

Lesson 5: More enlargements

Goals

- Describe (orally) a shape on a coordinate grid and its image under an enlargement, using coordinates to refer to points.
- Describe (orally) several enlargements of one shape with the same centre but different scale factors.
- Identify what information is needed to enlarge a polygon on a coordinate grid. Ask questions to elicit that information.

Learning Targets

- I can apply enlargements to polygons on a rectangular grid if I know the coordinates of the vertices and of the centre of enlargement.

Lesson Narrative

In previous lessons, students learned what an enlargement is and practised enlarging points and shapes on a circular grid, on a square grid, on a coordinate grid, and with no grid. In this lesson, they work on a coordinate grid and use the coordinates to communicate precisely the information needed to perform an enlargement. Students use the info gap structure. The student with the problem card needs to enlarge a polygon on the coordinate grid. In order to do so, they need to request the coordinates of the polygon's vertices and the centre of enlargement as well as the scale factor. After obtaining all of this information from the partner with the data card, the student performs the enlargement. The focus here is on deciding what information is needed and communicating clearly to request the information and explain why it is needed.

One important use of coordinates in geometry is to facilitate precise and concise communication about the location of points. This allows students to indicate where the centre of the enlargement is and also to communicate the vertices of the polygon that is enlarged.

Addressing

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Describe the effect of enlargements, translations, rotations, and reflections on two-dimensional shapes using coordinates.

Instructional Routines

- Information Gap Cards
 - Notice and Wonder
-

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Pre-printed slips, cut from copies of the blackline master

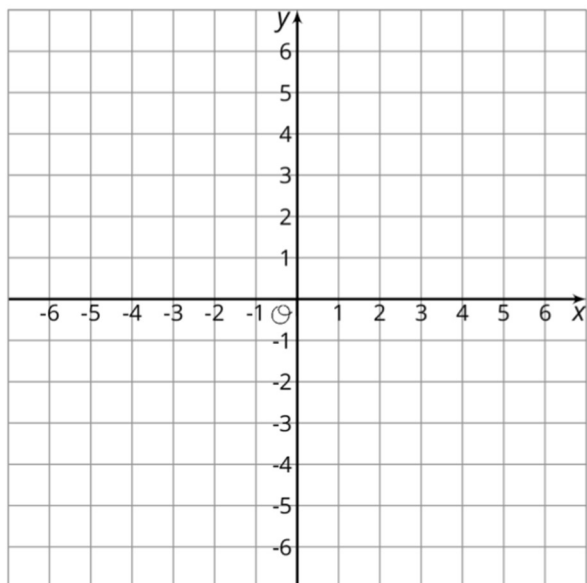
Info Gap: Enlargements

Info Gap: Enlargements

Problem card 1

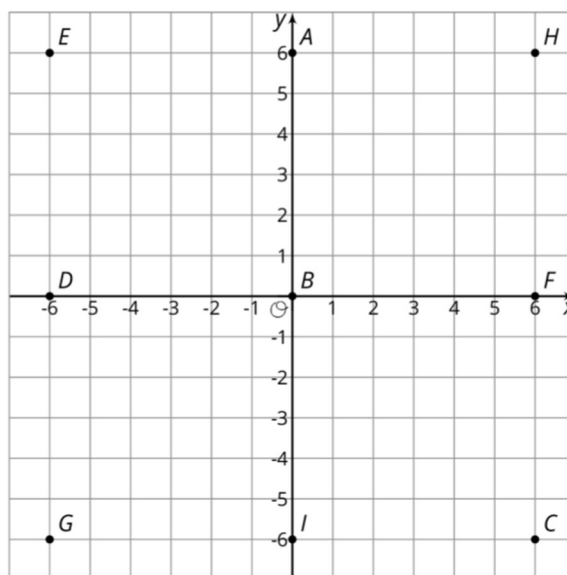
Polygon *AFID* is enlarged.

Draw the image of *AFID* under this enlargement.



