MEI Mathematics Education Innovation

Lesson 3: Interpreting division situations

Goals

- Create an equation and a diagram to represent a multiplication or division situation involving fractions, and coordinate these representations (orally).
- Explain (using words and other representations) how to find the unknown quantity in a multiplication or division situation involving fractions.
- Interpret a verbal description of a multiplication situation (in spoken or written language), and identify which quantity is unknown, i.e., the number of groups, the amount in one group, or the total amount.

Learning Targets

- I can create a diagram or write an equation that represents division and multiplication questions.
- I can decide whether a division question is asking “how many groups?” or “how many in each group?”.

Lesson Narrative

In an earlier lesson, students were reminded of the connection between multiplication and division. They revisited the idea of division as a way to find a missing factor, which can either be the number of groups, or the size of one group.

In this lesson, students interpret division situations in story problems that involve equal-size groups. They draw diagrams and write division and multiplication equations to make sense of the relationship between known and unknown quantities.

Building On

- Represent and solve problems involving multiplication and division.

Addressing

- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. In general, $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$. How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's explore situations that involve division.

3.1 Dot Image: Properties of Multiplication

Warm Up: 5 minutes

In this warm-up, students determine the number of dots in an image without counting and explain how they arrived at that answer. The goal is to prompt them to visualise and articulate different ways to decompose the dots, using what they know about arrays, multiplication, and area to arrive at the total number. To discourage counting and encourage students to focus on the structure of the dots, the image will be flashed and hidden a couple of times, rather than displayed the entire time.

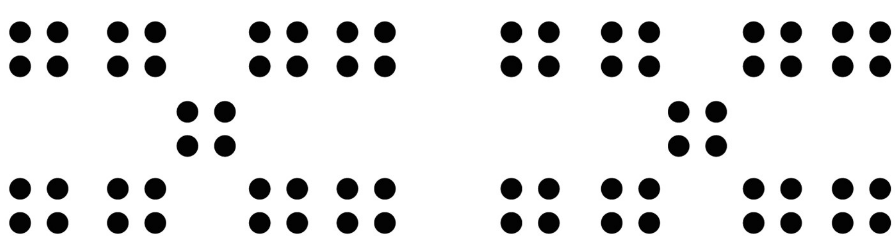
Launch

Tell students to keep their materials closed. Explain that you will show an image that contains dots for 3 seconds. Their job is to determine how many dots there are and explain how they saw them.

Display the image for 3 seconds and then hide it. Do this twice. Give students up to 1 minute of quiet think time after each flash. Encourage students who have one way of seeing the dots to think about another way while they wait.

Representation: Internalise Comprehension. Guide information processing and visualisation. To support working memory, show the image for a longer period of time or repeat the image flash as needed. Students may also benefit from being explicitly told not to count the dots, but instead to look for helpful structure within the image.
Supports accessibility for: Memory; Organisation

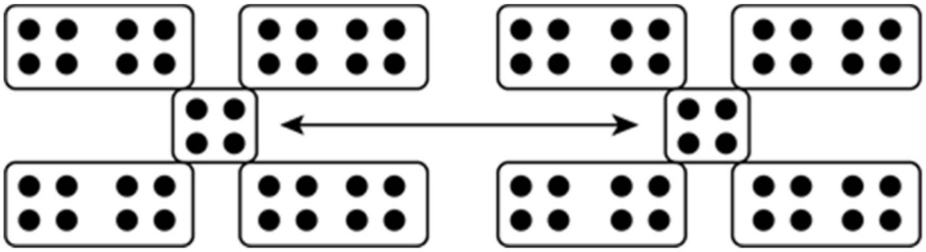
Student Task Statement



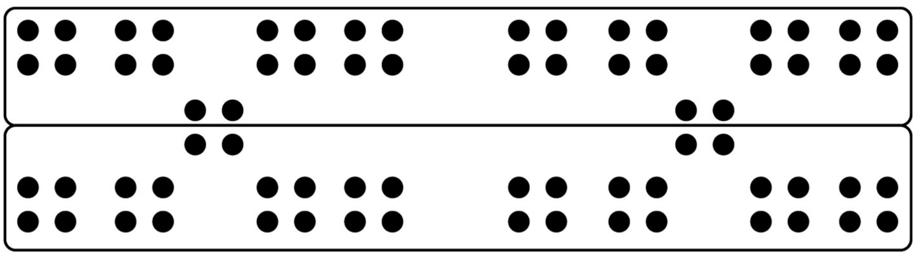
Student Response

72 dots. Possible strategies:

