Lesson 14: Nets and surface area

Goals

- Match polyhedra with their nets and justify (orally) that they match.
- Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.
- Visualise and identify the polyhedron that can be assembled from a given net.

Learning Targets

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson Narrative

Previously, students learned about polyhedra, analysed and defined their features, and investigated their physical representations. Students also identified the polygons that compose a polyhedron; they recognised a net as an arrangement of these polygons and as a two-dimensional representation of a three-dimensional shape.

This lesson extends students' understanding of polyhedra and their nets. They practice visualising the polyhedra that could be assembled from given nets and use nets to find the **surface area** of polyhedra.

Addressing

- Represent three-dimensional shapes using nets made up of rectangles and triangles, and use the nets to find the surface area of these shapes. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

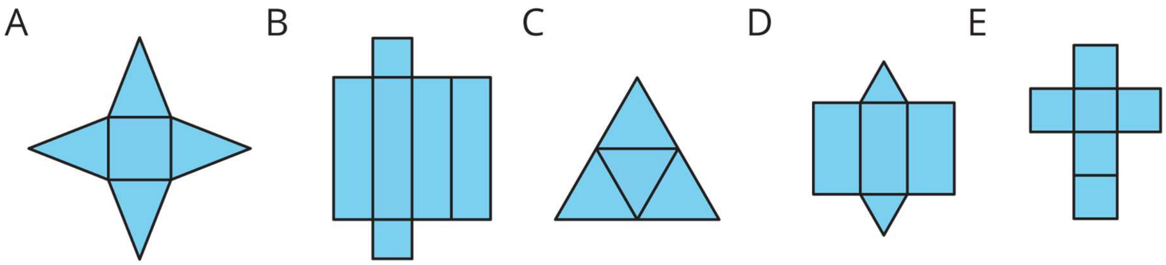
Glue or glue sticks

Nets of polyhedra

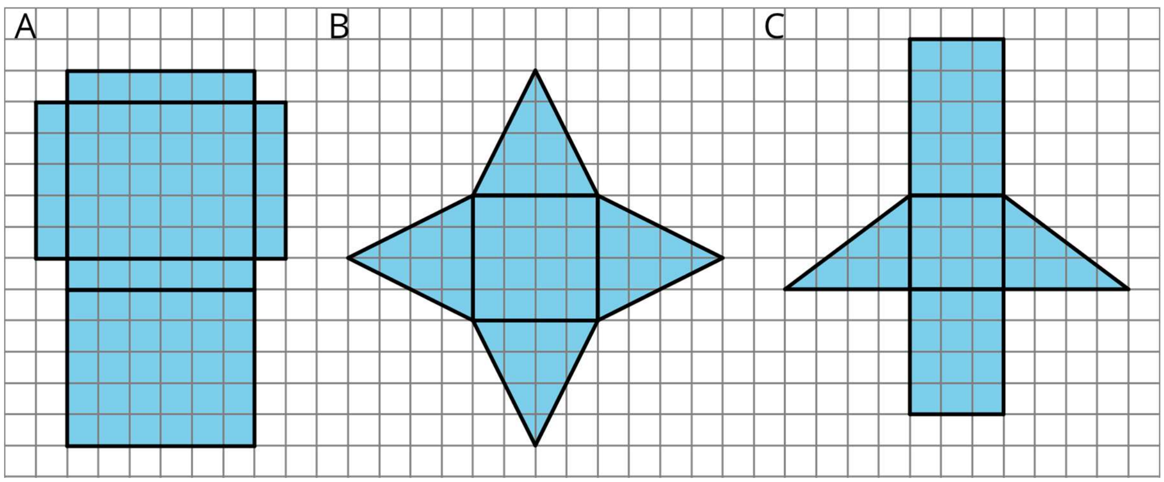
Sticky tape

Required Preparation

Warm-up Matching Nets activity



Using nets to find surface area activity



Prepare physical copies of the nets in the warm-up, in case needed to support students with visualisation. They can be enlarged as necessary.

Make copies of the nets in the blackline master for the main activity. Enlarge if necessary and add tabs. Prepare one set of 3 nets (A, B, and C) and some glue or sticky tape for each group of 3 students.

Student Learning Goals

Let's use nets to find the surface area of polyhedra.

