Lesson 12: Dividing decimals by whole numbers

Goals

- Compare and contrast (orally and using other representations) division problems with whole-number and decimal dividends
- Divide decimals by whole numbers, and explain the reasoning (orally and using other representations).
- Generalise (orally and in writing) that multiplying both the dividend and the divisor by the same factor does not change the quotient.

Learning Targets

- I can divide a decimal by a whole number.
- I can explain the division of a decimal by a whole number in terms of equal-sized groups.
- I know how multiplying both the dividend and the divisor by the same factor affects the quotient.

Lesson Narrative

This lesson serves two purposes. The first is to show that we can divide a decimal by a whole number the same way we divide two whole numbers. Students first represent a decimal dividend with base-ten diagrams. They see that, just like the units representing powers of 10, those for powers of 0.1 can also be divided into groups. They then divide using another method—partial quotients or long division—and notice that the principle of placing base-ten units into equal-size groups is likewise applicable.

The second is to uncover the idea that the value of a quotient does not change if both the divisor and dividend are multiplied by the same factor. Students begin exploring this idea in problems where the factor is a multiple of 10 (e.g. $8 \div 1 = 80 \div 10$). This work prepares students to divide two decimals in the next lesson.

Building On

- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- Apply and extend previous understandings of arithmetic to algebraic expressions.

- Fluently divide multi-digit numbers using the standard algorithm.

Instructional Routines

- Collect and Display
- Compare and Connect
- Discussion Supports
- Number Talk

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's divide decimals by whole numbers.

12.1 Number Talk: Dividing by 4

Warm Up: 5 minutes

The purpose of this number talk is to help students use the structure of base-ten numbers and the distributive property to solve a division problem involving decimals.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find each quotient mentally.

$80 \div 4$

$12 \div 4$

$1.2 \div 4$

$81.2 \div 4$

Student Response

- 20
- 3
- 0.3
- 20.3

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Highlight the use of the distributive property in finding $81.2 \div 4$. Students should recognise that since $81.2 = 80 + 1.2$, we have $81.2 \div 4 = (80 \div 4) + (1.2 \div 4)$. To make this clear, consider explaining that the division could be equivalently represented by $81.2 \times \frac{1}{4} = (80 + 1.2) \times \frac{1}{4} = (80 \times \frac{1}{4}) + (1.2 \times \frac{1}{4})$.

Speaking: Discussion Supports. Provide sentence frames to support students with explaining their strategies. For example, "I noticed that ______, so I ______." or "First, I ________ because ________." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the full group. Rehearsing provides opportunities to clarify their thinking.
Design Principle(s): Optimise output (for explanation)

12.2 Using Diagrams to Represent Division

15 minutes

Students have learned several effective methods to divide a whole number by a whole number, including cases when there is a remainder. The goal of this task is to introduce a method for dividing a decimal number by a whole number. Students notice that the steps in the division process are the same as when dividing a whole number by a whole number, whether the division is done with base-ten diagrams, as in this task, or using partial quotients or the division algorithm as in future tasks. Here, students need to think even more carefully about place value.

Throughout this activity, students rely on their understanding of equivalent expressions to interpret the ungrouping in Elena's process. For example, to ungroup a one into ten tenths means going between the expressions 1 and $0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1$.

Instructional Routines

- Collect and Display

Launch

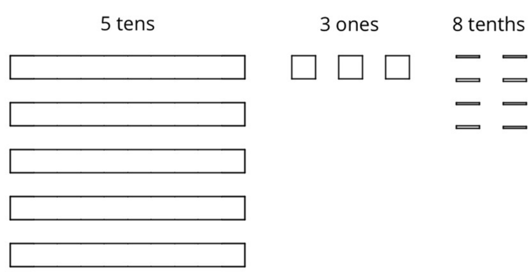
Give students 1–2 minutes of quiet think time for students to analyse Elena's work. Pause and discuss with the whole class. Select a couple of students to share their analyses of what Elena had done to divide a decimal by a whole number. Then, give students 8–9 minutes to complete the questions and follow with a whole-class discussion.

