplied the actual lengths, in metres, by 4 to find how many centimetres long the scale drawing should be.

   – Since every 1 cm represents 0.25 m, I divided the actual lengths, in metres, by 0.25 to find how many centimetres long the scale drawing should be.

Are You Ready for More?

If Noah wanted to draw another floor plan on which wall C was 20 cm, would 1 cm to 5 m be the right scale to use? Explain your reasoning.

Student Response

No. Sample explanations:

- If he used 1 cm to 5 m, wall C, which is 4 m long, will be less than 1 cm on the drawing—much smaller than what he wanted.

- If he used 1 cm to 5 m, a 20-cm segment would represent a 100-m long wall, which is not the length of wall C.

Activity Synthesis

Focus the class discussion on two things:

1. Different ways to express the same scale

2. The relationship between scaled and actual lengths

Invite a couple of students to share how they determined the scale of the drawing. Students are likely to come up with several variations, e.g., 4 cm to 1 m, 1 cm to 0.25 m, 16 cm to 4 m, etc. Discuss how all of these express the same relationship and are therefore equivalent, especially how 4 cm to 1 m is equivalent to 1 cm to 0.25 m (or 1 cm to $\frac{1}{4}$ m).

Explain that although we can express a scale in multiple but equivalent ways, 1) scales are often simplified to show the actual distance for 1 scaled unit, and 2) it is common to express at least one distance (usually the scaled distance) as a whole number or a benchmark fraction (e.g., $\frac{1}{4}$, $\frac{1}{2}$) or a benchmark decimal (e.g., 0.25, 0.5, 0.75).

Given their work on scaled copies, students may be inclined to say that the scaled and actual lengths are related by a scale factor of 4. Ask: "Are the actual lengths four times the lengths on the drawing? Why or why not?" Point out that because the units for the two quantities are different, multiplying a scaled length in centimetres (e.g., 2.5 cm) by 4 will yield another length in centimetres (10 cm), which is not the actual length. It is not essential for students to know that the scale factor here is 250. That work will be explored in an upcoming lesson.

Speaking, Listening: Compare and Connect. As students prepare a visual display of how they created the floor plan, look for students who expressed the scale in different ways. As students investigate each other's work, ask them to share what is especially clear about a particular approach. Then encourage students to explain why there are various yet equivalent ways to express the scale, such as 4 cm to 1 m and 1 cm to 0.25 m. Emphasise the language used to make sense of the different ways to express the same scale (e.g., Since 4 cm on the floor plan represents 1 m in the actual room, then 1 cm on the floor plan represents $\frac{1}{4}$ of 1 m, which is 0.25 m.) This will reinforce students' use of mathematical language related to equivalent scales.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

9.3 Two Maps of Utah

15 minutes

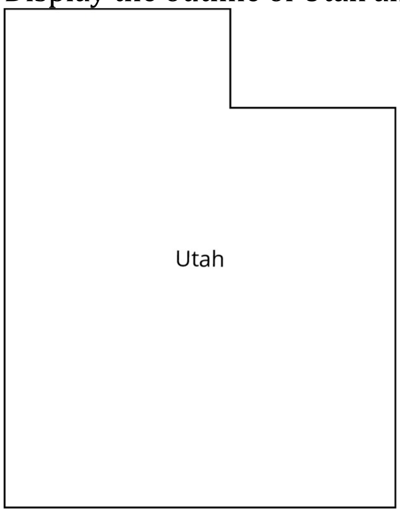
In the previous activity, students calculated the scaled distances they would need to create a scale drawing, but did not actually create the scale drawing (unless using the interactive resources). In this activity, they create two different scale drawings of the state of Utah and notice how the scale impacts the drawing. One of the reasons choice of a scale is important is that we want to see the appropriate level of detail within a fixed space.

Instructional Routines

- Clarify, Critique, Correct
- Notice and Wonder

Launch

Display the outline of Utah and ask students “What do you notice? What do you wonder?



Ask students to describe the shape: a rectangle with a smaller rectangle removed in the upper right corner.

Give students 5–6 minutes of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may get a shape that is not closed or does not have right angles if they did not measure carefully enough. Prompt them to double-check their measurement for a particular side of the state if you can easily tell which side is drawn incorrectly.

Students may think that a scale of 1 centimetre to 50 miles will produce a smaller scale drawing than a scale of 1 centimetre to 75 miles (because 50 is less than 75). Ask them how many centimetres it takes to represent 75 miles if 1 centimetre represents 50 miles (1.5) and how many centimetres it takes to represent 75 miles if 1 centimetre represents 75 miles (1).