14.1 Matching Nets

Warm Up: 10 minutes

This warm-up prompts students to match nets to polyhedra. It invites them to think about the polygons that make up a polyhedron and to mentally manipulate nets, which helps develop their visualisation skills.

Instructional Routines

- Think Pair Share

Launch

Give students 3 minutes of quiet think time to match nets to polyhedra and then another 2 minutes to discuss their response and reasoning with a partner. Encourage students to use the terminology they learned in prior lessons.

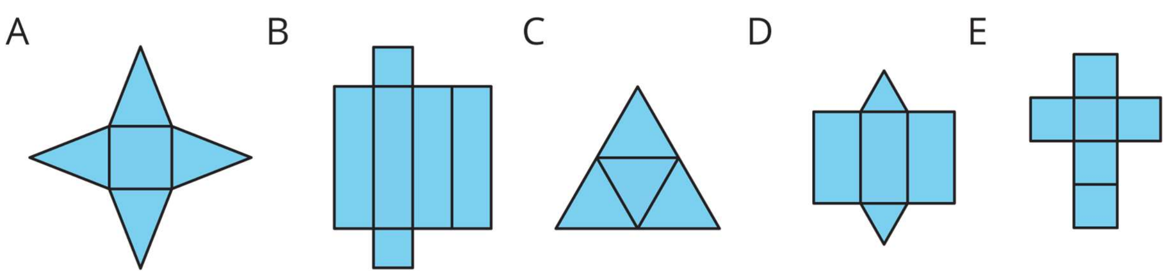
To support students who need more time or help in visualisation, prepare physical models of the polyhedra and copies of the nets from the blackline master. Pre-cut the nets or have scissors available so that students can assemble the nets and test their ideas.

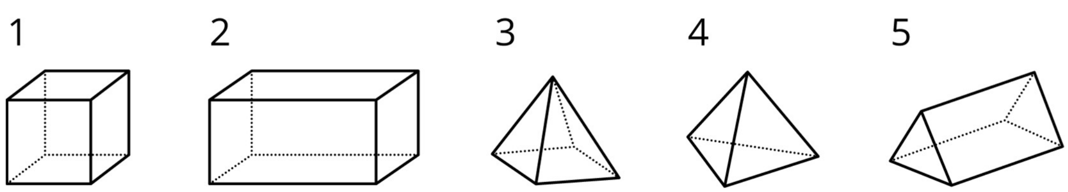
Anticipated Misconceptions

If students have trouble distinguishing between shapes A, C, and D, remind them that prisms and pyramids can both contain faces that are triangles. In a pyramid, all triangular faces that are not the base meet at a one vertex and have shared edges. In a prism, there can be a triangular base, but the other faces are quadrilaterals.

Student Task Statement

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.





Student Response

Net A is a square pyramid (3). It has five faces: one square and four triangles, just like the square pyramid (similar reasoning for each shape).

Net B is a cuboid (2).

Net C is a triangular pyramid (4).

Net D is a triangular prism (5).

Net E is a cube or square prism (1).

Activity Synthesis

Invite a few students to share their matching decisions and reasoning with the class. Ask students: “What clues did you use to help you match? How did you check if you were right?” If there is not unanimous agreement on any of the nets, ask students with differing opinions to explain their reasoning. Discuss to come to an agreement.

14.2 Using Nets to Find Surface Area

25 minutes

In this activity, students cut and assemble nets into polyhedra and learn to use nets to find surface area. The presence of a grid supports students in their calculations. It also reinforces the idea of area as the number of unit squares in a region and the connection between area and surface area. Students apply what they learned earlier about areas of triangles and parallelograms to find surface area.

As students make calculations, monitor their processes. Note those who work systematically to find surface area (e.g., by organising the measurements of each face, calculating the area of each face, and adding the areas together) and those who don't. Encourage students with disorganised or scattered work to take a more systematic approach. Demonstrate strategies such as labelling both the polygons on the net and portions of their work that pertain to those faces.

Also notice students who look for and use structure, for instance, by grouping certain polygons together and finding the area of the composite shape (e.g., a group of rectangles that have a common side length), or by identifying multiple copies of the same polygon and calculating the area once. Select them to share their work later.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Arrange students in groups of 3. Give each group one of each net (A, B, and C), sticky tape, and access to their geometry toolkits (especially scissors). Explain to students that they will cut out some nets, assemble them into polyhedra, and calculate their surface areas. Remind students that the surface area of a three-dimensional shape is the sum of the areas of all of its faces. Ask students to complete the first question before cutting anything.

