Lesson 11: Points on the coordinate grid

Goals

- Generalise about the signs of coordinates that represent locations in each "quadrant" of the coordinate grid.
- Plot a point given its coordinates or identify the coordinates of a given point on the coordinate grid.
- Recognise that the axes of the coordinate grid can be extended to represent negative numbers.

Learning Targets

- I can describe a coordinate grid that has four quadrants.
- I can plot points with negative coordinates on the coordinate grid.
- I know what negative numbers in coordinates tell us.

Lesson Narrative

In earlier lessons, students extended the number line to include negative numbers. In this lesson, students extend the coordinate axes to expand the coordinate grid. In a previous unit, students worked in the coordinate grid when they examined ratio and other relationships between two quantities with positive values. They now consider an expanded coordinate grid where negative numbers appear on both the vertical and horizontal axes. The crossing axes create the four regions of the coordinate grid, called **quadrants**. In this first lesson on the coordinate grid, students extend their understanding of the coordinate grid to points with negative coordinates. They gain experience by choosing and plotting points in order to hit targets or to manoeuvre through mazes in all four quadrants of the coordinate grid.

Building On

- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the grid located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Addressing

- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate grid; recognise that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate grid.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate grid. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards

- Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous years to represent points on the line and on the grid with negative number coordinates.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate grid. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

- Collect and Display
- Discussion Supports

Student Learning Goals

Let's explore and extend the coordinate grid.

11.1 Guess My Line

Warm Up: 10 minutes

The purpose of this warm-up is for students to review graphing and locating points in the first quadrant of the coordinate grid. Students observe the structure of horizontal and vertical lines when they compare points on the same line and notice which coordinate of the ordered pair changes and why.

Launch

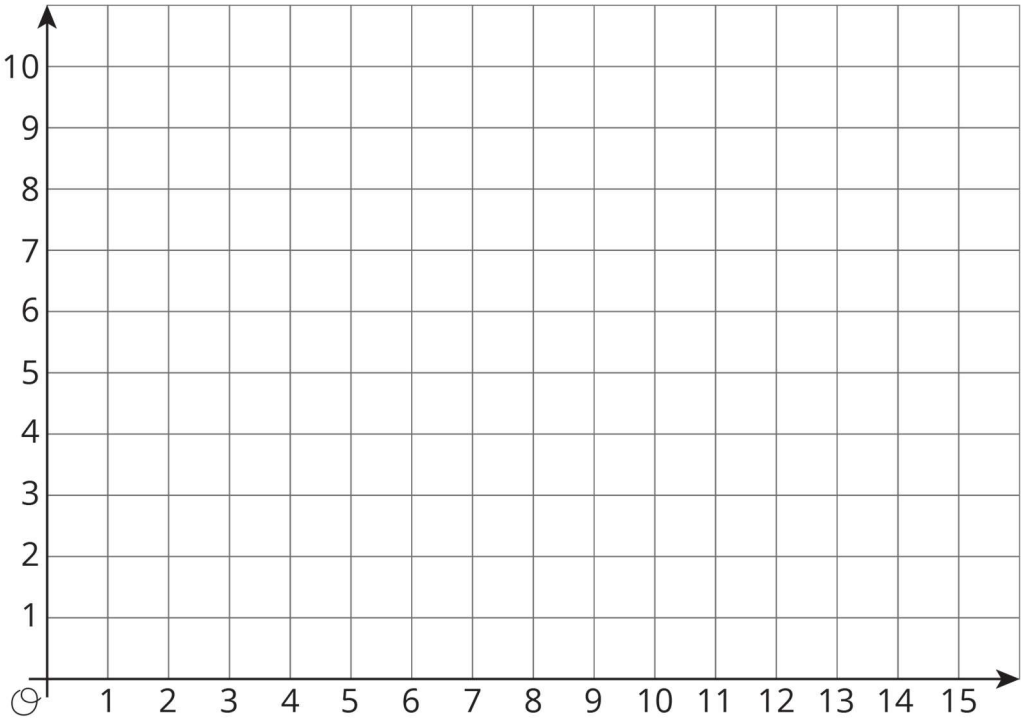
Arrange students in groups of 2. Display the coordinate grid for all to see and ask, "What do you notice about the grid? What do you wonder?" Invite a few students to share what they notice and wonder until a student has noticed there are no labels on the axes.
Ask, "How should we label the axes?" Otherwise, point out that the axes aren't labelled and ask them how they should be labelled. This lesson plan refers to the axes and coordinates with the standard x and y variables. Invite a student to read the directions for both questions in the task. Once confident that groups understand the directions, give groups 4 minutes to guess each other's points followed by a whole-class discussion.