Anticipated Misconceptions

Some students may stop dividing when they reach a remainder rather than ungrouping the remainder into smaller units. Remind them that they can continue to divide the remainder by ungrouping and to refer to Elena's worked-out example or those from earlier lessons, if needed.

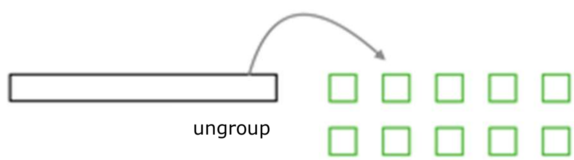
Student Task Statement

To find $53.8 \div 4$ using diagrams, Elena began by representing 53.8.



She placed 1 ten into each group, ungrouped the remaining 1 ten into 10 ones, and went on distributing the units.

This diagram shows Elena's initial placement of the units and the ungrouping of 1 ten.



1. Complete the diagram by continuing the division process. How would you use the available units to make 4 equal groups?

   As the units get placed into groups, show them accordingly and cross out those pieces from the bottom. If you ungroup a unit, draw the resulting pieces.

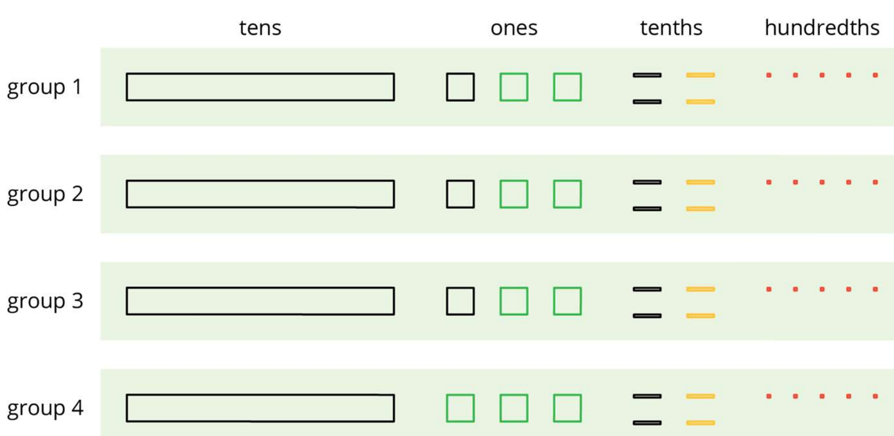
2. What value did you find for 53.8 ÷ 4? Be prepared to explain your reasoning.

3. Use long division to find 53.8 ÷ 4. Check your answer by multiplying it by the divisor 4.

4. Use long division to find 77.4 ÷ 5. If you get stuck, you can draw diagrams or use another method.

Student Response

1.



2. 13.45. Each group has 1 ten, 3 ones, 4 tenths, and 5 hundredths.

3. $53.8 \div 4 = 13.45$ and $(13.45) \times 4 = 53.8$

```
      1 3 . 4 5
    ___________
4  /  5 3 . 8
   - 4
   ----
     1 3
   - 1 2
   ---------
       1   8
     - 1   6
     ---------
           2 0
         - 2 0
         -------
             0
```

4. 15.48. Sample reasonings:

–

```
      1 5 . 4 8
    ___________
5  /  7 7 . 4
   - 5
   ----
     2 7
   - 2 5
   ---------
       2   4
     - 2   0
     ---------
           4 0
         - 4 0
         -------
             0
```

– There are five groups of 15 in 77 with 2 left over. There are 0.4 groups of 5 in 2.4 with 0.4 remaining. There are 0.08 groups of 5 in 0.40, so 77.4 can be divided into five equal groups of 15.48.

Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

A group of 4 craftsmen are paid 1 of each jewel. If they split the jewels evenly amongst themselves, which jewels does each craftsman get?

Student Response

Each craftsmen gets 1 orange, 1 green, and 1 indigo jewel. Together the four craftsmen must share 1 violet jewel.

Activity Synthesis

Ask a student or two to display and explain their work. Ask if others performed the division the same way and if there are disagreements.