Point out that the net has shaded and unshaded polygons. Explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are “flaps” to make it easier to glue or tape the polygons together. They will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold the net up and attach the various polygons together. A straightedge can be very helpful for making the creases.

Tell students that it is easy to miss or double-count the area of a face when finding surface area. Ask them to think carefully about how to record their calculations to ensure that all faces are accounted for, correct measurements are used, and errors are minimised.

When students have completed their calculations, ask them to compare and discuss their work with another student with the same polyhedron.

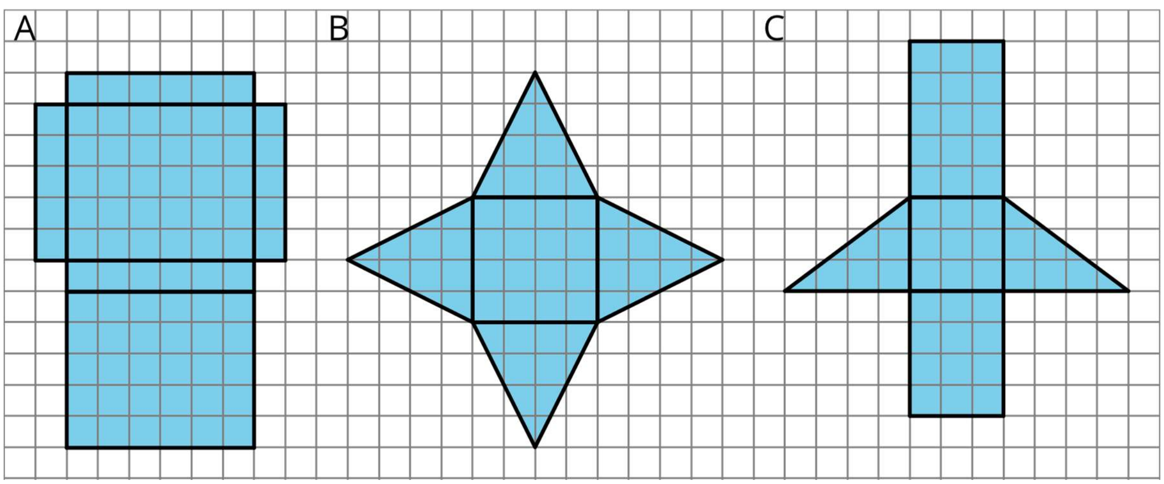
Representation: Develop Language and Symbols. Eliminate barriers and provide concrete manipulatives to connect symbols to concrete objects or values. Provide students with access to pre-cut nets and the polyhedra.
Supports accessibility for: Visual-spatial processing; Fine-motor skills

Anticipated Misconceptions

If students do not identity the specific type of prism or pyramid, remind them that they should also name each shape by the shape of their base.

Student Task Statement

1. Name the polyhedron that each net would form when assembled.



2. Your teacher will give you the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.

3. Find the **surface area** of each polyhedron. Explain your reasoning clearly.

Student Response

1. A: cuboid, B: square pyramid, C: triangular prism

2. No answer required.

3. Explanations vary. Sample responses:

    - A: The surface area is 82 square units. $2(6 \times 1) + 2(5 \times 1) + 2(6 \times 5) = 82$