Info Gap: Enlargements

Data card 1

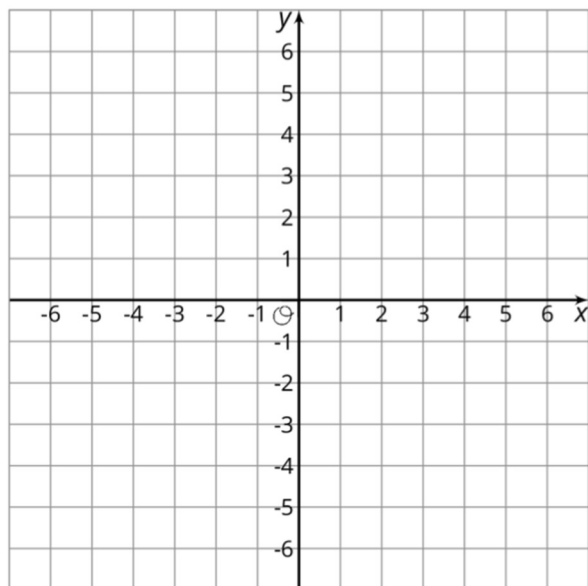


Centre of enlargement: (0,0)

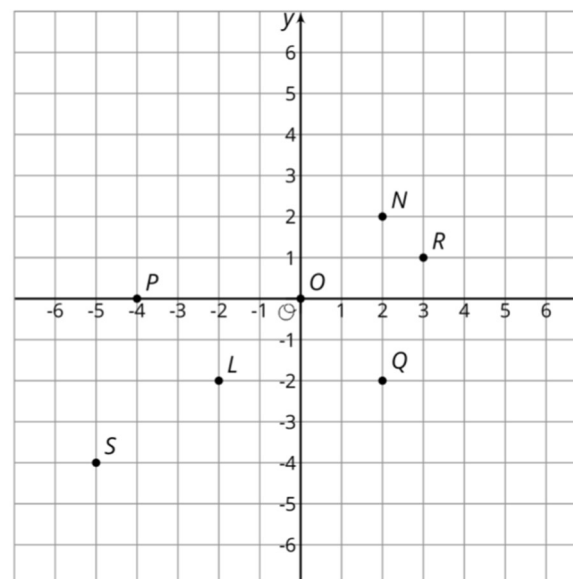
Scale factor: $\frac{1}{3}$

Info Gap: Enlargements
Problem card 2

Polygon $ONPQ$ is enlarged.
Draw the image of $ONPQ$ under this enlargement.



Info Gap: Enlargements
Data card 2



Centre of enlargement: $(-4, 0)$

Scale factor: $\frac{3}{2}$

Required Preparation

Print and cut out copies of the blackline master for the Info Gap activity (one set of two problem cards and two data cards per pair of students).

Student Learning Goals

Let's look at enlargements in the coordinate grid.

5.1 Many Enlargements of a Triangle

Warm Up: 5 minutes (there is a digital version of this activity)

Students have seen many examples of scale factors that are less than and greater than 1, but the goal of this warm up is to have students focus explicitly on how the size of the scale factor impacts the enlargement of a shape. Several enlargements of one shape, with the same centre but different scale factors, are displayed and students are asked to make observations about the shapes.

If using the digital activity, students will be able to scale a triangle by sliding the scale factor button. Students will informally make conjectures about the relationship between the original and scaled triangles. For example, the triangles are similar, the angles do not change, etc.

Instructional Routines

- Notice and Wonder

Launch

Show students the image from the task (or demonstrate the applet from the digital activity) and ask “What do you notice? What do you wonder?” Record their responses for all to see. Students may notice and wonder many things. Relevant mathematical things students may notice:

- There are triangles that look like enlargements of each other.
- The centre of enlargement is point P .
- The corresponding sides of the triangles are parallel to each other.
- There is a dashed line through each set of corresponding vertices.
- The corresponding angles appear to be congruent.

Things students may wonder:

- Which was the original triangle, and which are enlargements of it?
- What scale factors were used?

