Lesson 12: Using mean and MAD to make comparisons

Goals

- Compare (orally and in writing) the means and mean absolute deviations of different distributions, specifically those with the same MAD but different means.
- Interpret the mean and mean absolute deviation (MAD) in the context of the data.

Learning Targets

- I can say what the MAD tells us in a given context.
- I can use means and MADs to compare groups.

Lesson Narrative

In this lesson, students continue to develop their understanding of the mean and MAD as measures of centre and spread as well as interpret these values in context. They practise calculating the mean and the MAD for distributions; compare distributions with the same MAD but different means; and interpret the mean and MAD in the context of the data.

Addressing

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- Giving quantitative measures of centre (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- Relating the choice of measures of centre and variability to the shape of the data distribution and the context in which the data were gathered.

Building Towards

- Recognise that a measure of centre for a numerical data set summarises all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Instructional Routines

- Discussion Supports
- Notice and Wonder
- Number Talk

Student Learning Goals

Let's use mean and MAD to describe and compare distributions.

12.1 Number Talk: Decimal Division

Warm Up: 5 minutes

This number talk helps students to review strategies for dividing a decimal number by a whole number and to build their fluency. It also prepares them to calculate mean and MAD more efficiently. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the task. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$42 \div 12$

$2.4 \div 12$

$44.4 \div 12$

$46.8 \div 12$

Student Response

- 3.5. Possible strategy: $36 \div 12 + 6 \div 12 = 3.5$
- 0.2. Possible strategy: $2.4 \times 10 \div 12 = 2$ and $2 \div 10 = 0.2$
- 3.7. Possible strategy: $42 \div 12 + 2.4 \div 12 = 3.7$
- 3.9. Possible strategy: $42 \div 12 + 2.4 \div 12 + 2.4 \div 12 = 3.9$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

12.2 Which Player Would You Choose?

20 minutes

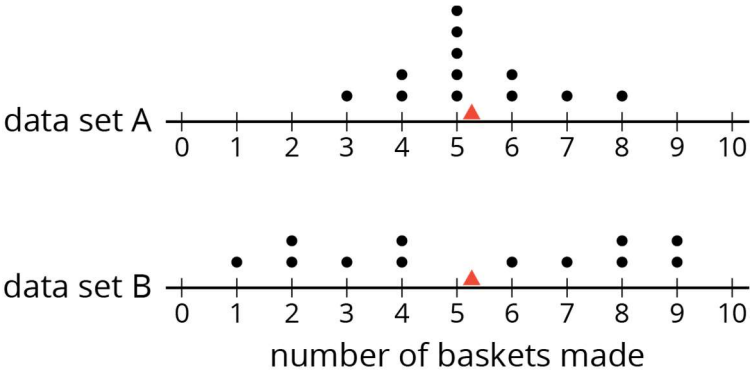
This activity allows students to practice calculating MAD and build a better understanding of what it tells us. Students continue to compare data sets with the same mean but different MADs and interpret what these differences imply in the context of the situation.

Instructional Routines

- Discussion Supports
- Notice and Wonder

Launch

Arrange students in groups of 3–4. Before students read the task statements, display the two dot plots in the task for all to see. Give students up to 1 minute to study the dot plots and share with their group what they notice and wonder about the plots.



Next, select a few students to share their observations and questions; it is not necessary to confirm or correct students' observations or answer their questions at this point. If no one noticed or wondered about what variable the data sets show, ask students what they think

the context for the data sets might be or what quantities they show. Explain to students that they will find more information in the task statement to help them compare and interpret the dot plots.

Give students 3–4 minutes of quiet work time to complete the first set of questions, and then 8–10 minutes to complete the second set with their group. Allow at least a few minutes for a whole-class discussion.

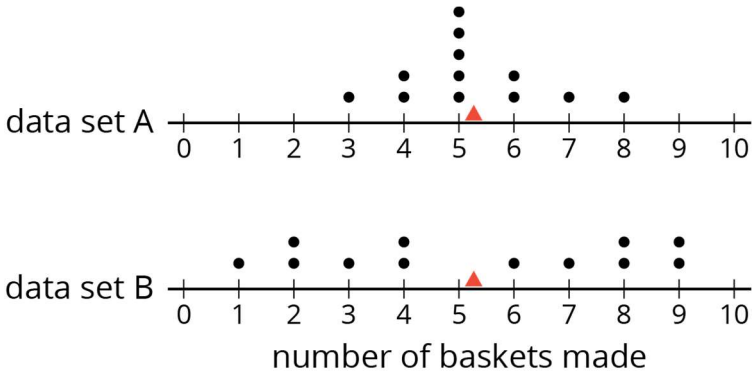
Representation: Internalise Comprehension. Activate or supply background knowledge about comparing and interpreting data sets in dot plots. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Andre and Noah joined Elena, Jada, and Lin in recording their basketball scores. They all recorded their scores in the same way: the number of baskets made out of 10 attempts. Each collected 12 data points.