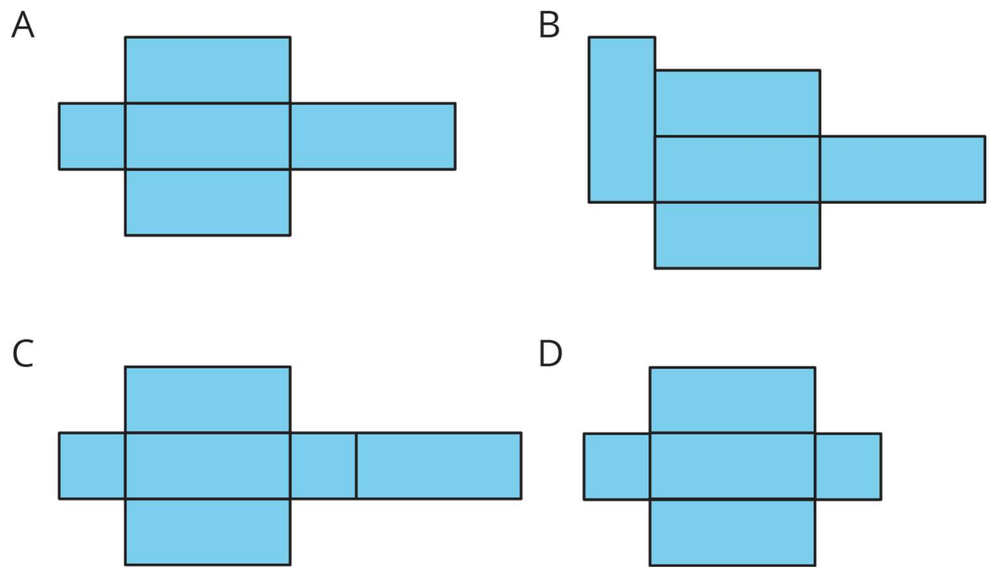
    - B: The surface area is 48 square units. $(4 \times 4) + 4(\frac{1}{2} \times 4 \times 4) = 48$

- C: The surface area is 48 square units. $(3 \times 5) + (3 \times 3) + (3 \times 4) + 2(\frac{1}{2} \times 3 \times 4) = 48$

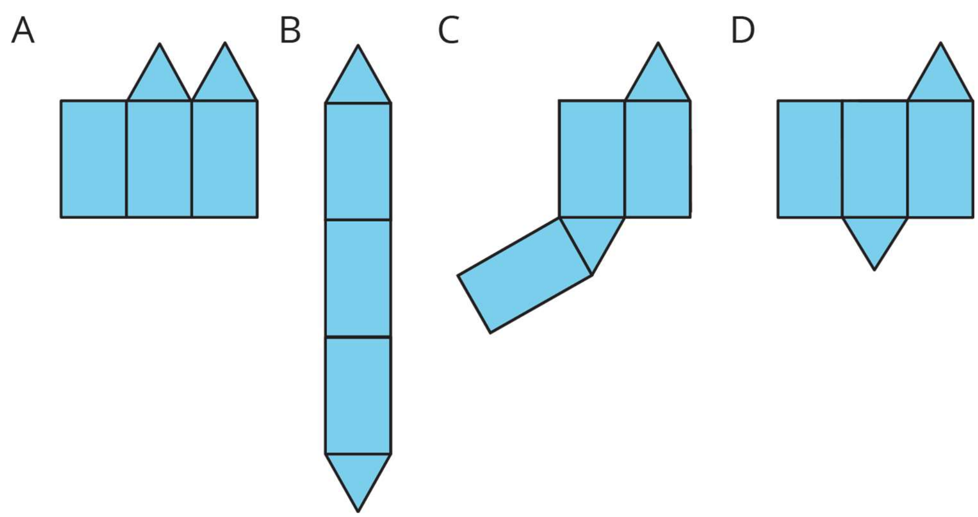
- C: The combined area of the three rectangular faces is 36 square units. $3 \times 12 = 36$. The combined area of the two right-angled triangles is 12 square units. $2(\frac{1}{2} \times 3 \times 4) = 12$. The surface area is 48 square units because $36 + 12 = 48$.

Are You Ready for More?

1. For each net, decide if it can be assembled into a cuboid.



2. For each net, decide if it can be folded into a triangular prism.



Student Response

1. Only C can be folded into a cuboid.

2. C and D can be folded into triangular prisms.

Activity Synthesis

For each polyhedron, select at least 2 students with correct calculations but different approaches to share their work, if possible.

For polyhedron A, select students who took the following approaches, in this sequence:

- Found the area of each rectangle separately
- Found the areas of pairs of identical rectangles (3 pairs total)
- Calculated the area of a group of connected rectangles with the same length or width (e.g., the four rectangles on the net with side length 6 units)

For polyhedron B, select students who:

- Found the area of each of the 5 polygons separately
- Found the area of the square, rearranged the 4 triangles into 2 parallelograms, and calculated the area of each parallelogram
- Calculated the area of the square and the area of 1 triangle, and multiplying the area of the triangle by 4

For polyhedron C, select students who:

- Found the area of each of the 5 polygons separately
- Rearranged the 2 right-angled triangles into a rectangle, and then found the area of each rectangle separately
- Calculated the area of each right-angled triangle and doubled it, and found the area of the group of connected rectangles with a width of 4 units

Point out that the reasoning strategies we used earlier in the unit still apply here. Even though we are working with three-dimensional shapes, surface area is a two-dimensional measure.