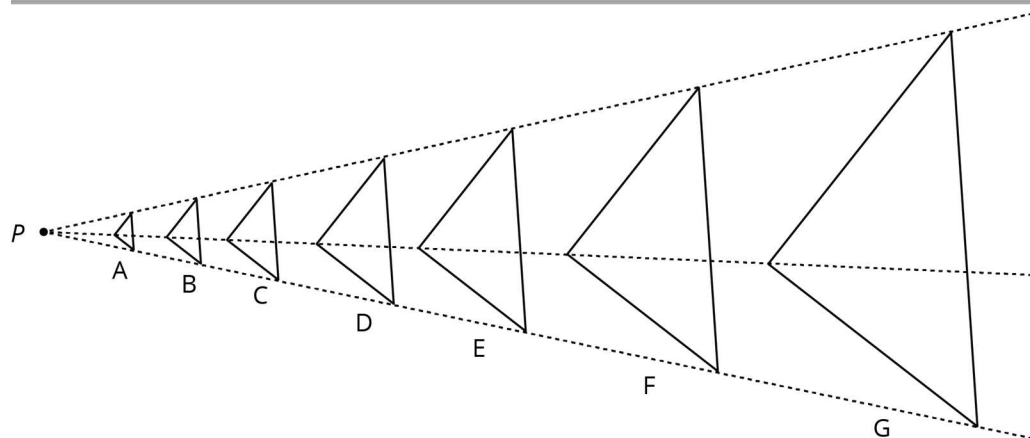
Give students 2 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

If students try to determine the actual scale factors between enlargements, let them know that they only need to make an overall observation about the sizes of the enlargements compared to triangle D.

Student Task Statement

All of the triangles are enlargements of triangle D. The enlargements use the same centre P , but different scale factors. What do triangles A, B, and C have in common? What do triangles E, F, and G have in common? What does this tell us about the different scale factors used?



Student Response

Triangles A, B, and C are smaller and closer to P than triangle D. Triangles E, F, and G are larger and farther away from P than triangle D. The scale factor used to enlarge triangle D is less than 1 for triangles A, B, and C. For E, F, and G, the scale factor is greater than 1.

Activity Synthesis

Important observations include

- The smaller triangles come from scale factors less than one while the bigger triangles correspond to scale factors larger than one
- The original triangle, itself, can be seen as coming from a scale factor of one
- All of the triangles have the same angles and orientation in the plane

A few other points worth discussing include

- Any of these triangles is an enlargement of any other with centre P .
- The lines from P contain “corresponding vertices” of the triangles (in the sense that these points map to one another via the enlargements taking one triangle to another).

5.2 Info Gap: Enlargements

20 minutes

This info gap activity gives students an opportunity to determine and request the information needed for an enlargement, and to realise that using coordinates greatly simplifies talking about specific points. In order to perform an enlargement, students will need to know the centre of enlargement (which can be communicated using the coordinate grid), the coordinates of the polygon that they are enlarging (also communicated using the coordinate grid), and the scale factor. With this information, they can find the enlargement as in previous activities.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for the information they need to solve the problem. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Listen for how students request (and supply) information about the centre of enlargement and the location of the polygon that is being enlarged. The coordinate grid helps name and communicate the location of points, which is essential in this activity. In addition to the location of points, listen for how students use “centre of enlargement” and “scale factor” in order to communicate this essential information.

Instructional Routines

- Information Gap Cards

Launch

Tell students they will continue to practise describing and drawing enlargements using coordinates. Explain the Info Gap and consider demonstrating the protocol if students are unfamiliar with it. Arrange students in groups of 2. Provide access to geometry toolkits. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for the second problem and instruct them to switch roles.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organisation *Conversing:* This activity uses *Information Gap* to give students a purpose for discussing information necessary to enlarge a polygon on a coordinate grid. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