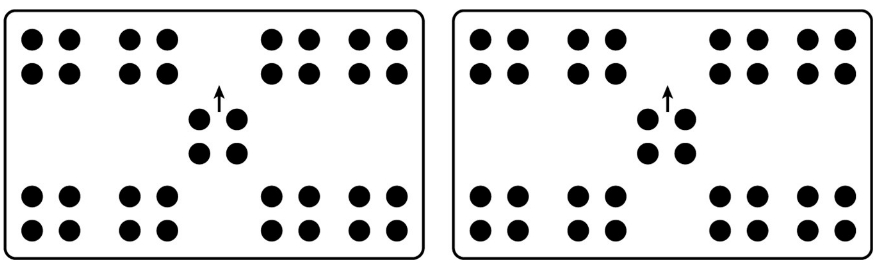
- $9 \times 8 = 72$



- $[(2 \times 4 \times 4) + 4] \times 2 = 72$



- $[(5 \times 4) + (4 \times 4)] \times 2 = 72$



Activity Synthesis

Select a couple of students to share the number of dots they saw. To focus students' reasoning on the structure of the dots, ask how they *saw* them, instead of how they *found* them. Record and display student explanations for all to see. Consider re-displaying the image to support students in their explanation.

To involve more students in the conversation, consider asking:

- Who can restate the way ___ saw the dots in different words?
- Did anyone see the dots the same way but would explain it differently?
- Does anyone want to add an observation to the way ____ saw the dots?

- Who saw the dots differently?
- Do you agree or disagree? Why?

3.2 Homemade Jams

20 minutes (there is a digital version of this activity)

This activity allows students to draw diagrams and write equations to represent simple division situations. Some students may draw concrete diagrams; others may draw abstract ones. Any diagrammatic representation is fine as long as it enables students to make sense of the relationship between the number of groups, the size of a group, and a total amount.

The last question is likely more challenging to represent with a diagram. Because the question asks for the number of jars, and because the amount per jar is a fraction, students will not initially know how many jars to draw (unless they know what $6\frac{3}{4} \div \frac{3}{4}$ is). Suggest that they start with an estimate, and as they reason about the problem, add jars to (or remove jars from) their diagram as needed.

As students work, monitor for the range of diagrams that students create. Select a few students whose work represent the range of diagrams to share later.

Students in digital classrooms can use an applet to make sense of the problems, but it is still preferable that they create their own diagrams.

Instructional Routines

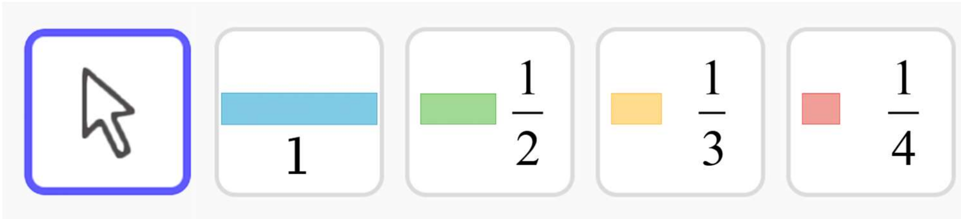
- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell the class that you will read the three story problems, and ask them to be prepared to share at least one thing they notice and one thing they wonder. After reading, give them a minute to share their observation and question with their partner.

Clarify that their job is to draw a diagram and write a multiplication equation to express the relationship in each story and then answer the question. Give students 7–8 minutes of quiet work time, followed by 2–3 minutes to share their work with their partner.

If the applet is used to complete the activity or for class discussion, note that the toolbar includes coloured rectangles that represent fractional parts. Encourage students to drop the fractional parts in the work space on the left of the window, and then use the Move tool (the arrow) to drag them into the jars. The blocks do snap, but the grid is very fine so this may be challenging for some students. Troubleshooting tip: if two blocks get stuck together, delete them. Try not to overlap blocks when adding them to the work space.



Student Task Statement

Draw a diagram, and write a multiplication equation to represent each situation. Then answer the question.