Anticipated Misconceptions

Some students may not remember that the first coordinate in an ordered pair corresponds to the horizontal coordinate and the second coordinate in the ordered pair corresponds to the vertical coordinate. Display the ordered pair (x, y) or (horizontal,vertical) to remind students of the order.

Student Task Statement

1. Choose a horizontal or a vertical line on the grid. Draw 4 points on the line and label each point with its coordinates.



2. Tell your partner whether your line is horizontal or vertical, and have your partner guess the locations of your points by naming coordinates.

   If a guess is correct, put an X through the point. If your partner guessed a point that is on your line but not the point that you plotted, say, "That point is on my line, but is not one of my points."

   Take turns guessing each other's points, 3 guesses per turn.

Student Response

1. Answers vary. Possible response: Vertical line: (2,3), (2,4), (2,7), (2,8)

2. Answers vary.

Activity Synthesis

The key takeaway of this discussion is that points on the same horizontal line share the same y coordinate and points on the same vertical line share the same x coordinate. Ask 3 or 4 students to share the coordinates of their 4 points. After each student shares, ask the rest of the class if the given points are on the same horizontal or vertical line and to explain how they know. To help guide the conversation, consider asking some of the following questions:

- "How do you know the points fall on the same line?"
- "How do you know the points are on a horizontal or vertical line?"
- "Could you name other points on the same line?"
- "How far is each of the points from one another?"
- "How far is each point from the x-axis and y-axis?"

11.2 The Coordinate Grid

15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to extend the vertical and horizontal axes to include 4 quadrants just as they extended the number line to include negative numbers. Students are introduced to the term **quadrant.** Students plot and label coordinates using ordered pairs and identify their quadrants.

Instructional Routines

- Discussion Supports

Launch

Introduce the concept of the 4-region coordinate grid by explaining that, just like we extended the number line include negative numbers, we can extend both the number lines of the coordinate grid (the axes) to include negative coordinates. Use the word **quadrant** to describe the four regions of the coordinate grid. It may be helpful to explain that the prefix "quad-" means 4 and give other examples from English and other languages that use the prefix (quadriceps, quadrilateral). Give students 10 minutes to work followed by whole-class discussion.

Classes using the digital version have an interactive applet to use. Instead of naming the coordinates, students enter them into the Input Bar. If correct, the new points will hit the targets and turn them black.

Representation: Develop Language and Symbols. Activate or supply background knowledge. During the launch, take time to review terms that students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their

understanding of: vertical number line, horizontal number line.
Supports accessibility for: Conceptual processing; Language