Highlight the benefits of approaching the problems systematically, e.g., by labelling parts, listing measurements and computations in order, etc.

Speaking: Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

In this lesson, we matched nets to the polyhedra, assembled polyhedra from nets, and used nets to find **surface area**. Discuss:

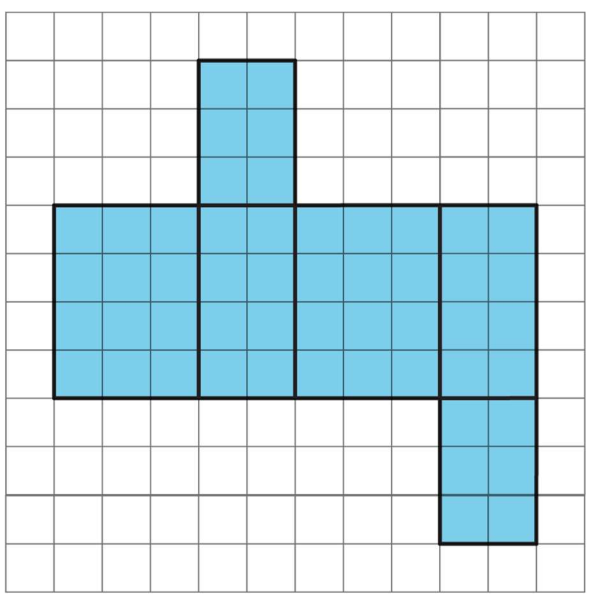
- "How do we use a net to find surface area?" (We calculate the area of each polygon on the net and add all the areas.)
- "How is finding surface area using a net different from finding surface area by looking at a picture of a polyhedron—as we had done with the filing cabinet, or by studying the actual object—as we had done with the multi-link cubes?" (A net allows us to see all the faces of a polyhedron at once. When working from a picture or drawing, we need to visualise the hidden faces. Working with an actual polyhedron could help, but again we are not looking at all the faces at once; we have to rotate the object and might miss or double-count a face.)
- "When using a net, how do we keep track of your calculations or make sure all faces are accounted for?" (We can label all the polygons and the calculations.)
- "Are there ways to simplify the calculations? Or is it best to find the area of each polygon one at a time?" (Sometimes we can simplify the process by combining polygons and finding the area of the combined region—e.g., a group of rectangles with the same side length. If there are several polygons that are identical, we can find the area of one polygon and multiply it by the number of identical polygons in the net.)