1. Mai had 4 jars. In each jar, she put $2\frac{1}{4}$ cups of homemade blueberry jam. Altogether, how many cups of jam are in the jars?

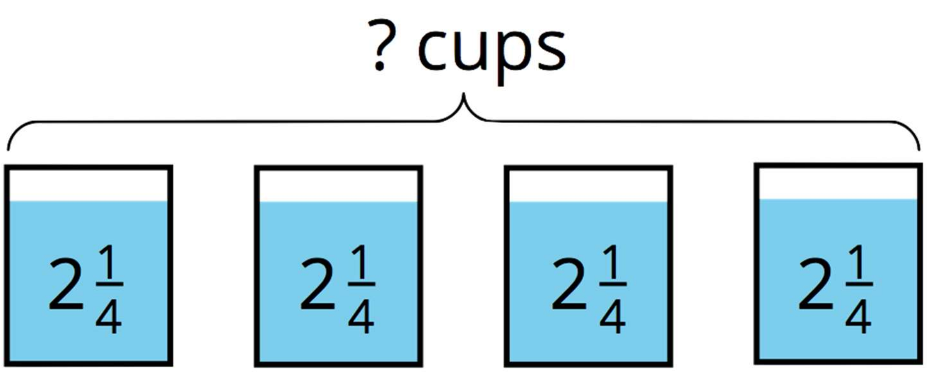
2. Priya filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?

3. Han had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

Student Response

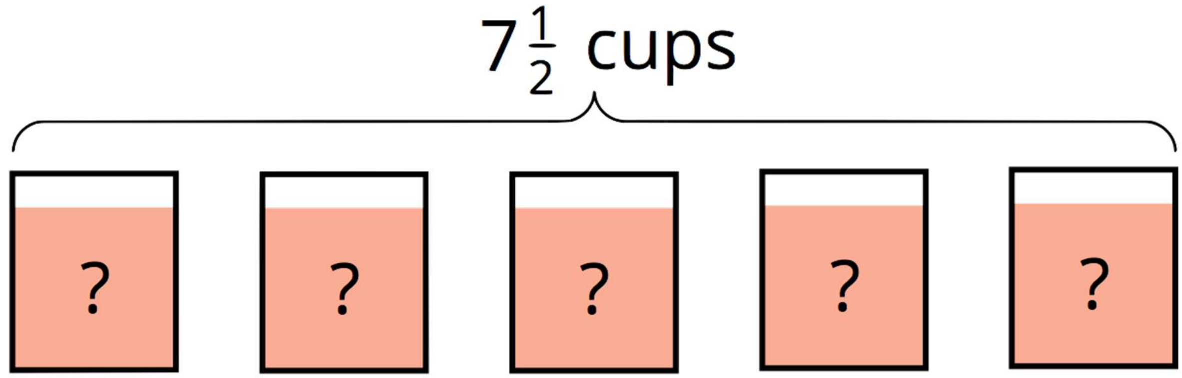
Equations and diagrams vary.