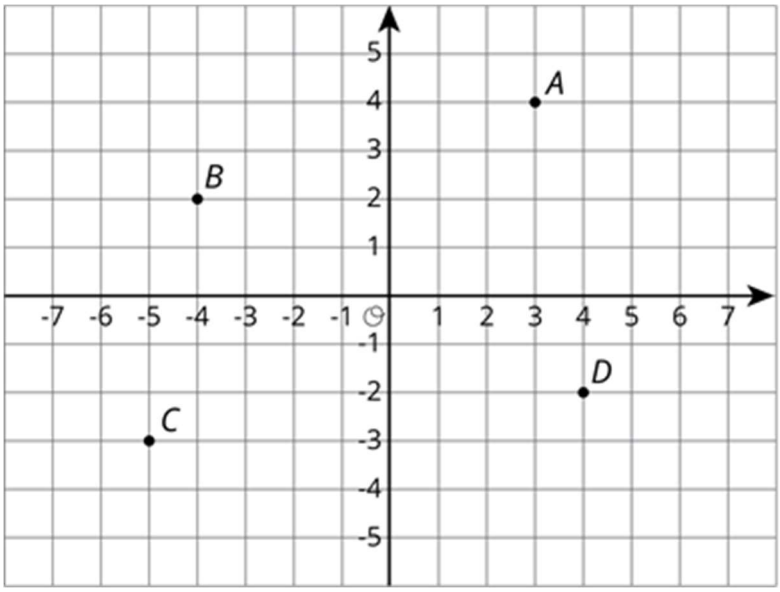
Anticipated Misconceptions

Students may confuse the order of the coordinate pairs when plotting points. Direct students to look at their work with vertical and horizontal lines in the warm up to remind them which coordinate is vertical.

Students may locate -4.5 between -3 and -4, rather than between -4 and -5. Direct them to the number lines used in earlier lessons to remind them of locating non integer numbers on number lines.

Student Task Statement

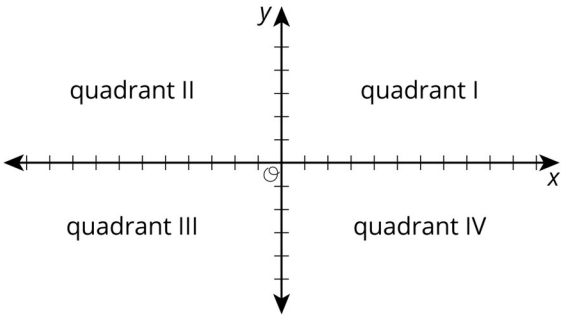
1. Label each point on the coordinate grid with an ordered pair.



2. What do you notice about the locations and ordered pairs of B, C, and D? How are they different from those for point A?

3. Plot a point at (-2,5). Label it E. Plot another point at (3,-4.5). Label it F.

4. The coordinate grid is divided into four **quadrants**, I, II, III, and IV, as shown here.



$G = (5,2)$

$H = (-1,-5)$

$I = (7,-4)$

a. In which quadrant is point G located? Point H? Point I?
b. A point has a positive y-coordinate. In which quadrant could it be?

Student Response

1.

a. $A = (3,4)$

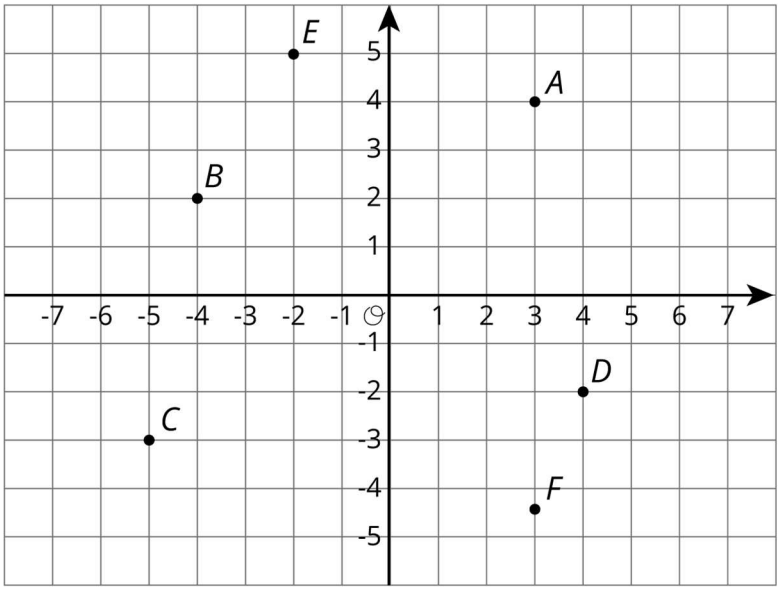
b. $B = (-4,2)$

c. $C = (-5,-3)$

d. $D = (4,-2)$

2. Answers vary. Sample response: They all have at least one negative coordinate.

3.



4.

a. Point G is in quadrant I, H is in quadrant III, and I is in quadrant IV.

b. Quadrant I or quadrant II.