   - Andre's mean number of baskets was 5.25, and his MAD was 2.6.
   - Noah's mean number of baskets was also 5.25, but his MAD was 1.

   Here are two dot plots that represent the two data sets. The triangle indicates the location of the mean.



   a. Without calculating, decide which dot plot represents Andre's data and which represents Noah's. Explain how you know.

   b. If you were the captain of a basketball team and could use one more player on your team, would you choose Andre or Noah? Explain your reasoning.

2. A year 9 student decided to join Andre and Noah and kept track of his scores. His data set is shown here. The mean number of baskets he made is 6.

year 9 student	6	5	4	7	6	5	7	8	5	6	5	8
distance from 6												

a. Calculate the MAD. Show your reasoning.

b. Draw a dot plot to represent his data and mark the location of the mean with a triangle (Δ).

c. Compare the year 9 student's mean and MAD to Noah's mean and MAD. What do you notice?

d. Compare their dot plots. What do you notice about the distributions?

e. What can you say about the two players' shooting accuracy and consistency?

Student Response

1.

a. Dot plot A represents Noah's data, and dot plot B represents Andre's data. Sample explanation: I know because Andre's data has a larger MAD than Noah's, so the data would be more spread out than Noah's.

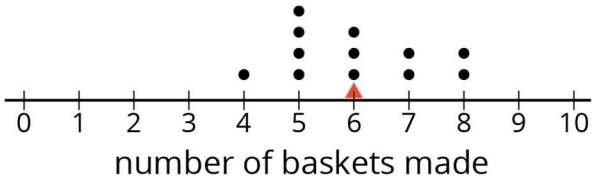
b. Answers vary. Sample response: I would choose Noah. Both players made, on average, about 5 out of 10 baskets, but Noah is more consistent, so he's less likely to miss more than 6 out of 10 shots. Andre scored some high points a few times, but he also scored some very low ones.

2.

year 9 student	6	5	4	7	6	5	7	8	5	6	5	8
distance from 6	0	1	2	1	0	1	1	2	1	0	1	2

a. MAD: $\frac{0+1+2+1+0+1+1+2+1+0+1+2}{12} = 1$

b. Dot plot for the year 9 student:



c. Answers vary. Sample response: The year 9 student has a mean of 6, which is larger than Noah's mean of 5.25. Both Noah and the year 9 student have the same value for their MAD. Although their means are different, their data is similarly spread around their means.

d. Answers vary. Sample response: Both dot plots show roughly the same spread; the points are clustered toward the middle and are within 2–3 units away from the mean.

e. Answers vary. Sample response: The year 9 student has a higher mean, so on average, he shoots more accurately than Noah does. The same MAD value suggests that the two players are equally consistent. Each person typically scores within 1 point of their mean, rather than scoring some very high scores and some very low ones.

Are You Ready for More?

Invent a data set with a mean of 7 and a MAD of 1.

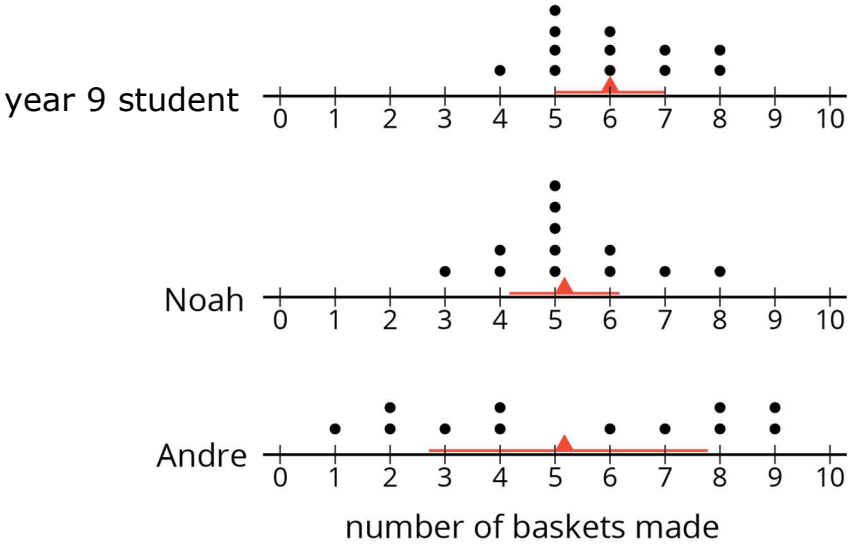
Student Response

Answers vary. Sample response: take the year 9 student's results and add 1 to each value.