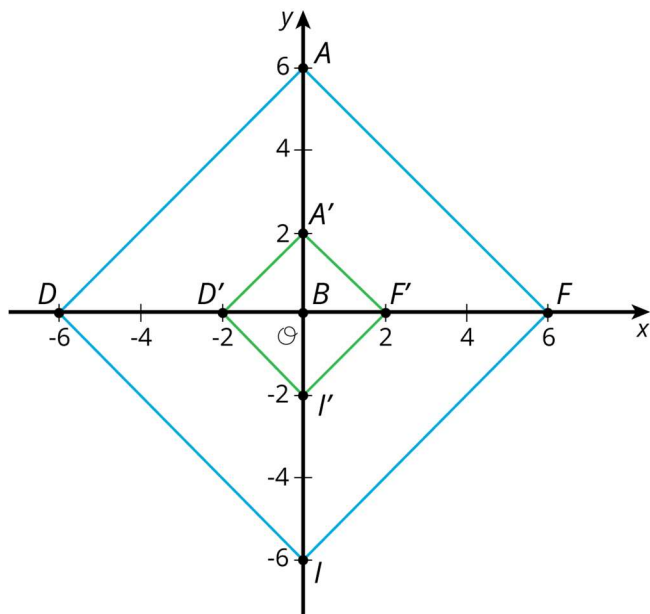
If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

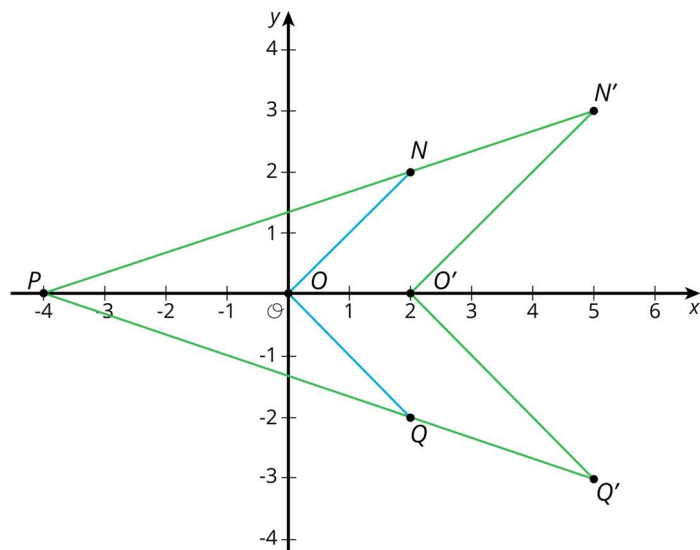
Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Problem 1. $A' = (0,2)$, $F' = (2,0)$, $I' = (0,-2)$, $D' = (-2,0)$. Image to help visualise:

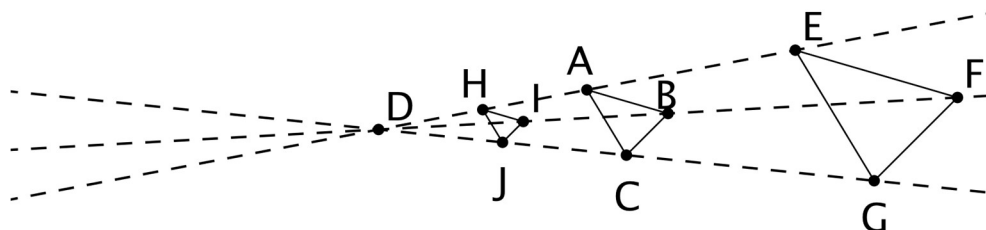


Problem 2. $O' = (2,0)$, $N' = (5,3)$, $P' = (-4,0)$, $Q' = (5,-3)$. Image to help visualise:



Are You Ready for More?

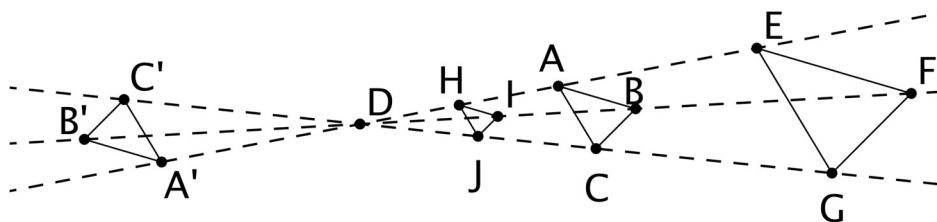
Triangle EFG was created by enlarging triangle ABC using a scale factor of 2 and centre D .
Triangle HIJ was created by enlarging triangle ABC using a scale factor of $\frac{1}{2}$ and centre D .



1. What would the image of triangle ABC look like under an enlargement with scale factor 0?
2. What would the image of the triangle look like under enlargement with a scale factor of -1? If possible, draw it and label the vertices A' , B' , and C' . If it's not possible, explain why not.
3. If possible, describe what happens to a shape if it is enlarged with a negative scale factor. If enlarging with a negative scale factor is not possible, explain why not.

Student Response

1. If the distance from D to any vertex were multiplied by 0, the product would be 0. So, all three vertices of ABC would move to point D .



2.

3. The enlarged point goes to the other side of the point of enlargement.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Some guiding questions:

- “Other than the answer, what information would have been nice to have?”
- “How did using coordinates help in talking about the problem?”
- “If this same problem had a shape on a grid without coordinates, how would you talk about the points?”
- “What if there had been no grid at all? Would you still have been able to request or provide the needed information to perform the transformation?”

Highlight that coordinates allow us to unambiguously provide the location of a polygon’s vertices. In addition, reinforce the idea that in order to perform an enlargement, we need to know the scale factor and the centre of enlargement. The coordinate grid again provides an efficient means to communicate the centre of enlargement.

Lesson Synthesis

Ask students to think about the question: “Why does anyone bother putting coordinate axes on a grid? Why are coordinates useful? What are they good for?”

Here are the points to emphasise: Coordinates are an exceptionally powerful tool for communicating the location of points in the plane. There is only one point 3 units to the left of the origin and 2 units up from the origin, the point (3,2). The location of a polygon is determined by the location and order of its vertices. On a coordinate grid, these can be communicated by giving their coordinates. When we perform an enlargement, we also need to know the centre of enlargement (another point) and the scale factor (a number). On the coordinate grid, all of the information we need to enlarge a polygon can be communicated unambiguously with some numbers!

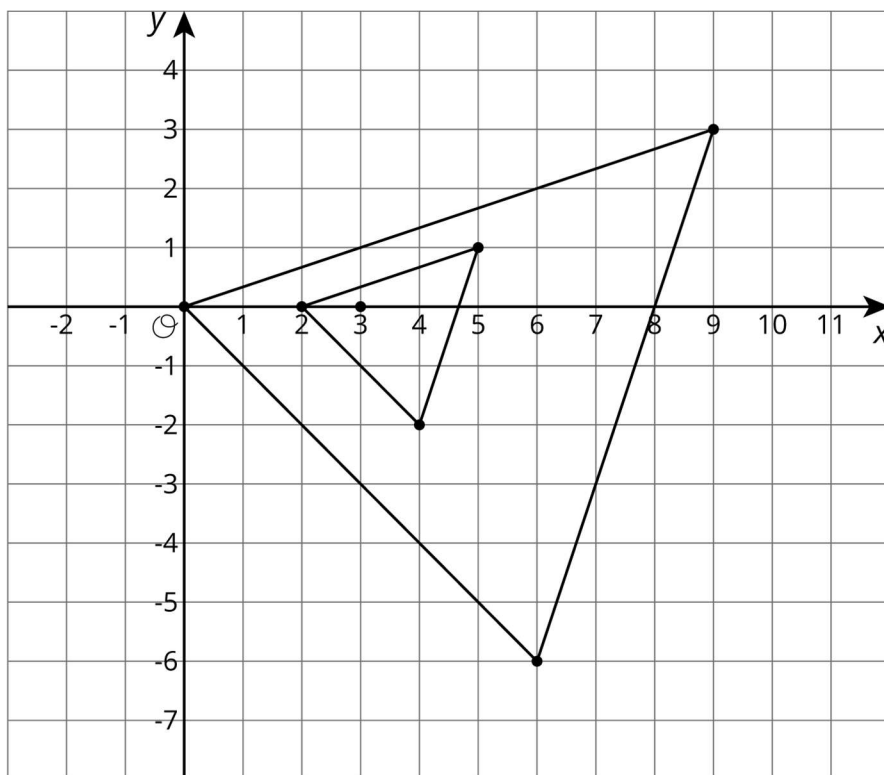
Coordinates in the plane are much like an address in the city: they tell you where to go unambiguously. Because the plane is laid out in a grid, these “addresses” are particularly simple, consisting of two directed numbers. We will use coordinates much more in this unit and beyond, not only to describe individual points but also to describe relationships (like proportional relationships and other new types of relationships).

5.3 Identifying an enlargement

Cool Down: 5 minutes

Student Task Statement

The smaller triangle is enlarged to create the larger triangle. The centre of enlargement is plotted, but not labelled.



Describe this enlargement. Be sure to include all of the information someone would need to perform the enlargement.

Student Response

Answers vary. Information that must be included:

- The centre of enlargement is $(3,0)$.
- The scale factor is 3.
- The triangle being enlarged has vertices at $(2,0)$, $(4,-2)$, and $(5,1)$.

Student Lesson Summary

One important use of coordinates is to communicate geometric information precisely. Let's consider a quadrilateral $ABCD$ in the coordinate grid. Performing an enlargement of $ABCD$ requires three vital pieces of information:

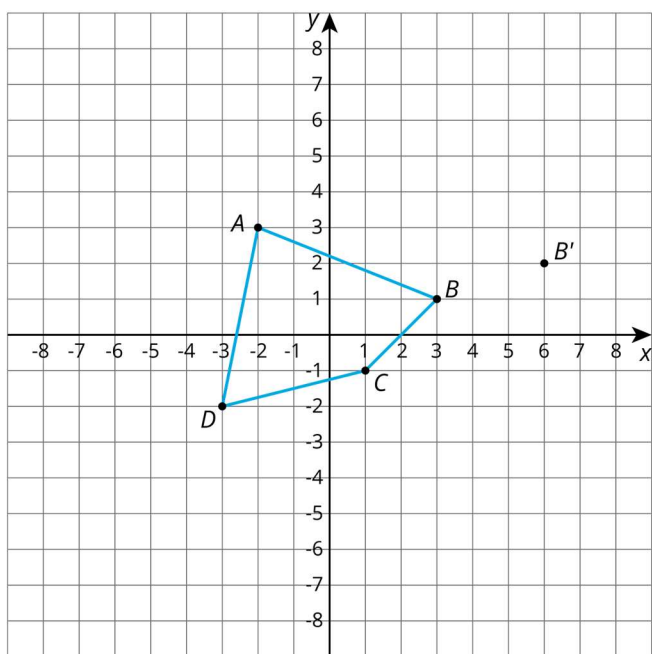
1. The coordinates of A , B , C , and D
2. The coordinates of the centre of enlargement, P
3. The scale factor of the enlargement

With this information, we can enlarge the vertices A , B , C , and D and then draw the corresponding line segments to find the enlargement of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the centre of enlargement.

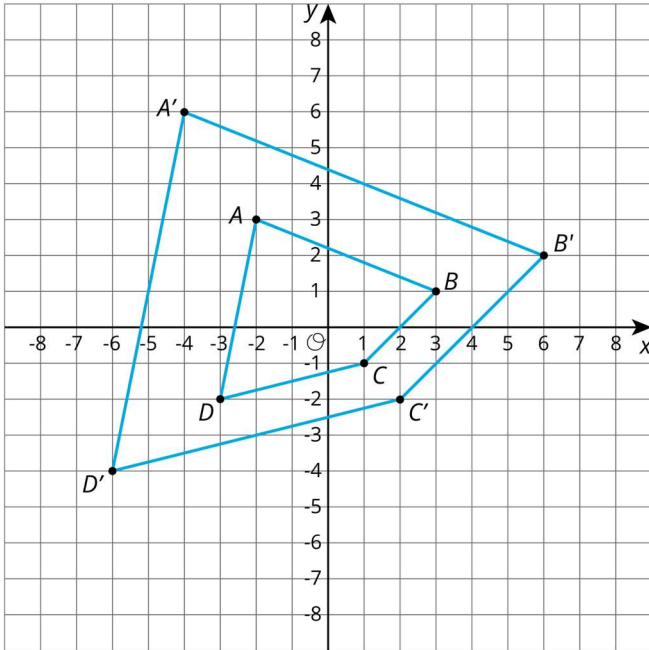
Lesson 5 Practice Problems

1. Problem 1 Statement

Quadrilateral $ABCD$ is enlarged with centre $(0,0)$, taking B to B' . Draw $A'B'C'D'$.

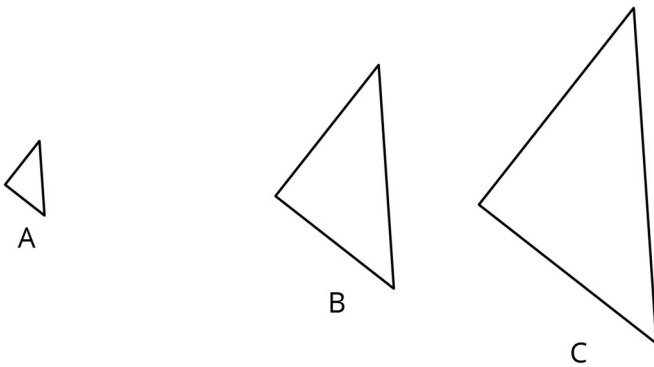


Solution



2. Problem 2 Statement

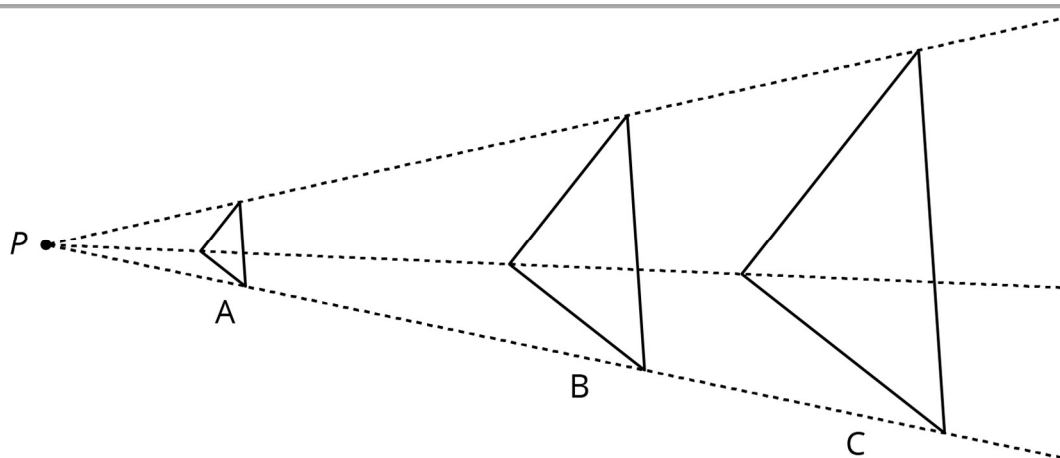
Triangles B and C have been built by enlarging triangle A .



- Find the centre of enlargement.
- Triangle B is an enlargement of A with approximately what scale factor?
- Triangle A is an enlargement of B with approximately what scale factor?
- Triangle B is an enlargement of C with approximately what scale factor?

Solution

- The centre of enlargement is here:

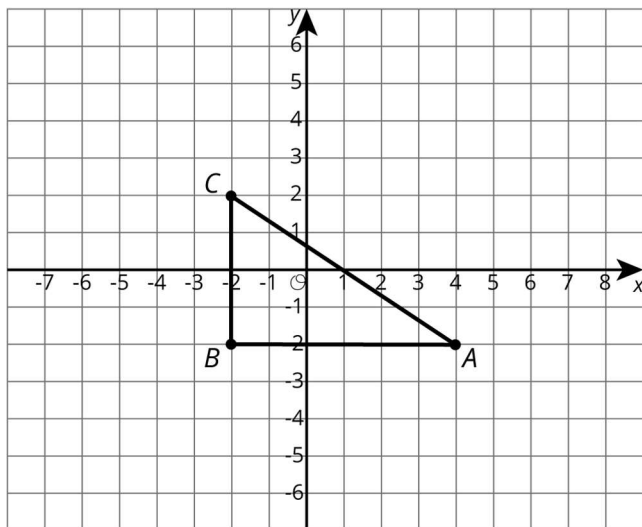


- b. 3
- c. $\frac{1}{3}$
- d. $\frac{2}{3}$

3. Problem 3 Statement

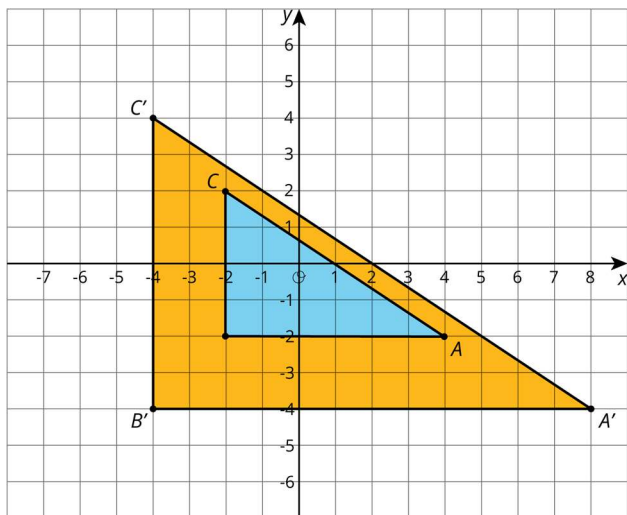
Here is a triangle.

- a. Draw the enlargement of triangle ABC , with centre $(0,0)$, and scale factor 2. Label this triangle $A'B'C'$.
- b. Draw the enlargement of triangle ABC , with centre $(0,0)$, and scale factor $\frac{1}{2}$. Label this triangle $A''B''C''$.
- c. Is $A''B''C''$ an enlargement of triangle $A'B'C'$? If yes, what are the centre of enlargement and the scale factor?

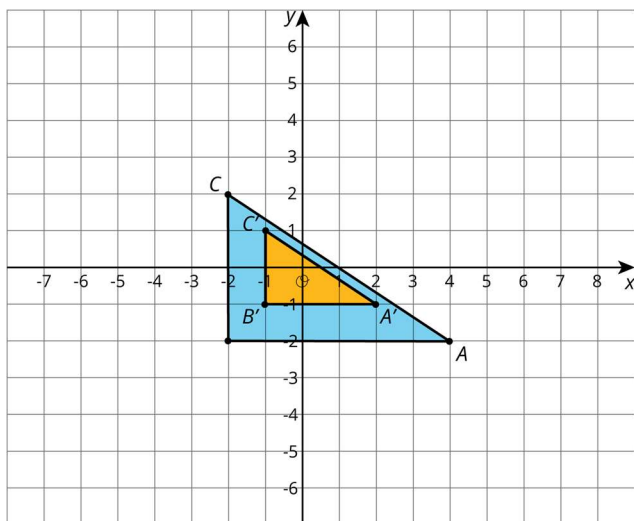


Solution

a.



b.



c. Yes, $A''B''C''$ is an enlargement of $A'B'C'$ with centre $(0,0)$ and scale factor $\frac{1}{4}$.

4. Problem 4 Statement

Triangle DEF is a right-angled triangle, and angle D is 28° . What are the other two angles?

Solution

90° and 62°



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