Activity Synthesis

The most important idea for students to understand is that by extending the two number lines that form the coordinate axes for the first quadrant, we now have 4 quadrants. We describe points in these quadrants using negative and positive numbers as the x and y coordinates. Focus on responses from question 4 for discussion. Invite students to share their reasoning about how to identify the quadrants for the points G, H, and I. As time allows, consider asking the following questions:

- "If a point has a negative x-coordinate, what quadrant could it be in?"
- "If a point has a negative y-coordinate, what quadrant could it be in?"
- "If a point has a positive x-coordinate, what quadrant could it be in?"

To involve more students in the conversation, consider asking:

- "Do you agree or disagree? Why?"
- "Who can restate ___'s reasoning in a different way?"
- "Does anyone want to add on to ____'s reasoning?"

Speaking: Discussion Supports. To support students in producing statements about features of the quadrants that have negative and/or positive numbers provide sentence frames for students to use such as: "Point ____ is in quadrant ____ because ____." This will support class discussion in providing low entry points for students to start speaking about their reasoning.
Design Principle(s): Support sense-making

11.3 Coordinated Archery

15 minutes (there is a digital version of this activity)

In this activity, students select and describe points in different regions of the coordinate grid using ordered pairs. Students must name specific coordinates in order to hit different parts of an archery target embedded in a coordinate grid. All points within the archery target contain negative coordinates, allowing students to practise and build on what they were introduced to in the previous activity.

This activity was inspired by one created by Nathan Kraft https://teacher.desmos.com/activitybuilder/custom/563d705f36a7843710aba2ce.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2. It may be necessary to introduce students to what an archery target looks like and how it is scored. More points are scored the closer to the

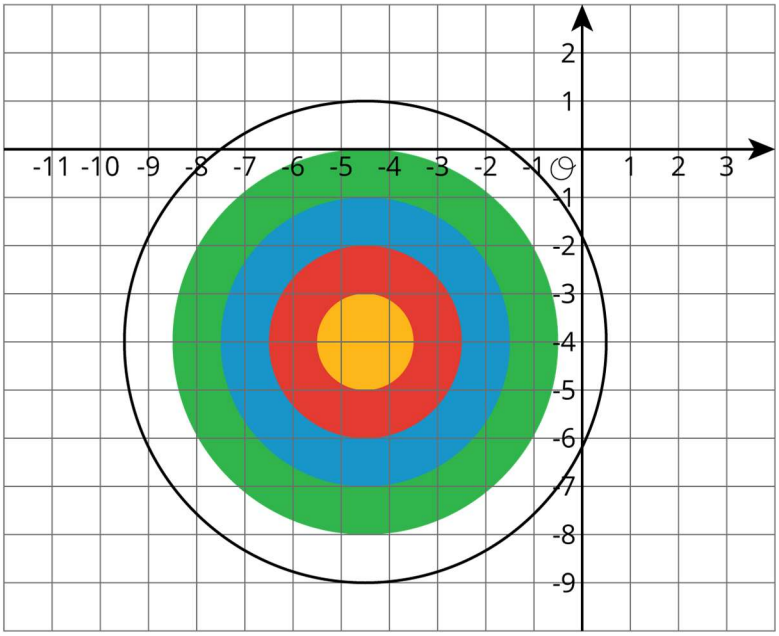
centre the arrow lands. Remind students to label the axes with x and y so that they can accurately describe the coordinates as x-coordinates or y-coordinates. Give students 8 minutes of quiet work time followed by 2 minutes of partner discussion for students to check whether their partner has made valid choices for coordinates. Follow with a whole-class discussion.

Students using the digital activity will be able to plot points using an applet and then determine if their points are in the desired area. To enter points, students enter an ordered pair of coordinates in the table.