Activity Synthesis

Select a couple of students to share their responses to the first set of questions about how they matched the dot plots to the players (Andre and Noah) and how they knew.

Then, display a completed table and the MAD for the second set of questions. Give students a moment to check their work. To facilitate discussion, help students connect MAD and the spread of data, and enable them to make comparison, consider displaying all three dot plots at the same scale and using a line segment to represent the MAD on each dot plot, as shown here.



Invite a few students to share their observations about how the means and MADs of Noah and the year 9 student compare. Discuss:

- "How are the distributions of points related to the mean? How are they related to the MAD?"
- "Which might be more desirable for a basketball team: a lower mean or a higher mean? Why?"
- "Which might be more desirable: a lower MAD or a higher MAD? Why?"
- "Of the three students, which one would you want on your team? Why?"

Expect students to choose different players to be on their team, but be sure they support their preferences with a reasonable explanation. Students should walk away understanding that, in this context, a higher MAD indicates more variability and less consistency in the number of shots made.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language and build a better understanding of MAD.
Design Principle(s): Support sense-making

12.3 Swimmers Over the Years

10 minutes

In this activity, students continue to practise interpreting the mean and the MAD and to use them to answer statistical questions. A new context is introduced, but students should continue to consider both the centre and variability of the distribution as ways of thinking about what is typical for a set of data and how consistent the data tends to be.

Instructional Routines

- Discussion Supports

Launch

Give students 5–7 minutes of quiet work time. Ask students to consider drawing a triangle and a line segment on each dot plot in the last question to represent the mean and MAD for each data set (as was done in an earlier lesson).

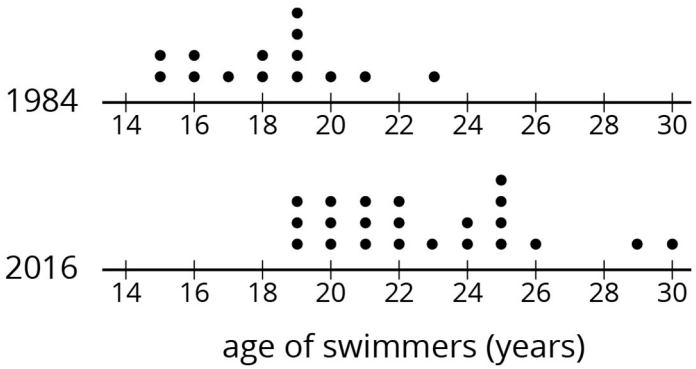
Student Task Statement

In 1984, the mean age of swimmers on the U.S. women's swimming team was 18.2 years and the MAD was 2.2 years. In 2016, the mean age of the swimmers was 22.8 years, and the MAD was 3 years.

1. How has the mean age of the women on the U.S. swimming team changed from 1984 to 2016? Explain your reasoning.

2. Are the swimmers on the 1984 team closer in age to one another than the swimmers on the 2016 team are to one another? Explain your reasoning.

3. Here are dot plots showing the ages of the women on the U.S. swimming team in 1984 and in 2016. Use them to make two other comments about how the women's swimming team has changed over the years.



Student Response

1. Answers vary. Sample response: The mean age of the women's swimming team has increased by about 4.6 years over the past three decades, from 18.2 to 22.8 years old. The swimmers representing the U.S. in 2016 are older, on average, than those in 1984.

2. Yes. Explanations vary. Sample response: In 2016, the mean distance from the mean age was 3 years. In 1984, the mean absolute deviation was 2.2 years, which means that the swimmers were closer to one another in age.

3. Answers vary. Students could comment on the number of observations, the range, shape, or distribution of data, or other features of the dot plots. Sample responses:

- The 2016 team was much older than the 1984 team. In 2016, about half the swimmers were 22 or younger, while in 1984, all but one swimmer was 22 or younger.
- The youngest swimmers in 2016 were 19 years, 4 years older than the youngest swimmers in 1984.
- The oldest swimmer in 2016 was 7 years older than the oldest swimmer in 1984.

Activity Synthesis

Have a student display dot plots with the means and MADs marked on them. Invite several students to share their comments about how the composition of the swimming team has changed over the three decades. Some discussion questions:

- "What can you say about the size of the team? Has it changed?"