1. Multiplication equation: $4 \times \left(2\frac{1}{4}\right) = ?$ Sample diagram:



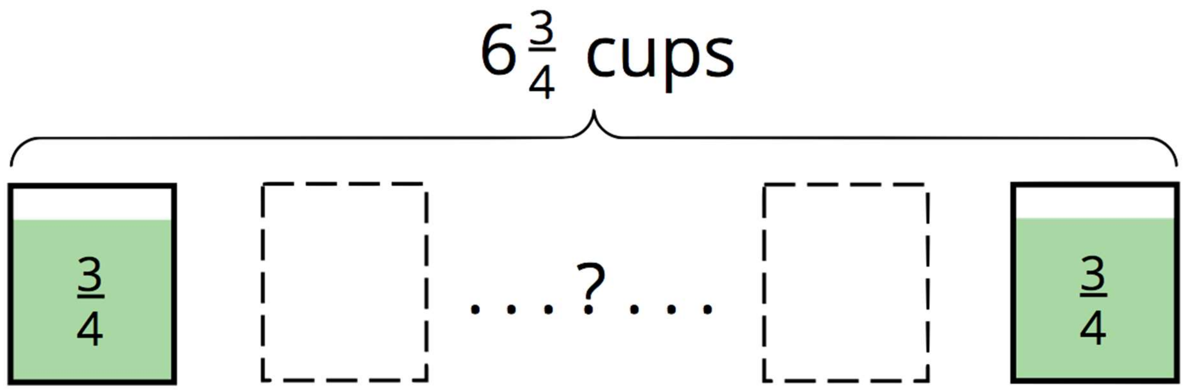
   Answer: 9 cups

2. Multiplication equation: $5 \times ? = 7\frac{1}{2}$. Sample diagram:



Answer: $1\frac{1}{2}$ cups per jar

3. Multiplication equation: $? \times \frac{3}{4} = 6\frac{3}{4}$. Sample diagram:



Answer: 9 jars

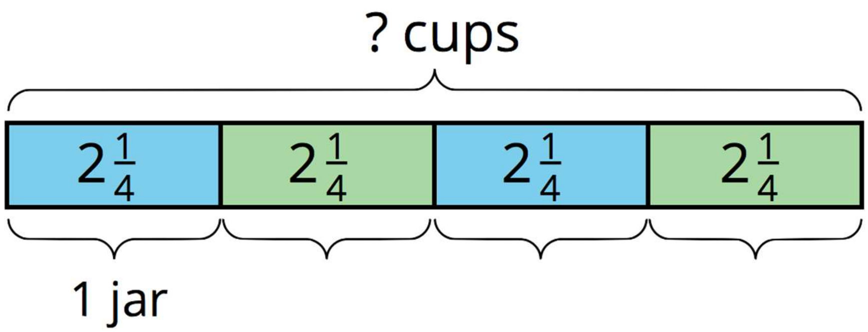
Activity Synthesis

Select previously identified students to share their diagrams, sequenced from the more concrete (e.g., pictures of jars and cups) to the more abstract (e.g., rectangles, bar models). Display the diagrams and equations for all to see. Ask them how they used the diagrams to answer the questions (if at all).

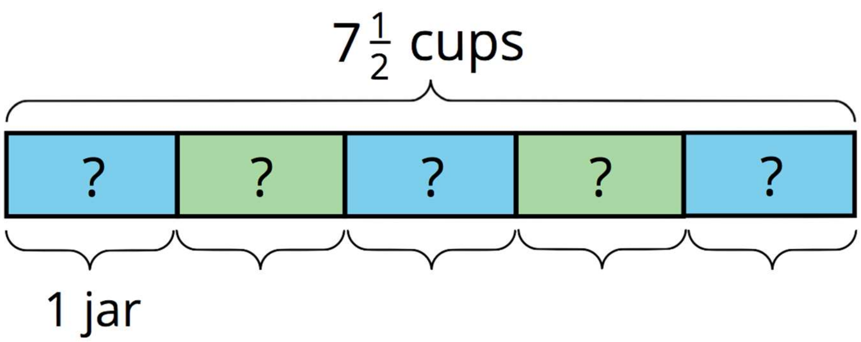
If bar models such as the ones here are not already shown and explained by a student, display them for all to see. Help students make sense of the diagrams and connecting them to multiplication and division by discussing questions such as these:

- "In each diagram, what does the '?' represent?" (The unknown amount)
- "What does the length of the entire bar represent?" (The total amount, which is sometimes known.)
- "What does each rectangular part represent?" (One jar)

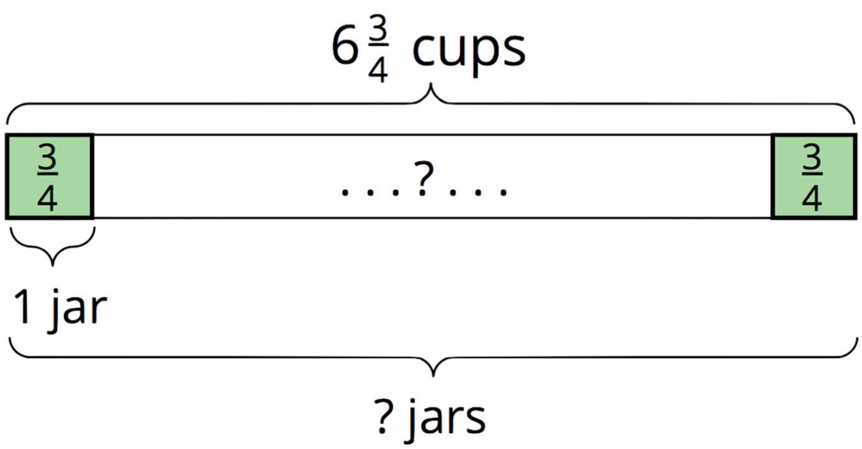
- "What does the number in each rectangle represent?" (The amount in each jar)
- "How do the three parts of each multiplication equation relate to the diagram?" (The first factor refers to the number of rectangles. The second factor refers to the amount in each rectangle. The product is the total amount.)
- "The last diagram doesn't represent all the jars and shows a question mark in the middle of the bar. Why might that be?" (The diagram shows an unknown number of jars, which was the question to be answered.)