Listening, Representing: Collect and Display. As students work, listen for, collect and display vocabulary and phrases students use to describe the coordinates, such as x-coordinate, y-coordinate, and quadrant. Update the list with student language and questions from student discussions that describe where to place the points on the graph. Remind students to borrow language from the display as needed. Teachers can use the list to check for student progress and misunderstandings. This will help students develop mathematical language for the class discussions that describe the location of points.
Design Principle(s): Support sense-making

Student Task Statement

Here is an image of an archery target on a coordinate grid. The scores for landing an arrow in the coloured regions are shown.



- Yellow: 10 points
- Red: 8 points
- Blue: 6 points

- Green: 4 points
- White: 2 points

Name the coordinates for a possible landing point to score:

1. 6 points
2. 10 points
3. 2 points
4. No points
5. 4 points
6. 8 points

Student Response

Answers vary. Sample responses (these work for both the applet and the printed version above):

1. (-2,-4)
2. (-4.5,-4)
3. (0,-6)
4. (-10,-10)
5. (-4.5,0)
6. (-3,-4)

Are You Ready for More?

Pretend you are stuck on a coordinate grid. You can only take vertical and horizontal steps that are one unit long.

1. How many ways are there to get from the point (-3, 2) to (-1,-1) if you will only step down and to the right?
2. How many ways are there to get from the point (-1,-2) to (4,0) if you can only step up and to the right?
3. Make up some more problems like this and see what patterns you notice.

Student Response

1. 10 paths
2. 21 paths

3. Answers vary. This problem is mostly an exercise in careful counting, although some students may realise that, for example, the first problem amounts to counting how many ways there are to arrange two steps to the right and three steps down. For some, this may be an easier representation of the problem.

Activity Synthesis

The main goal of discussion is to allow students to describe points on the grid that involve negative coordinates. Display the archery target for all to see. Ask students to share their responses for coordinates in the various regions of the target and record them for all to see. Record the points exactly as students describe them and push students to be precise if there was a mistake. Ask whether it's possible to get even closer to the exact centre of the target. If not mentioned by students, suggest using decimals or fractions as coordinates.

Lesson Synthesis

In this lesson, students extended the axes of the coordinate grid to include negative coordinates. Consider asking students the following questions to summarise the main ideas of the lesson:

- What are the names of the quadrants and where are they on the coordinate grid? (The quadrants are called I, II, III, and IV. They go in counter clockwise order from top right to top left to bottom left to bottom right.)
- What quadrant is the point (-4,5) in? How do you know? (That point is in quadrant II. The x-value is negative, so the point is left of the y-axis, and the y-value is positive so the point is above the x-axis.)
- What quadrant is the point (5,-4) in? How do you know? (That point is in quadrant IV. The x-value is positive, so the point is right of the y-axis, and the y-value is negative, so the point is below the x-axis.)

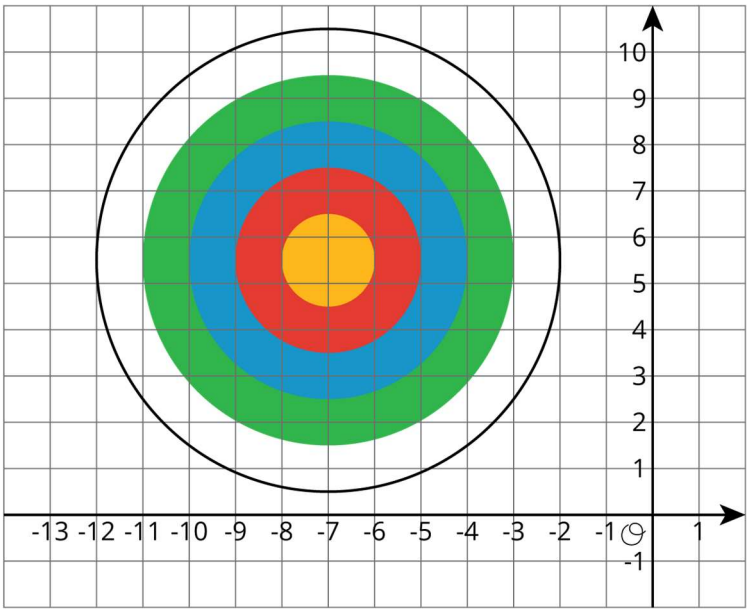
If time allows, ask students to make up their own challenges for the class with the targets. Invite other students to pick points that meet the requirement of the challenges. For example, one challenge could be to hit a point that is exactly between two colours. Another might be to hit the target in the bullseye 3 times on a horizontal line.