14.3 Unfolded

Cool Down: 5 minutes

Student Task Statement

1. What kind of polyhedron can be assembled from this net?



2. Find the surface area (in square units) of the polyhedron. Show your reasoning.

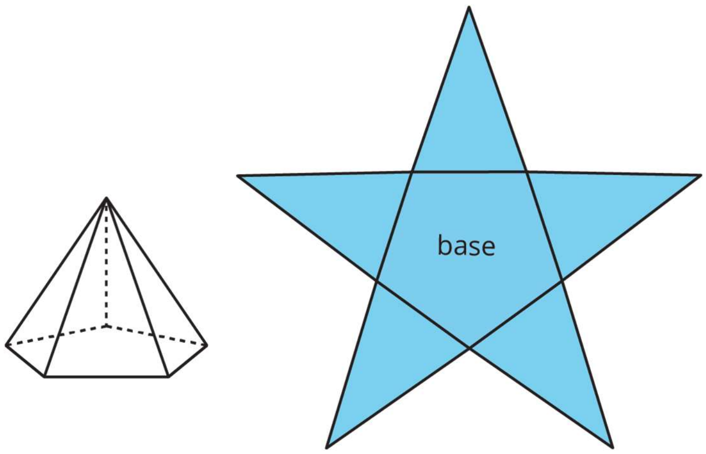
Student Response

1. It would assemble into a cuboid.

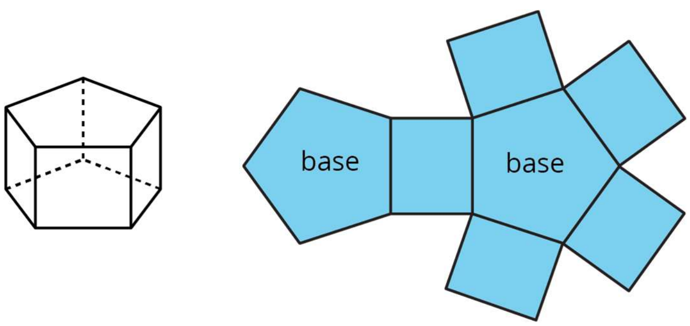
2. The surface area would be 52 square units. $2(3 \times 4) + 2(2 \times 4) + 2(2 \times 3) = 52$

Student Lesson Summary

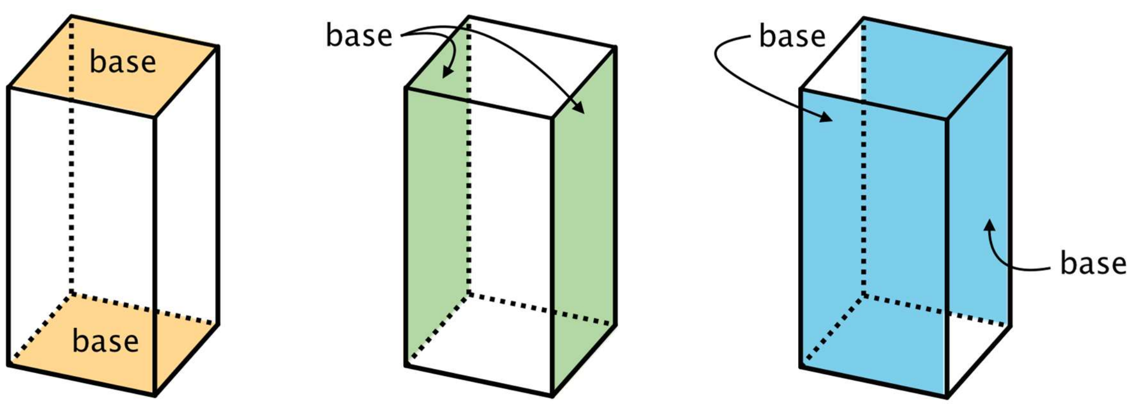
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



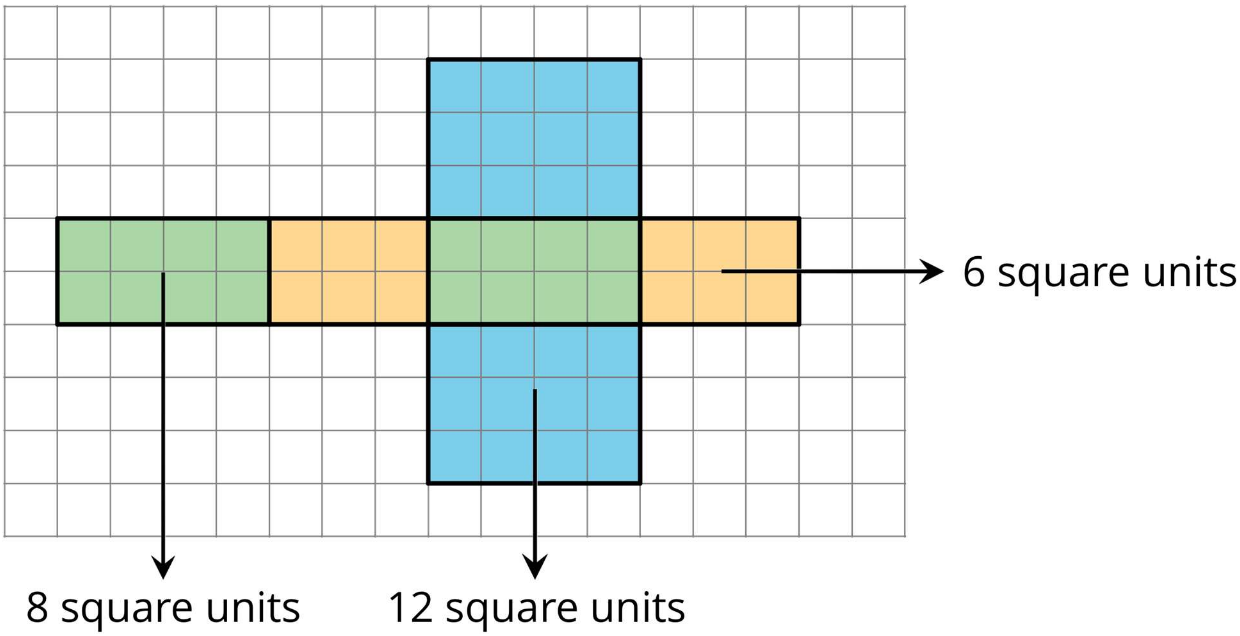
A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.