Student Task Statement

A rectangle around Utah is about 270 miles wide and about 350 miles tall. The upper right corner that is missing is about 110 miles wide and about 70 miles tall.

1. Make a scale drawing of Utah where 1 centimetre represents 50 miles.

   Make a scale drawing of Utah where 1 centimetre represents 75 miles.

2. How do the two drawings compare? How does the choice of scale influence the drawing?

Student Response

1. A rectangle approximately 5.4 centimetres wide and 7 centimetres tall, missing an upper right corner which is approximately 2.2 centimetres wide and 1.4 centimetres

tall and a rectangle approximately 3.6 centimetres wide and 4.7 centimetres tall, missing an upper right corner which is approximately 1.5 centimetres wide and 1 centimetre tall.

2. The measurements in the 1 cm to 50 mile scale drawings are larger than the measurements in the 1 cm to 75 mile scale drawing. This makes sense because when 1 centimetre represents 50 miles, it takes 1.5 centimetres to represent 75 miles.

Activity Synthesis

Ask students what the two scale drawings share in common. Answers include: they both represent Utah, they both have the same shape, and they both can be used to measure distances in the actual state of Utah.

Ask students how the two scale drawings differ. The one at a scale of 1 centimetre to 50 miles is larger than the one at a scale of 1 centimetre to 75 miles.

Some students may notice that the scale drawing at a scale of 1 centimetre to 75 miles is actually a scaled copy of the other drawing, with a scale factor of 1.5. If so, ask them to share their observation linking scale drawings with scale copies.

Representation: Internalise Comprehension. Use colour coding and annotations to highlight connections between representations in a problem. For example, annotate a display of the two scale drawings to make visible what the drawings share in common, and how they differ.
Supports accessibility for: Visual-spatial processing Reading, Speaking: Clarify, Critique, Correct. Before presenting the correct scale drawings of Utah, present an incorrect drawing and written explanation. For example, present a rectangle approximately 5.4 cm wide and 7 cm tall that is missing an upper right corner which is approximately 1.4 cm wide and 2.2 cm tall, and provide the statement: "Since 1 cm represents 50 miles, I divided 110 and 70 each by 50, and got 2.2 and 1.4. The small rectangle that is missing is 2.2 cm tall and 1.4 cm wide." Ask students to identify the error, critique the reasoning, and revise the statement so that the drawing is a scale drawing of Utah. This will remind students about the characteristics of scale drawings discussed in previous lessons and how to determine whether a drawing qualifies as a scale drawing of an actual object.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Lesson Synthesis

The size of the scale determines the size of the drawing. You can have different-sized scale drawings of the same actual object, but the size of the actual object doesn't change.

- "Suppose there are two scale drawings of the same house. One uses the scale of 1 cm to 2 m, and the other uses the scale 1 cm to 4 m. Which drawing is larger? Why?" (The one with the 1 cm to 2 m scale is larger, because it takes 2 cm on the drawing to represent 4 m of actual length.)

- "Another scale drawing of the house uses the scale of 5 cm to 10 m. How does its size compare to the other two?" (It is the same size as the drawing with the 1 cm to 2 m scale.)

Sometimes two different scales are actually equivalent, such as 5 cm to 10 m and 1 cm to 2 m. It is common to write a scale so that it tells you what one unit on the scale drawing represents (for example, 1 cm to 2 m).

9.4 Drawing a Pool

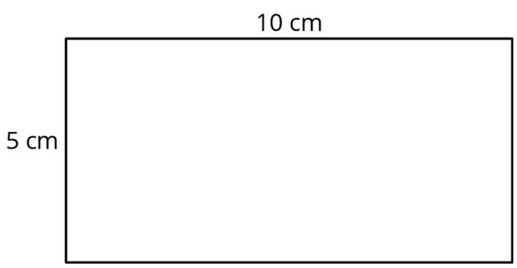
Cool Down: 5 minutes

Student Task Statement

A rectangular swimming pool measures 50 metres in length and 25 metres in width.