11.4 Target Practice

Cool Down: 5 minutes

Student Task Statement

Here are the scores for landing an arrow in the coloured regions of the archery target.



- Yellow: 10 points
- Red: 8 points
- Blue: 6 points
- Green: 4 points
- White: 2 points

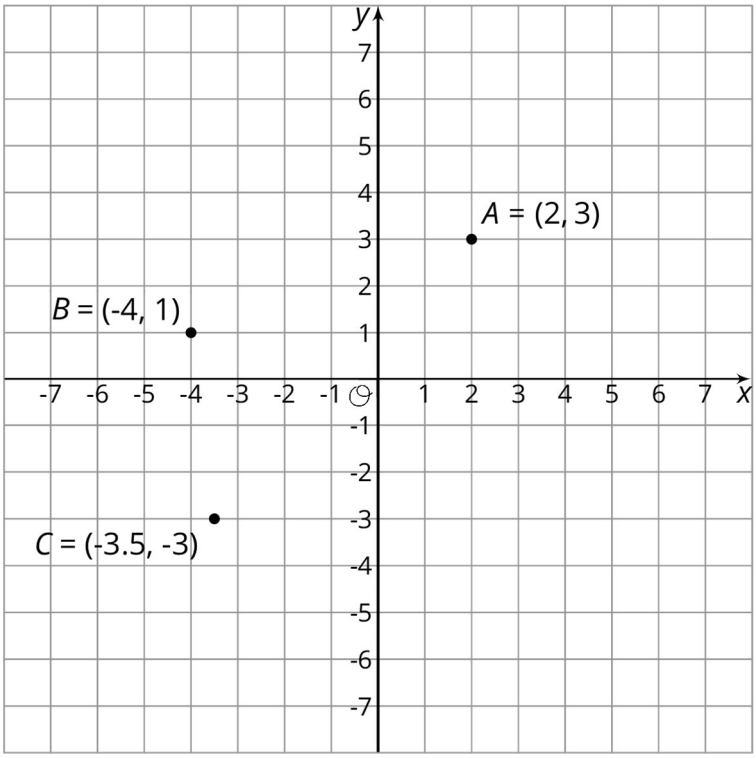
1. Andre shot three arrows and they landed at (-5,4), (-8,7) and (1,6). What is his total score? Show your reasoning.
2. Jada shot an arrow and scored 10 points. She shot a second arrow that landed directly below the first one but scored only 2 points. Name two coordinates that could be the landing points of her two arrows.

Student Response

1. (-5,4) is 6 points, (-8,7) is 8 points, (1,6) is 0 points. The total is 14 points.
2. Answers vary. Sample responses:
   - (-7,5) and (-7,1)
   - (-6.5,5.5) and (-6.5,1.1)

Student Lesson Summary

Just as the number line can be extended to the left to include negative numbers, the x- and y-axis of a coordinate grid can also be extended to include negative values.



The ordered pair (x, y) can have negative x- and y-values. For $B = (-4,1)$, the x-value of -4 tells us that the point is 4 units to the left of the y-axis. The y-value of 1 tells us that the point is one unit above the x-axis.

The same reasoning applies to the points A and C. The x- and y-coordinates for point A are positive, so A is to the right of the y-axis and above the x-axis. The x- and y-coordinates for point C are negative, so C is to the left of the y-axis and below the x-axis.