- "Overall, has the team gotten older, younger, or stayed about the same? How do you know?"
- "Has the age of the youngest swimmer changed? What about the age of the oldest swimmer?"
- "Has the team become more diverse in ages, in general? Or have the swimmers become more alike in their age? How do you know?"

Students should recognise that the higher mean and MAD for the 2016 swimming team tell us that the team, on the whole, has gotten older and more diverse in ages. In 1984, 18.2 years was a typical age for the swimmers. A typical age for the 2016 swimmers was 22.8 years and there was a wider range of ages represented.

Speaking, Listening: Discussion Supports. Use this routine to support students as they produce statements about interpreting the mean and the MAD. Display sentence frames for students to use such as: "I know the mean age changed ___ because….”; "Over the three decades, the _____ of the swimming team has changed by _____,”; "I know the swimmers' ages in the year ____ are closer to one another because….” This will help students to explain their interpretations of the data sets.
Design Principle(s): Support sense-making; Optimise output (for explanation)

Lesson Synthesis

In this lesson, we look at what different means and MADs tell us in situations.

- "Suppose the mean height of the students in a class is 60 inches and the MAD is 2.5 inches. How do the mean and the MAD tell us about what is typical for the students' heights?"
- "How do two distributions compare if they have the same means but different MADs?" (Same centre, different variability or spread.)
- "How do two distributions compare if they have the same MADs but different means?" (Same variability or spread, different centres.)

12.4 Travel Times Across the World

Cool Down: 5 minutes

Student Task Statement

Ten year 7 students in five different countries were asked about their travel times to school. Their responses were organised into five data sets. The mean and MAD of each data set is shown in the table.

	mean (minutes)	MAD (minutes)
United States	9	4.2

Australia	18.1	7.9
South Africa	23.5	16.2
Canada	11	8
New Zealand	12.3	5.5

1. Which group of students has the greatest variability in their travel times? Explain your reasoning.

    a. The mean of the data set for New Zealand is close to that of Canada. What does this tell us about the travel times of students in those two data sets?

    b. The MAD of the data set for New Zealand is quite different than that of Canada. What does this tell us about the travel times of students in those two data sets?

2. The data sets for Australia and Canada have very different means (18.1 and 11 minutes) but very similar MADs. What can you say about the travel times of the students in those two data sets?

Student Response

1. South Africa, because it has the largest MAD.

    Answers vary. Sample responses:

    a. The mean travel times for the students in New Zealand and Canada are similar. The students in New Zealand travel only about 1 minute longer than those in Canada.

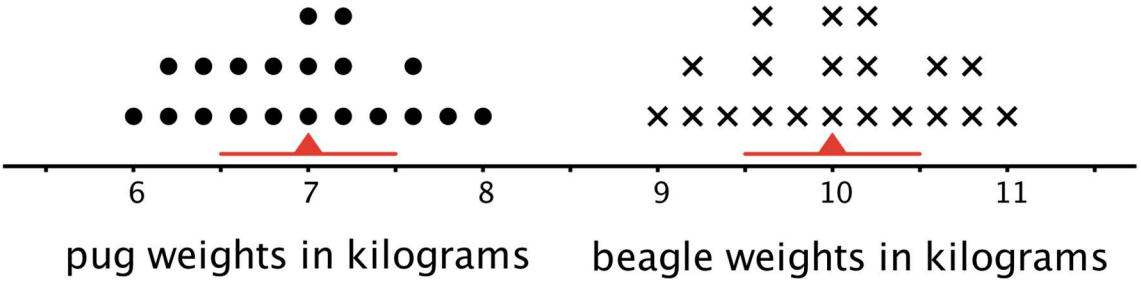
    b. The travel times of the students in New Zealand are less variable than those of the Canadian students. On average, the travel times of the students in Canada are 8 minutes longer or shorter than the mean. For New Zealand students, the travel times differ from the mean by a mean of 5.5 minutes.

2. Answers vary. Sample response: On average, the students in Australia have a longer commute to school than students in Canada, but the travel times of students in both countries have the same variability. The data points are, on average, about 8 minutes from the mean.

Student Lesson Summary

Sometimes two distributions have different means but the same MAD.

Pugs and beagles are two different dog breeds. The dot plot shows two sets of weight data—one for pugs and the other for beagles.



- The mean weight for pugs is 7 kilograms, and the MAD is 0.5 kilogram.
- The mean weight for beagles is 10 kilograms, and the MAD is 0.5 kilogram.

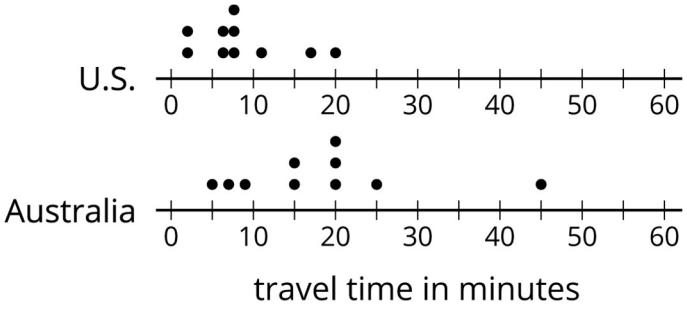
We can say that, in general, the beagles are heavier than the pugs. A typical weight for the beagles in this group is about 3 kilograms heavier than a typical weight for the pugs.

The variability of pug weights, however, is about the same as the variability of beagle weights. In other words, the weights of pugs and the weights of beagles are equally spread out.

Lesson 12 Practice Problems

Problem 1 Statement

The dot plots show the amounts of time that ten U.S. students and ten Australian students took to get to school.



Which statement is true about the MAD of the Australian data set?

a. It is significantly less than the MAD of the U.S. data set.

b. It is exactly equal to the MAD of the U.S. data set.

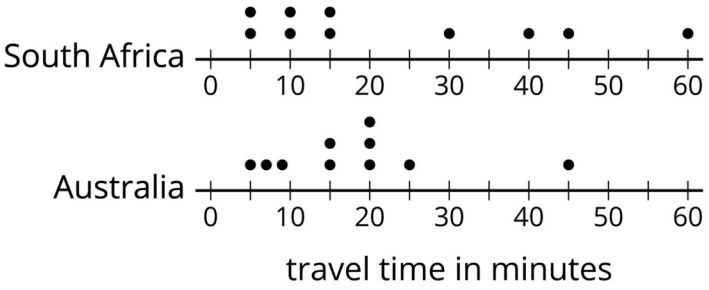
c. It is approximately equal to the MAD of the U.S. data set.

d. It is significantly greater than the MAD of the U.S. data set.

Solution C

Problem 2 Statement

The dot plots show the amounts of time that ten South African students and ten Australian students took to get to school. Without calculating, answer the questions.



a. Which data set has the smaller mean? Explain your reasoning.

b. Which data set has the smaller MAD? Explain your reasoning.

c. What does a smaller mean tell us in this context?

d. What does a smaller MAD tell us in this context?

Solution

Responses vary. Sample responses:

a. The mean of the Australian data set is smaller. The balance point of the Australian data set is less than 20 minutes. The balance point for the South African set is probably larger because four of the points are much larger than 20.

b. The MAD for the Australian data set is smaller. The data points are closer together and closer to the centre of the distribution.

c. A smaller mean tells us that the travel times of the students in the group are shorter overall.

d. A smaller MAD tells us the travel times of the students in the group are closer to each other, closer to the mean, and more alike overall.

Problem 3 Statement

Two secondary school basketball teams have identical records of 15 wins and 2 losses. Sunnyside School's mean score is 50 points and its MAD is 4 points. Shadyside School's mean score is 60 points and its MAD is 15 points.

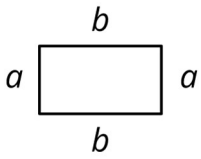
Lin read the records of each team's score. She likes the team that had nearly the same score for every game it played. Which team do you think Lin likes? Explain your reasoning.

Solution

Sunnyside School. Explanations vary. Sample explanation: The smaller MAD indicates that most of the scores for the games were close to the mean of 50 points.

Problem 4 Statement

Jada thinks the perimeter of this rectangle can be represented with the expression $a + a + b + b$. Andre thinks it can be represented with $2a + 2b$. Do you agree with either of them? Explain your reasoning.



Solution

They are both correct. $2a + 2b$ and $a + a + b + b$ are equivalent expressions because $2a$ means $a + a$ and $2b$ means $b + b$.

Problem 5 Statement

Draw a number line.

a. Plot and label three numbers between -2 and -8 (not including -2 and -8).

b. Use the numbers you plotted and the symbols $<$ and $>$ to write three inequality statements.

Solution

a. Answers vary. Sample response: any three numbers between -2 and -8.

b. Answers vary. Sample response: $-2 > -8$, $-2 > -7$, and $-6 < -3$.

Problem 6 Statement

Adult elephant seals generally weigh about 5 500 pounds. If you weighed 5 elephant seals, would you expect each seal to weigh exactly 5 500 pounds? Explain your reasoning.

Solution

No. Not every seal will weigh the same amount. There will be variability in the data.