Then, focus the discussion on the connections between a division problem with a whole-number dividend (such as 62 ÷ 5) and that with a decimal dividend (such as 53.8 ÷ 4). Discuss:

- How is the division problem 53.8 ÷ 4 similar to 62 ÷ 5 from a previous lesson?
  - In both problems, when we get to the final place value (tenths for 53.8 ÷ 4 and one for 62 ÷ 5), there is still a remainder.
  - In both problems, to complete the division and find the quotient we need to introduce a new place value (hundredths for 53.8 ÷ 4, and tenths for 62 ÷ 5).
  - We have to ungroup at every step in both division problems.
- How is the division problem 53.8 ÷ 4 different to 62 ÷ 5 from a previous lesson?
  - There is already a decimal in 53.8: we had to write the decimal point for 62 ÷ 5.
  - The quotient 53.8 ÷ 4 goes to the hundredths place, so there is an extra step and an additional place value.

If we were to rewrite 62 ÷ 5 as 62.0 ÷ 5 (which is what is needed in order to complete the division), then the two division problems look similar. The biggest difference between 53.8 ÷ 4 and 62 ÷ 5 is that the former problem has an answer in the hundredths while the answer to the latter only has tenths.

Representing, Speaking, Listening: Collect and Display. As students discuss how the division problem 53.8 ÷ 4 is related to 62 ÷ 5 from a previous lesson, create a two-column display with “similarities” and “differences” for the headers. Circulate through the groups and record student language in the appropriate column. Look for phrases such as “whole-

number dividend," "decimal dividend," and "place value." This will help students compare and contrast different types of division problems by recognising the structure of the numbers.
Design Principle(s): Support sense-making; Maximise meta-awareness

12.3 Dividends and Divisors

15 minutes

In this activity, students study some carefully chosen quotients where the dividends are decimal numbers. The key goal here is to notice that there are other quotients of whole numbers that are equivalent to these quotients of decimals. In other words, when the dividend is a terminating decimal number, we can find an equivalent quotient whose dividend is a whole number. In combination with the previous task, this gives students the tools they need to divide a decimal number by a decimal number.

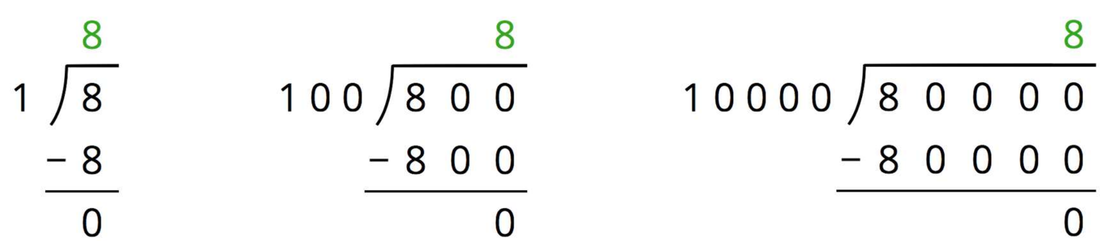
Students notice that, when working with a fraction, multiplying the numerator and denominator in a fraction by 10 does not change the value of the fraction. They use this insight to develop a way to divide decimal numbers in subsequent activities. This work develops students' understanding of equivalent expressions by emphasising that, for example, $8 \div 1 = (8 \times 10) \div (1 \times 10)$. Eventually, students will recognise the equivalence of $8 \div 1$ to statements such as $(8 \times y) \div (1 \times y)$. However, in this activity, students only examine situations where the dividend and divisor are multiplied by powers of 10.

Instructional Routines

- Compare and Connect

Launch

Display the following image of division calculations for all to see.



Ask students what quotient each calculation shows. ($8 \div 1$, $800 \div 100$, and $8\,000 \div 10\,000$. Give students 1–2 minutes to notice and wonder about the dividends, divisors, and quotients in the three calculations. Ask them to give a signal when they have at least one observation and one question. If needed, remind students that the 8, 800, and 80 000 are the dividends and the 1, 100, and 10 000 are the divisors.