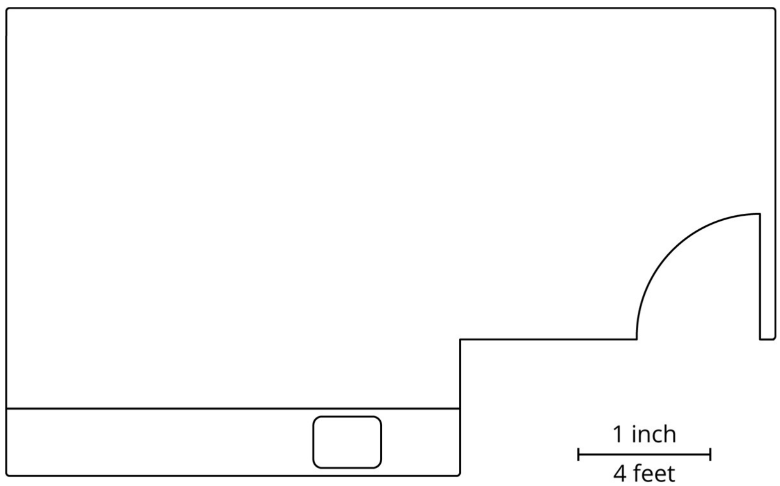
1. Make a scale drawing of the swimming pool where 1 centimetre represents 5 metres.
2. What are the length and width of your scale drawing?

Student Response



Student Lesson Summary

If we want to create a scale drawing of a room's floor plan that has the scale "1 inch to 4 feet," we can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for our drawing.



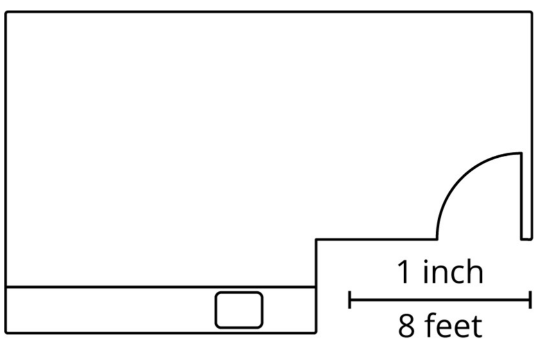
Suppose the longest wall is 15 feet long. We should draw a line 3.75 inches long to represent this wall, because $15 \div 4 = 3.75$.

There is more than one way to express this scale. These three scales are all equivalent, since they represent the same relationship between lengths on a drawing and actual lengths:

- 1 inch to 4 feet
- $\frac{1}{2}$ inch to 2 feet
- $\frac{1}{4}$ inch to 1 foot

Any of these scales can be used to find actual lengths and scaled lengths (lengths on a drawing). For instance, we can tell that, at this scale, an 8-foot long wall should be 2 inches long on the drawing because $\frac{1}{4} \times 8 = 2$.

The size of a scale drawing is influenced by the choice of scale. For example, here is another scale drawing of the same room using the scale 1 inch to 8 feet.



Notice this drawing is smaller than the previous one. Since one inch on this drawing represents twice as much actual distance, each side length only needs to be half as long as it was in the first scale drawing.

Lesson 9 Practice Problems

Problem 1 Statement

An image of a book shown on a website is 1.5 inches wide and 3 inches tall on a computer monitor. The actual book is 9 inches wide.

a. What scale is being used for the image?

b. How tall is the actual book?

Solution

a. 1 inch to 6 inches

b. 18 inches

Problem 2 Statement

The flag of Colombia is a rectangle that is 6 ft long with three horizontal strips.

- The top stripe is 2 ft tall and is yellow.
- The middle stripe is 1 ft tall and is blue.
- The bottom stripe is also 1 ft tall and is red.

a. Create a scale drawing of the Colombian flag with a scale of 1 cm to 2 ft.

b. Create a scale drawing of the Colombian flag with a scale of 2 cm to 1 ft.