$$4 \times 2\frac{1}{4} = ?$$



$$4 \times 2\frac{1}{4} = ?$$



$$? \times \frac{3}{4} = 6\frac{3}{4}$$

Highlight that the last two situations can be described with division: $7\frac{1}{2} \div 5$ and $6\frac{3}{4} \div \frac{3}{4}$.

Representing: Discussion Supports. Use this routine to support whole-class discussion. As students discuss the questions listed in the Activity Synthesis, label the display of the diagrams and equations accordingly. Annotate the display to illustrate connections between equivalent parts of each representation. For example, next to each question mark, write “unknown amount.”
Design Principle(s): Support sense-making

3.3 Making Granola

15 minutes

In this activity, students continue to investigate division problems in context, think of them in terms of equal-size groups, and represent them using diagrams and equations.

As students work, monitor the different diagrams and interpretations for each problem. Select students whose diagrams are especially clear in showing the meanings of division to share later.

Both division problems result in quotients that are not whole numbers. As students work, encourage them to use their multiplication equations to check the answers to the division problems.

Instructional Routines

- Think Pair Share

Launch

Keep students in the same groups. Some students may not be familiar with granola and oats, so show or explain what they are.

Give students 7–8 minutes of quiet work time, and 2–3 minutes to discuss their responses with a partner. During partner discussion, ask them to compare their equations and diagrams in the first question, and their interpretations of division in the second question.

Representation: Internalise Comprehension. Represent the same information through different modalities using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help organise the information provided. Some students may benefit from support to be able to draw abstract diagrams. For example, demonstrate how 4 ounces of oats can be represented with 1 scoop in a drawing.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Some students may round their answers to the nearest whole number rather than including the fractions of a scoop or a batch. Ask students to consider if it is possible to have a part of a scoop, a part of a batch, or a part of a unit of weight. Encourage them to think about how to show a part of a whole unit on a diagram.

Student Task Statement

1. Consider the problem: To make 1 batch of granola, Kiran needs 26 ounces of oats. The only measuring tool he has is a 4-ounce scoop. How many scoops will it take to measure 26 ounces of oats?

    a. Will the answer be more than 1 or less than 1?

    b. Write a multiplication equation and a division equation that represent this situation. Use "?" to represent the unknown quantity.

    c. Find the unknown quantity. If you get stuck, consider drawing a diagram.

2. The recipe calls for 14 ounces of mixed nuts. To get that amount, Kiran uses 4 bags of mixed nuts.

    a. Write a mathematical question that might be asked about this situation.

    b. What might the equation $14 \div 4 = ?$ represent in Kiran's situation?

    c. Find the quotient. Show your reasoning. If you get stuck, consider drawing a diagram.

Student Response

a. More than 1

b. $? \times 4 = 26$ (or $4 \times ? = 26$) and $26 \div 4 = ?$

c. The missing quantity is $6\frac{1}{2}$ scoops. Reasoning varies. Sample reasoning: I know 6×4 is 24, and $\frac{1}{2}$ of 4 is 2, so it would take $6 + \frac{1}{2}$ or $6\frac{1}{2}$ scoops to make 26 ounces.

a. Answers vary. Sample responses:

- How many ounces of mixed nuts are in each bag?
- Did he use up all 4 bags, or are there leftover mixed nuts?
- How many bags will he need if he is doubling the recipe?

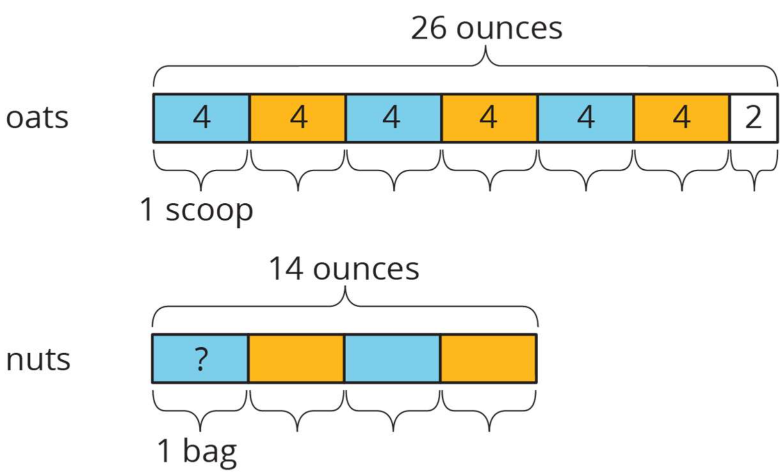
b. How many ounces of mixed nuts are in each bag?

c. $3\frac{1}{2}$ ounces. Reasoning varies. Sample reasoning: $14 \div 2 = 7$, so there are 7 ounces in 2 bags of mixed nuts, or $3\frac{1}{2}$ ounces in 1 bag of mixed nuts. $7 \div 2 = 3\frac{1}{2}$.

Activity Synthesis

The aim of the discussion is to consolidate students' understanding of the two interpretations of division ("how many groups?" and "how much in a group?"). Ask students who drew effective diagrams to display and explain them. For each interpretation, write a multiplication equation and discuss what each factor represents in the context (e.g., the number of batches, scoops, or bags, vs. how much is in each batch, scoop, or bag).

If no students drew bar models to represent the situations, show the following. Reiterate that each rectangle represents one group (one scoop or 1 bag), the number inside represents the amount in one group, and the number of rectangles tells us how many groups there are.



Lesson Synthesis

In this lesson, we solved problems that involved multiplication and division. Reiterate to students that in division situations that involve equal-size groups, we are not always looking for the same unknown. There are typically three pieces of information involved: the number of groups, the size of each group, and the total amount. Knowing what information we have and what is missing can help us answer questions.

Present a few more story problems. Ask students: "What information is unknown in each situation?"

- Flour is sold in 3-pound bags. How many pounds are in 7 bags? (The total amount is unknown.)
- Five tickets to a play cost £38. What does each ticket cost if they all cost the same? (The amount for one person is unknown.)
- One quart is equal to 32 ounces. How many quarts are in 128 ounces? (The number of groups is unknown.)

For each problem, discuss: "What multiplication equation can we write? What diagram can we draw to represent the quantities?"

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3.4 Rice and Beans

Cool Down: 5 minutes

Student Task Statement

1. Here are three problems. Select **all** problems that can be solved using division.

    a. Jada cut 4 pieces of ribbon that were equal in length. She used a total of 5 feet of ribbon. How long, in feet, was each piece of ribbon she cut?

    b. A chef bought 3 bags of beans. Each bag contains $1\frac{2}{5}$ kilograms of beans. How many kilograms of beans did she buy?

    c. A printer takes $2\frac{1}{2}$ seconds to print a flyer. It took 75 seconds to print a batch of flyers without stopping. How many flyers were in the batch?

2. Consider the problem: Andre poured 27 ounces of rice into 6 containers. If all containers have the same amount of rice, how many ounces are in each container?

    a. Write an equation to represent the situation. Use a "?" to represent the unknown quantity.

    b. Find the unknown quantity. Show your reasoning.

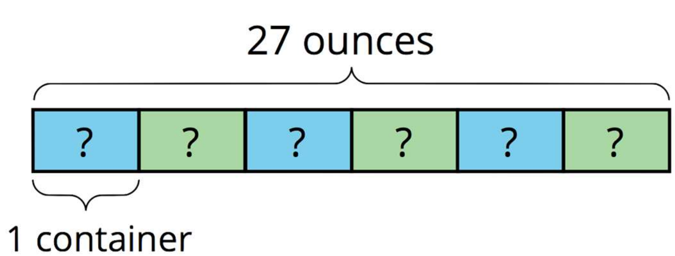
Student Response

1. A and C

    a. $6 \times ? = 27$ or $27 \div 6 = ?$

    b. $4\frac{1}{2}$ ounces. Reasoning varies. Sample reasoning:

    - 3 groups of 9 make 27. Splitting each of the 3 groups into 2 gives us 6 groups. Each group is half of 9, which is $4\frac{1}{2}$.
    - 27 divided by 6 is 4 with a remainder of 3. That means 27 is made of 6 groups of size 4 and a remainder of 3. Splitting the remainder of 3 into 6 groups makes 6 groups of size $\frac{1}{2}$. That means that 27 is made of 6 groups of size $4\frac{1}{2}$.

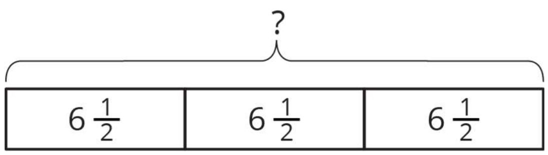


$6 \times ? = 27$

Student Lesson Summary

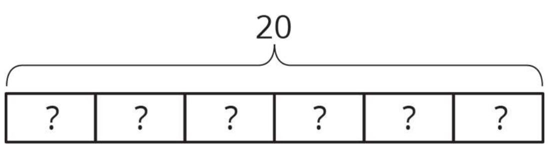
If a situation involves equal-sized groups, it is helpful to make sense of it in terms of the number of groups, the size of each group, and the total amount. Here are three examples to help us better understand such situations.

- Suppose we have 3 bottles with $6\frac{1}{2}$ ounces of water in each, and the total amount of water is not given. Here we have 3 groups, $6\frac{1}{2}$ ounces in each group, and an unknown total, as shown in this diagram:

  We can express this situation as a multiplication problem. The unknown is the product, so we can simply multiply the 2 known numbers to find it. $3 \times 6\frac{1}{2} = ?$

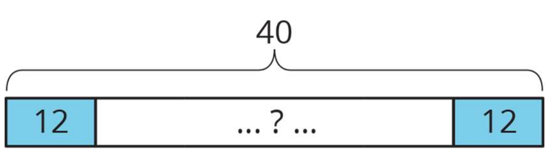
- Next, suppose we have 20 ounces of water to fill 6 equal-sized bottles, and the amount in each bottle is not given. Here we have 6 groups, an unknown amount in each, and a total of 20. We can represent it like this:

  This situation can also be expressed using multiplication, but the unknown is a factor, rather than the product: $6 \times ? = 20$

  To find the unknown, we cannot simply multiply, but we can think of it as a division problem: $20 \div 6 = ?$

- Now, suppose we have 40 ounces of water to pour into bottles, 12 ounces in each bottle, but the number of bottles is not given. Here we have an unknown number of groups, 12 in each group, and a total of 40.

  Again, we can think of this in terms of multiplication, with a different factor being the unknown: $? \times 12 = 40$

  Likewise, we can use division to find the unknown: $40 \div 12 = ?$

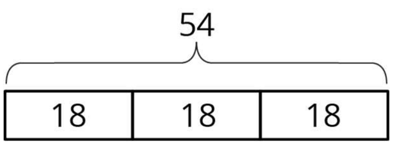
Whenever we have a multiplication situation, one factor tells us *how many groups* there are, and the other factor tells us *how much is in each group*.

Sometimes we want to find the total. Sometimes we want to find how many groups there are. Sometimes we want to find how much is in each group. Anytime we want to find out how many groups there are or how much is in each group, we can represent the situation using division.

Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Write a multiplication equation and a division equation that this diagram could represent.

   Solution

   Multiplication: $3 \times 18 = 54$ (or $18 \times 3 = 54$), division: $54 \div 18 = 3$ (or $54 \div 3 = 18$)

2. **Problem 2 Statement**

   Consider the problem: Mai has £36 to spend on movie tickets. Each movie ticket costs £4.50. How many tickets can she buy?

   a. Write a multiplication equation and a division equation to represent this situation.

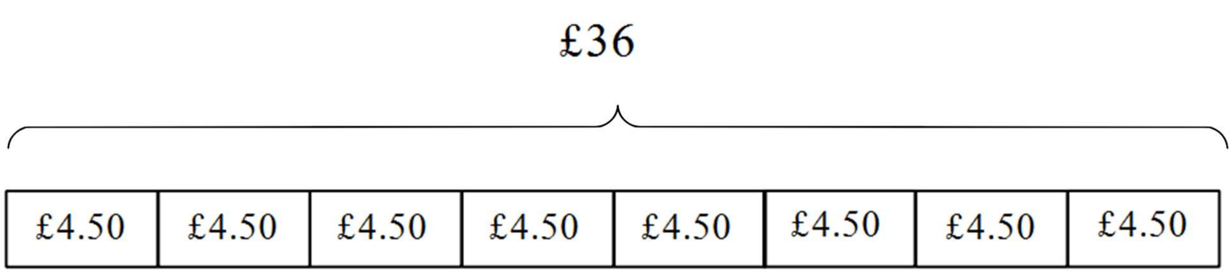
   b. Find the answer. Draw a diagram, if needed.

   c. Use the multiplication equation to check your answer.

   Solution

   a. Multiplication: $? \times (4.50) = 36$ (or equivalent), division: $36 \div 4.50 = ?$ (or equivalent)

   b. Mai can buy 8 movie tickets.

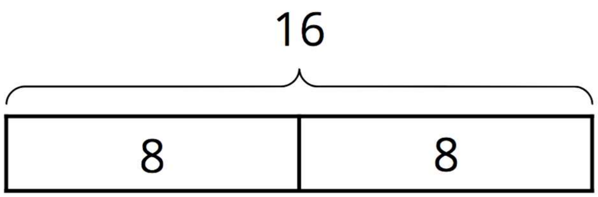
   

c. 8 is correct because $8 \times (4.50) = 36$.

3. **Problem 3 Statement**

Kiran said that this diagram can show the solution to $16 \div 8 =?$ or $16 \div 2 =?$, depending on how we think about the equations and the "?".

Explain or show how Kiran is correct.



Solution

The diagram can illustrate $16 \div 8 =?$ if we interpret the equation and the "?" to mean: "How many groups of 8 are in 16?" The diagram can illustrate $16 \div 2 =?$ if we interpret the equation and the "?" to mean: "What is in each group if 16 is divided into 2 equal groups?"

4. **Problem 4 Statement**

Write a sentence describing a situation that could be represented by the equation $4 \div 1\frac{1}{3} =?$.

Solution

Answers vary. Sample responses:

- A group of friends met for lunch and got 4 small pizzas to share. Each person had $1\frac{1}{3}$ pizzas. How many friends went for lunch?
- A baker is filling equal-sized containers with sugar. Four pounds of sugar fill $1\frac{1}{3}$ containers. How many pounds fit in each container?

5. **Problem 5 Statement**

Noah said, "When you divide a number by a second number, the result will always be smaller than the first number."

Jada said, "I think the result could be larger or smaller, depending on the numbers."

Do you agree with either of them? Explain or show your reasoning.

Solution

I agree with Jada. Explanations vary. Sample explanation: If number is divided by a number that is between 0 and 1, then the result is bigger than the first number. For example, $1 \div 0.1 = 10$, which is bigger than 1. But $1 \div 2 = 0.5$, which is smaller than 1.

6. **Problem 6 Statement**

Mini muffins cost £3.00 per dozen.

– Andre says, "I have £2.00, so I can afford 8 muffins."

– Elena says, "I want to get 16 muffins, so I'll need to pay £4.00."

Do you agree with either of them? Explain your reasoning.

Solution

They are both correct. Each muffin costs 25 pence because $3 \div 12 = 0.25$. Andre can afford 8 muffins because $2 \div 0.25 = 8$, and Elena will need £4 because $16 \times 0.25 = 4$.

7. **Problem 7 Statement**

A family has a monthly budget of £2 400. How much money is spent on each category?

a. 44% is spent on housing.

b. 23% is spent on food.

c. 6% is spent on clothing.

d. 17% is spent on transportation.

e. The rest is put into savings.

Solution

a. £1 056, because $(0.44) \times 2\,400 = 1\,056$

b. £552, because $(0.23) \times 2\,400 = 552$.

c. £144, because $(0.06) \times 2\,400 = 144$.

d. £408, because $(0.17) \times 2\,400 = 408$.

e. £240, because there is 10% remaining for savings, and $(0.1) \times 2\,400 = 240$.