Invite a few students to share their observations and questions. They are likely to notice:

- Each calculation shows that the value of the corresponding quotient is 8; it is the same for all three calculations.

- All calculations have an 8 in the dividend and a 1 in the divisor.
- All calculations take one step to solve.
- Each divisor is 100 times the one to the left of it.
- Each dividend is 100 times the one to the left of it.
- Each dividend and each divisor have 2 more zeros than in the calculation immediately to their left.

They may wonder:

- Why are the quotients equal even though the divisors and dividends are different?
- Would 80 ÷ 10 and 8 000 ÷ 10 also produce a quotient of 8?
- Are there other division expressions with an 8 in the dividend and a 1 in the divisor and no other digits but zeros that would also produce a quotient of 8?

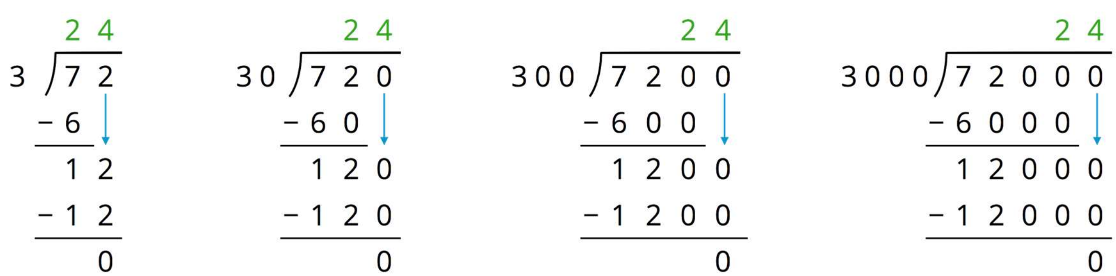
Without answering their questions, tell students that they'll analyse the sizes of dividends and divisors more closely to help them reason about quotients of numbers in base ten.

Give students 7–8 minutes of quiet work time to answer the four questions followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisation and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organisation; Attention

Student Task Statement

Analyse the dividends, divisors, and quotients in the calculations, and then answer the questions.



1. Complete each sentence. In the calculations shown:

   – Each dividend is ______ times the dividend to the left of it.

   – Each divisor is ______ times the divisor to the left of it.

– Each quotient is __________________ the quotient to the left of it.

2. Suppose we are writing a calculation to the right of 72 000 ÷ 3 000. Which expression has a quotient of 24? Be prepared to explain your reasoning.

   a. 72 000 ÷ 30 000

   b. 720 000 ÷ 300 000

   c. 720 000 ÷ 30 000

   d. 720 000 ÷ 3 000

3. Suppose we are writing a calculation to the left of 72 ÷ 3. Write an expression that would also give a quotient of 24. Be prepared to explain your reasoning.

4. Decide which of the following expressions would have the same value as 250 ÷ 10. Be prepared to share your reasoning.

   a. 250 ÷ 0.1

   b. 25 ÷ 1

   c. 2.5 ÷ 1

   d. 2.5 ÷ 0.1

   e. 2 500 ÷ 100

   f. 0.25 ÷ 0.01

Student Response

1.

   – 10 times

   – 10 times

   – “equal to” or “the same size as”

2. c. 720 000 ÷ 30 000

3. 7.2 ÷ 0.3

4. B, D, E, and F

Activity Synthesis

Ask students to write a reflection using the following prompt:

What happens to the value of the quotient when both the divisor and the dividend are multiplied by the same power of 10? Use examples to show your thinking.

The goal of this discussion is to make sure students understand that the value of a quotient does not change when both the divisor and the dividend are multiplied by the same power of ten. Ask students to explain why $\frac{25}{20} = \frac{250}{200}$. Possible responses include:

- Both the numerator and denominator of $\frac{250}{200}$ have a factor of 10, so the fraction can be written as $\frac{25}{20}$.
- Both fractions are equivalent to $\frac{5}{4}$.
- Dividing 250 by 200 and 25 by 20 both give a value of 1.25.

Tell students that their observations here will help them divide decimals in upcoming activities.