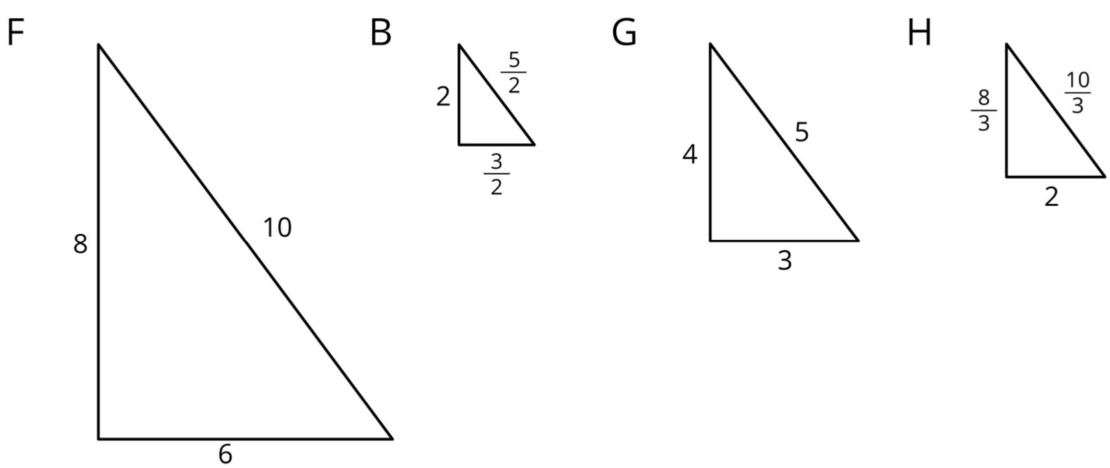
Solution

a. The flag will be 3 cm long and 2 cm tall. The yellow rectangle is 1 cm tall and the red and blue rectangles are each 0.5 cm tall.

b. The flag will be 12 cm long and 8 cm tall. The yellow rectangle is 4 cm tall and the red and blue rectangles are each 2 cm tall.

Problem 3 Statement

These triangles are scaled copies of each other.



For each pair of triangles listed, the area of the second triangle is how many times larger than the area of the first?

a. triangle G and triangle F

b. triangle G and triangle B

c. triangle B and triangle F

d. triangle F and triangle H

e. triangle G and triangle H

f. triangle H and triangle B

Solution

a. 4

b. $\frac{1}{4}$

c. 16

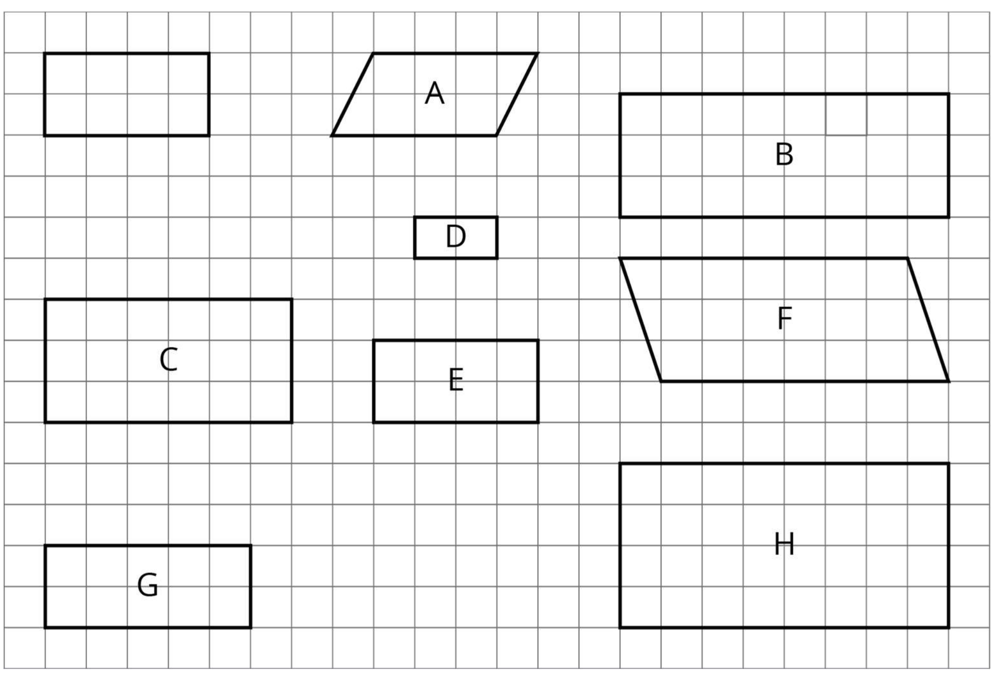
d. $\frac{1}{9}$

e. $\frac{4}{9}$

f. $\frac{9}{16}$

Problem 4 Statement

Here is an unlabelled rectangle, followed by other quadrilaterals that are labelled.



a. Select **all** quadrilaterals that are scaled copies of the unlabelled rectangle. Explain how you know.

b. On graph paper, draw a different scaled version of the original rectangle.

Solution

a. C, D, E, and H. Sample explanation: The length and width of each copy is related to the length and width of the original by the same factor and the corresponding angles are unchanged.

b. Drawings vary. Sample response: