Lesson 16: Is a smartphone smart enough to go to the Moon?

Goals

- Use standard form to compare quantities in context, and describe (orally) how using standard form helps with making comparisons between very large and very small quantities.

Learning Targets

- I can use standard form to compare different amounts and answer questions about real-world situations.

Lesson Narrative

In this culminating lesson, students use standard form as a tool for making comparisons. Students compare old hardware to new hardware using various digital media as a form of measurement. For example, students compare floppy drives to modern technology by measuring how many floppy drives it would take to store a high-definition film. Students must identify the essential features of the questions and reason qualitatively and abstractly in order to answer them in context.

Addressing

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Instructional Routines

- Compare and Connect

- Discussion Supports

Required Materials

Copies of blackline master

Old Hardware, New Hardware

1966 Apollo Guidance Computer	Storage	75 kilobytes
	Processing Speed	2 megahertz
	Memory	4 kilobytes

2007 Smartphone	Storage	4 gigabytes
	Processing Speed	400 megahertz
	Memory	128 megabytes

1977 Desktop Computer	Storage	140 kilobytes
	Processing Speed	1 megahertz
	Memory	4 kilobytes

2016 Smartphone	Storage	32 gigabytes
	Processing Speed	4 processors, 2.2 gigahertz each
	Memory	3 gigabytes

2001 Desktop Computer	Storage	20 gigabytes
	Processing Speed	1.1 gigahertz
	Memory	128 megabytes

2016 Desktop Computer	Storage	1 terabyte
	Processing Speed	4 processors, 3 gigahertz each
	Memory	8 gigabytes

2007 Desktop Computer	Storage	500 gigabytes
	Processing Speed	2 processors, 2 gigahertz each
	Memory	4 gigabytes

Required Preparation

Print the Old Hardware, New Hardware blackline master. Prepare 1 copy for every 2 students.

Student Learning Goals

Let’s compare digital media and computer hardware using standard form.

16.1 Old Hardware, New Hardware

20 minutes

Students perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Students use standard form and choose units of appropriate size for measurements of very large or very small quantities.

As students work, look for those who use standard form to make their calculations and estimations easier. Consider asking them to share their work later.

Instructional Routines

- Compare and Connect

Launch

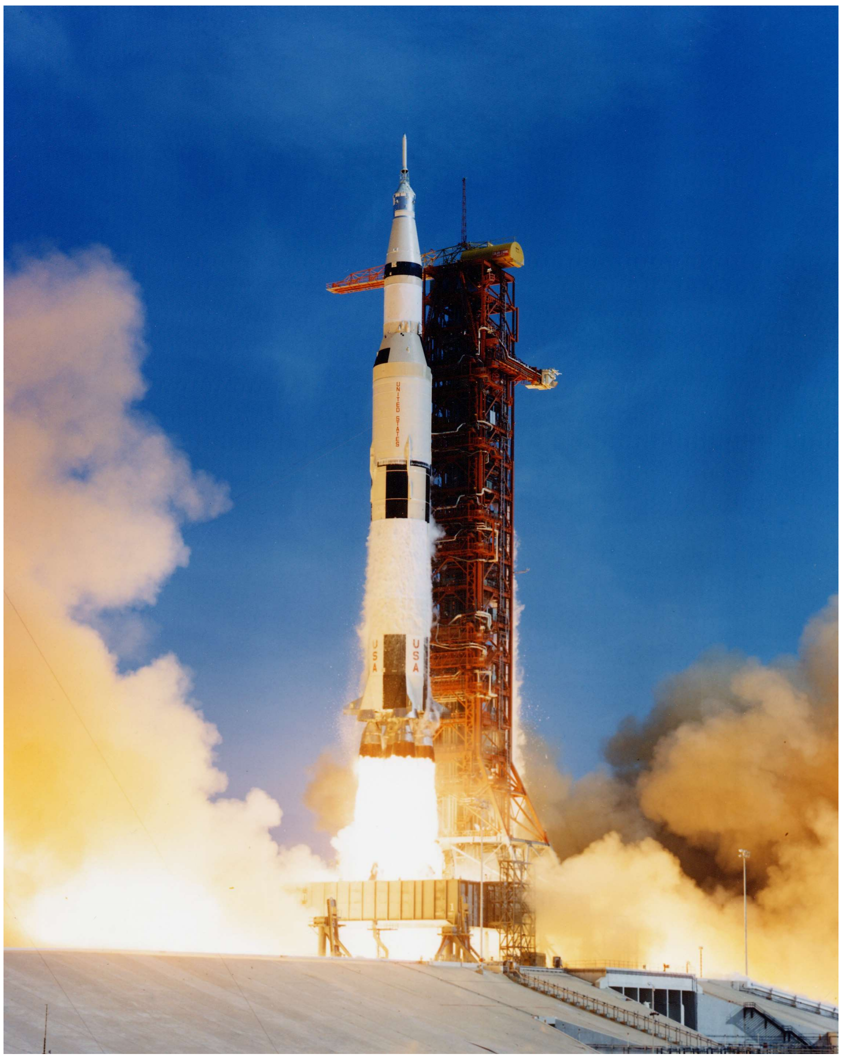
Arrange students in groups of 2. Display or distribute the included blackline master containing computer hardware specifications over time for all to see throughout the activity. Give students 15–20 minutes to work before a brief whole-class discussion.