In a cuboid, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a cuboid shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.

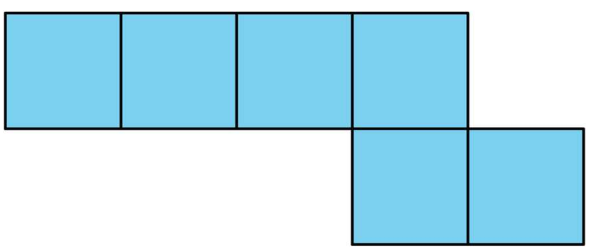


The **surface area** of the cuboid is 52 square units because $8 + 8 + 6 + 6 + 12 + 12 = 52$.

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.

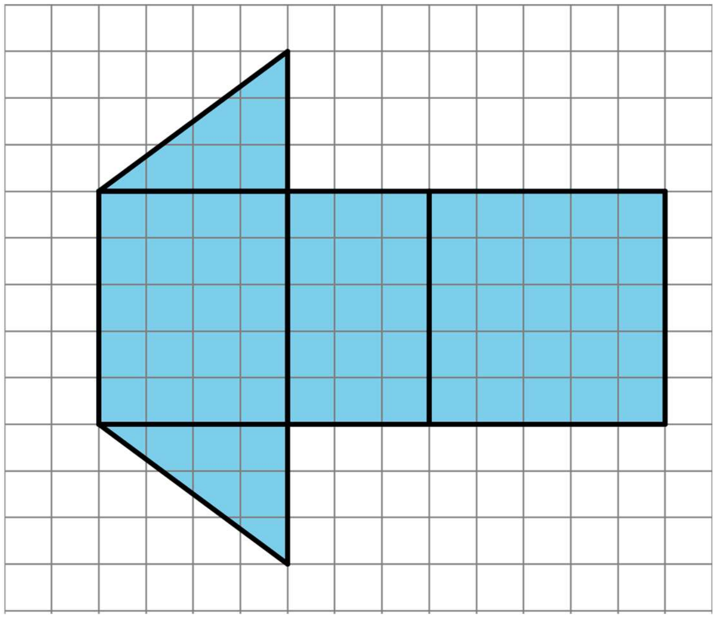


Solution

No. Sample explanation: The four squares placed side by side can only be folded in one way to meet up with one another, making a cube without a top and bottom. One of the remaining two squares can be folded to make the top or bottom, but the other one cannot be used.

2. **Problem 2 Statement**

   a. What polyhedron can be assembled from this net? Explain how you know.



   b. Find the surface area of this polyhedron. Show your reasoning.