Speaking, Listening: Compare and Connect. After students have answered the four questions in this activity, ask them to work in groups of 2–4 and identify what is similar and what is different about the approaches they used in analysing the dividends, divisors, and quotients for the last question. Lead a whole-class discussion that draws students attention to what worked or did not work well when deciding which expressions have the same value as 250 ÷ 10 (i.e., using powers of 10, long division, ungrouping, etc). Look for opportunities to highlight mathematical language and reasoning involving multiplying or dividing by powers of 10. This will foster students' meta-awareness and support constructive conversations as they develop understanding of equivalent quotients.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

In this lesson, we saw that we can divide a decimal by a whole number the same way we divide two whole numbers. We used base-ten diagrams to show how this is true.

- How do we use base-ten diagrams to show division of a decimal by a whole number, for example, 0.8 ÷ 5? (We can draw eight of a type of figures to represent 8 tenths and distribute them into 5 groups.)
- What do we do with remainders? For example, in the case of 0.8 ÷ 5, how do we deal with the remainder of 3 tenths? (We can ungroup each tenth into 10 hundredths and distribute the 30 hundredths into 5 equal groups. Each group would have 6 hundredths.)
- How do we know what the quotient of 0.8 ÷ 5 is? (It is the value of each group: 1 tenth and 6 hundredths, or 0.16.)

We also thought about division of decimals a different way—by multiplying both the dividend and divisor by the same power of 10, which gives us an equivalent division expression.

- What are some division expressions that are equivalent to $30 \div 0.1$? ($300 \div 1$; $3\,000 \div 10$)
- What happens to the value of a quotient when both the divisor and the dividend are multiplied by the same power of 10? (The value does not change.)

12.4 The Same Quotient

Cool Down: 5 minutes

Student Task Statement

1. Use long division to find the value of $43.5 \div 3$. If you get stuck, you can draw base-ten diagrams. Be sure to say what each type of figure represents in your diagrams.

2. Explain why all of these expressions have the same value.

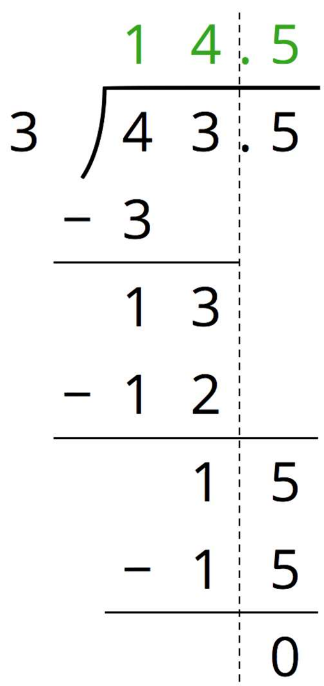
   $100 \div 5$

   $10 \div 0.5$

   $1 \div 0.05$

Student Response

1. 14.5



2. Answers vary. Sample reasonings:

   – Each dividend is 20 times the divisor.

   – Each dividend is 10 times the next dividend. Each divisor is 10 times the next divisor. The quotient of each pair of numbers is therefore the same.

Student Lesson Summary

We know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because:

- The numerator and denominator of $\frac{60}{40}$ are each 10 times those of $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 600 divided by 400 is 1.5, and 60 divided by 40 is also 1.5.

Just like fractions, division expressions can be equivalent. For example, the expressions $540 \div 90$ and $5\,400 \div 900$ are both equivalent to $54 \div 9$ because:

- They all have a quotient of 6.
- The dividend and the divisor in $540 \div 90$ are each 10 times the dividend and divisor in $54 \div 9$. Those in $5,400 \div 900$ are each 100 times the dividend and divisor in $54 \div 9$. In both cases, the quotient does not change.

This means that an expression such as $5.4 \div 0.9$ also has the same value as $54 \div 9$. Both the dividend and divisor of $5.4 \div 0.9$ are $\frac{1}{10}$ of those in $54 \div 9$.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, 1 000, etc.) can be particularly useful for dividing decimals, as we will see in an upcoming lesson.

Lesson 12 Practice Problems

1. **Problem 1 Statement**

Here is a diagram representing a base-ten number. The large rectangle represents a unit that is 10 times the value of the square. The square represents a unit that is 10 times the value of the small rectangle.

Here is a diagram showing the number being divided into 5 equal groups.

a. If a large rectangle represents 1 000, what division problem did the second diagram show? What is its answer?

b. If a large rectangle represents 100, what division problem did the second diagram show? What is its answer?

c. If a large rectangle represents 10, what division problem did the second diagram show? What is its answer?

Solution

a. 1 320 ÷ 5. The answer is 264.

b. 132 ÷ 5. The answer is 26.4.

c. 13.2 ÷ 5. The answer is 2.64.

2. **Problem 2 Statement**

a. Explain why all of these expressions have the same value.

4.5 ÷ 0.09

45 ÷ 0.9

450 ÷ 9

4 500 ÷ 90

b. What is the common value?

Solution

a. Answers vary. Sample response: The expressions all have the same value because the numerator and denominator are *both* being multiplied by 10 to get from one expression to the one above it. This does not affect the quotient (because dividing the 10 in the numerator by the 10 in the denominator results in 1).

b. 50

3. **Problem 3 Statement**

Use long division to find each quotient.

a. $7.89 \div 2$

a. $39.54 \div 3$

a. $0.176 \div 5$

Solution

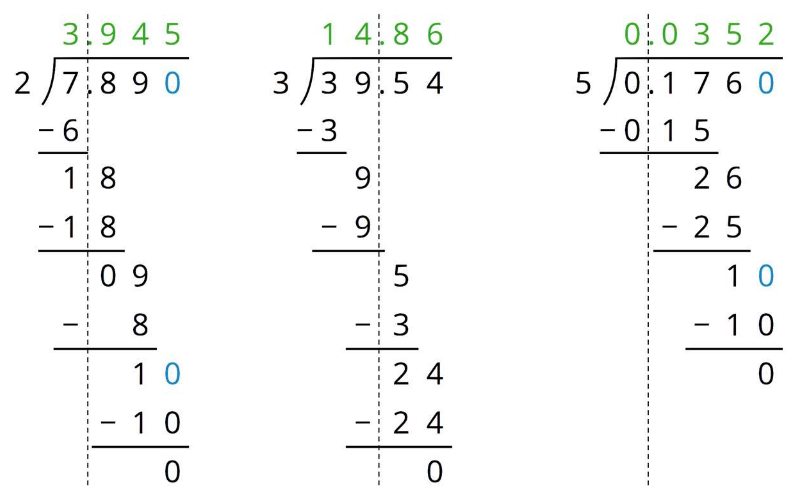
a. 3.945

b. 13.18

c. 0.0352

Possible calculations:

a. b. c.



4. **Problem 4 Statement**

Four students set up a lemonade stand. At the end of the day, their profit is £17.52. How much money do they each have when the profit is split equally? Show or explain your reasoning.

Solution

£4.38. Answers vary. Sample explanation: Four people are sharing £17.52 equally, so each person gets £17.52 ÷ 4. Each person can be given £4, and then £1.52 remains. Each person can be given £0.30, and then £0.32 remains. So they each get £0.08 more. That means each person gets a total of $4 + 0.30 + 0.08$ or £4.38.

5. **Problem 5 Statement**

   a. A standard sheet of paper in the United States is 11 inches long and 8.5 inches wide. Each inch is 2.54 centimetres. How long and wide is a standard sheet of paper in centimetres?

   b. A standard sheet of paper in Europe is 21.0 cm wide and 29.7 cm long. Which has the greater area, the standard sheet of paper in the United States or the standard sheet of paper in Europe? Explain your reasoning.

Solution

   a. 27.94 cm by 21.59 cm

   b. The European paper. Reasoning varies. Sample reasoning: The difference in the length is substantially larger than the difference in width, so the European paper probably has a larger area. Calculating shows the standard European paper is 623.7 sq cm while the standard United States paper is 603.2246 sq cm.