Student Task Statement

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

Your teacher will give you advertisements for different devices from 1966 to 2016 (blackline masters). Choose one device and compare that device with the Apollo Guidance Computer. If you get stuck, consider using standard form to help you do your calculations.

For reference, storage is measured in bytes, processor speed is measured in hertz, and memory is measured in bytes. Kilo stands for 1 000, mega stands for 1 000 000, giga stands for 1 000 000 000, and tera stands for 1 000 000 000 000.



1. Which one can store more information? How many times more information?

2. Which one has a faster processor? How many times faster?

3. Which one has more memory? How many times more memory?

Student Response

1977 Desktop:

- Storage is $\frac{28}{15}$ times as much as Apollo
- Processing is $\frac{1}{2}$ as much as Apollo
- Memory is the same as Apollo

2001 Desktop:

- Storage is $2.\dot{6} \times 10^5$ times as much as Apollo
- Processing is 550 times as much as Apollo
- Memory is 32 000 times as much as Apollo

2007 Desktop:

- Storage is $6.\dot{6} \times 10^6$ times as much as Apollo
- Processing is 2 000 times as much as Apollo
- Memory is 10^6 times as much as Apollo

2007 Smartphone:

- Storage is $5.\dot{3} \times 10^4$ times as much as Apollo
- Processing is 200 times as much as Apollo
- Memory is 32 000 times as much as Apollo

2016 Smartphone:

- Storage is $4.2\dot{6} \times 10^5$ times as much as Apollo
- Processing is 4 400 times as much as Apollo
- Memory is 7.5×10^5 times as much as Apollo

2016 Desktop:

- Storage is $1.\dot{3} \times 10^7$ times as much as Apollo

- Processing is 6 000 times as much as Apollo
- Memory is 2×10^6 times as much as Apollo

Activity Synthesis

Select students who chose various devices to share their results. A key insight to take away would be how rapidly technology improves and how modern smartphones are much, much more sophisticated than the computer that put people on the Moon.

Speaking, Listening: Compare and Connect. Ask students to create a visual display of their strategy and result for comparing the Apollo Guidance Computer and the device they selected. Invite students to take a tour of the displays and identify "what is the same and what is different about each approach". Draw students' attention to the ways the values were compared using different strategies (e.g., using estimation, calculating differences using standard form versus expanded form). In this discussion, emphasise the mathematical language used to make sense of the different strategies to compare the values. These exchanges strengthen students' mathematical language use and reasoning when comparing large and small quantities.
Design Principle(s): Maximise meta-awareness

16.2 A Bit More on Bytes

25 minutes

Students use standard form as a tool to understand the relative scale of different units. They practise modelling skills by identifying essential elements of the problems and gathering relevant information before computing.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Instruct students to first read through the problems and decide on what information they need to solve each problem. Record relevant information for all students to see. Only record information when students have asked for it. Possible information students will ask for include:

- Mai's dad's computer holds 500 gigabytes of storage space.
- A kilobyte is 1 000 bytes, a megabyte is 1 000 000 bytes, and a gigabyte is 1 000 000 000 bytes.
- 1 character is roughly 1 byte.
- An emoji is roughly 4 bytes.
- A full-length, high-definition film is around 8 gigabytes and runs 2 hours.

- A person sleeps about 8 hours in a night.

Give 15–20 minutes of work time before a brief whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge of working with very large numbers. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

For each question, think about what information you would need to figure out an answer. Your teacher may provide some of the information you ask for. Give your answers using standard form.