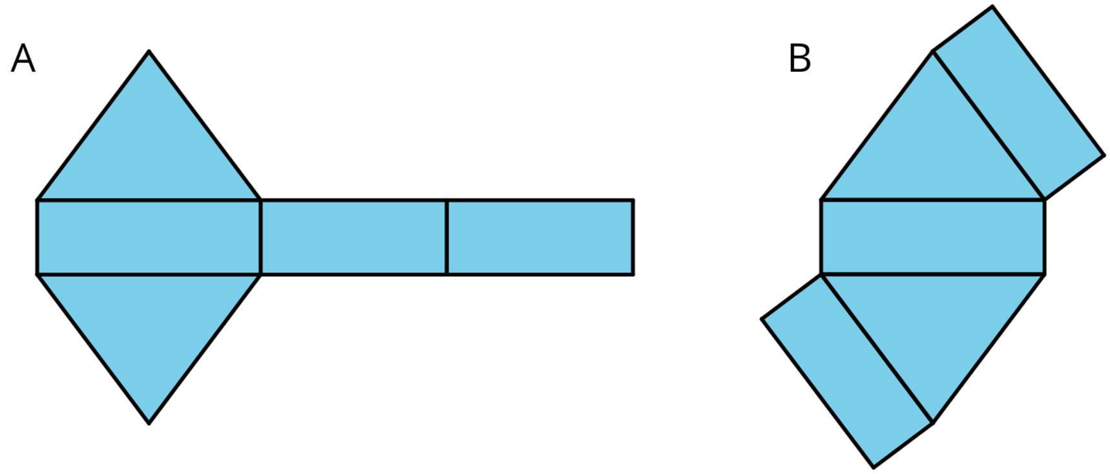
Solution

   a. A triangular prism. Sample explanation: There are two identical triangles that are the bases. The rest of the faces are rectangles.

   b. 72 square units. Sample reasoning: The area of the three rectangles are 20, 15, and 25 square units. The area of the two triangles are $2(\frac{1}{2} \times 4 \times 3)$ or 12 square units. $20 + 15 + 25 + 2(6) = 72$.

3. **Problem 3 Statement**

Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.



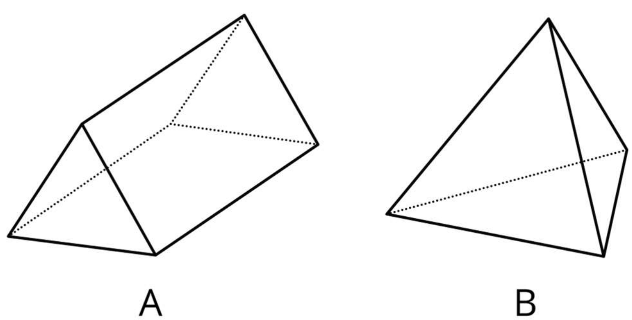
Solution

Agree. Sample reasoning: Both nets are composed of the same set of polygons. The positions of the one rectangular face are different, but when assembled, that face will meet the same edge of three other polygons.

4. **Problem 4 Statement**

Here are two three-dimensional shapes.

Tell whether each of the following statements describes shape A, shape B, both, or neither.



a. This shape is a polyhedron.

b. This shape has triangular faces.

c. There are more vertices than edges in this shape.

d. This shape has rectangular faces.

e. This shape is a pyramid.

f. There is exactly one face that can be the base for this shape.

g. The base of this shape is a triangle.

h. This shape has two identical and parallel faces that can be the base.

Solution

a. Both

b. Both

c. Neither

d. Shape A

e. Shape B

f. Neither

g. Both

h. Shape A

5. **Problem 5 Statement**

Select **all** units that can be used for surface area.

a. square metres

b. feet

c. centimetres

d. cubic inches

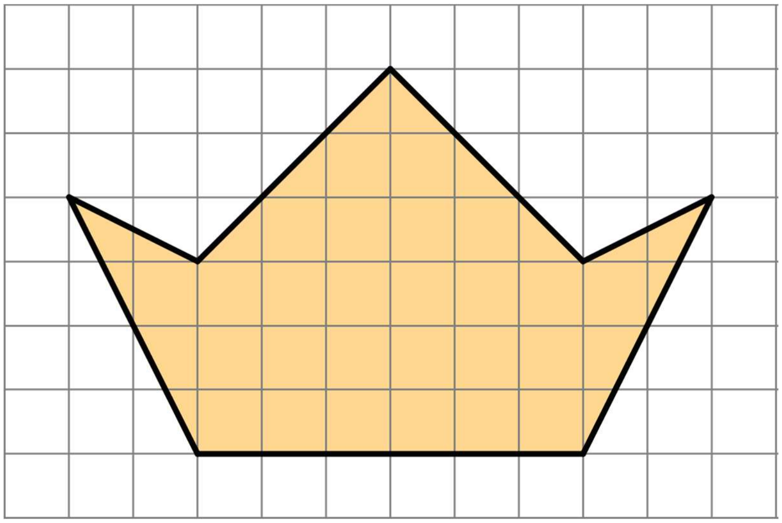
e. square inches

f. square feet

Solution ["A", "E", "F"]

6. **Problem 6 Statement**

Find the area of this polygon. Show your reasoning.



Solution

33 square units. Reasoning varies.