1. Mai found an 80's computer magazine with an advertisement for a machine with hundreds of kilobytes of storage! Mai was curious and asked, "How many kilobytes would my dad's new 2016 computer hold?"

2. The old magazine showed another ad for a 750-kilobyte floppy disk, a device used in the past to store data. How many gigabytes is this?

3. Mai and her friends are actively involved on a social media service that limits each message to 140 characters. She wonders about how the size of a message compares to other media.

   Estimate how many messages it would take for Mai to fill up a floppy disk with her 140-character messages. Explain or show your reasoning.

4. Estimate how many messages it would take for Mai to fill a floppy disk with messages that only use emojis (each message being 140 emojis). Explain or show your reasoning.

5. Mai likes to go to the movies with her friends and knows that a high-definition film takes up a lot of storage space on a computer.

   Estimate how many floppy disks it would take to store a high-definition movie. Explain or show your reasoning.

6. How many seconds of a high-definition movie would one floppy disk be able to hold?

7. If you fall asleep watching a movie streaming service and it streams movies all night while you sleep, how many floppy disks of information would that be?

Student Response

1. 500 million kilobytes. Mai's dad's computer can hold 500 gigabytes, which is 500×10^9 bytes. A kilobyte is 10^3 bytes, so his computer holds $\frac{500\times10^9}{10^3}$ or 500×10^6 kilobytes.

2. 0.00075 gigabytes. A floppy drive holds 750 kilobytes, which is 750×10^3 bytes. As a fraction of a gigabyte (10^9 bytes), divide $\frac{750\times10^3}{10^9} = 750 \times 10^{-6} = 0.00075$.

3. About 5 000 messages, because: $\frac{7.5\times10^5 \text{ bytes per floppy}}{1.4\times10^2 \text{ bytes per message}} \approx 5 \times 10^3$ messages.

4. About 1 250 messages. Emojis take up 4 times as much storage as a character, so there will be 4 times fewer messages. $\frac{5\,000}{4} = 1250$.

5. About 10 000 floppy disks, because: $\frac{8\times10^9 \text{ bytes per movie}}{7.5\times10^5 \text{ bytes per floppy}} \approx 1 \times 10^4$ floppy disks.

6. 0.72 seconds. A 2-hour film is 120 minutes, which is 7 200 seconds. Therefore, $\frac{7.2\times10^3 \text{ seconds per movie}}{10^4 \text{ floppy disks per movie}} = 7.2 \times 10^{-1} = 0.72$ seconds.

7. About 40 000 floppy disks. Eight hours is equivalent to 4 movies, which is 40 000 floppy disks.

Are You Ready for More?

Humans tend to work with numbers using powers of 10, but computers work with numbers using powers of 2. A "binary kilobyte" is 1 024 bytes instead of 1 000, because $1\,024 = 2^{10}$. Similarly, a "binary megabyte" is 1 024 binary kilobytes, and a "binary gigabyte" is 1 024 binary megabytes.

1. Which is bigger, a binary gigabyte or a regular gigabyte? How many more bytes is it?

2. Which is bigger, a binary terabyte or a regular terabyte? How many more bytes is it?

Student Response

1. A binary gigabyte is about 74 million more bytes (74 megabytes) than a regular gigabyte. A binary gigabyte is equal to 1 024 binary megabytes, which is equal to 1 024 binary kilobytes, which is equal to 1 024 bytes. So, a binary gigabyte is $1\,024^3$ (or 1 073 741 824) bytes.

2. A binary terabyte is about 100 billion more bytes (100 gigabytes) than a regular terabyte. A binary terabyte is 1 024 times a binary gigabyte, so a binary terabyte would be $1\,024^4$ (or 1 099 511 627 776) bytes.

Activity Synthesis

In a whole-class discussion, ask students what they might have found surprising or interesting when comparing different digital media and different hardware. If time permits, discuss how standard form helps to make those comparisons.

Speaking: Discussion Supports. As students explain what they noticed about the differences in digital media and hardware, press for details in students' ideas by requesting that students challenge an idea, elaborate on an idea, or make explicit their process for calculating or comparing the values. Revoice student ideas to model mathematical language use in order to clarify, apply appropriate language, and involve more students. This will help students produce and make sense of the language needed to communicate their own ideas.

Design Principle(s): Support sense-making; Optimise output (for explanation)