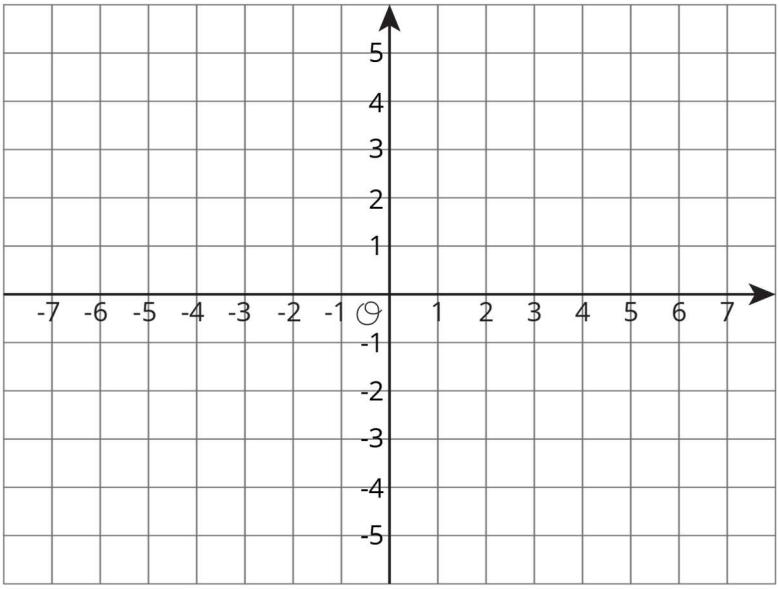
Glossary

- Quadrant

Lesson 11 Practice Problems

1. **Problem 1 Statement**

   a. Graph these points on the coordinate grid: (-2,3), (2,3), (-2,-3), (2,-3).

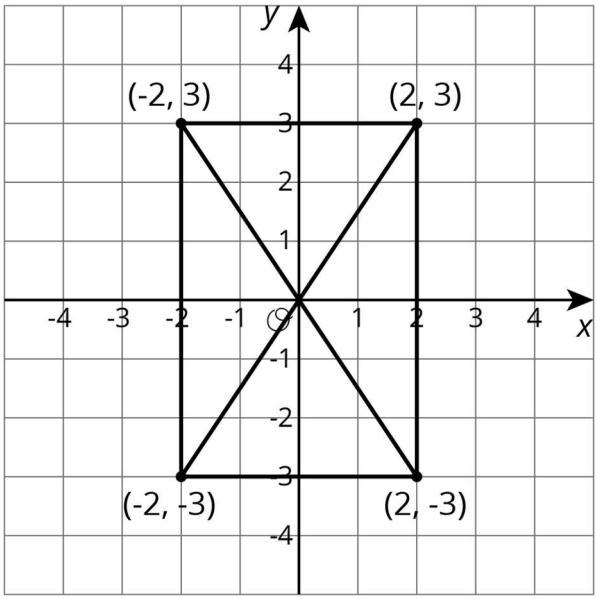


   b. Connect all of the points. Describe the shape.

Solution

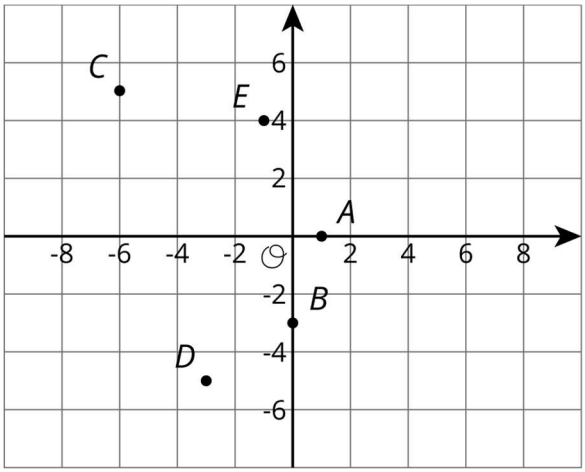
   a. The points $A = (-2,3)$, $B = (2,3)$, $C = (-2,-3)$, $D = (2,-3)$ are shown on a coordinate grid with segments between them.

   b.



2. **Problem 2 Statement**

Write the coordinates of each point.



Solution

$A = (1,0), B = (0,-3), C = (-6,5), D = (-3,-5), E = (-1,4)$

3. **Problem 3 Statement**

These three points form a horizontal line: (-3.5,4), (0,4), and (6.2,4). Name two additional points that fall on this line.

Solution

Answers vary. Any answer that has a y-coordinate of 4 is on the line.

4. **Problem 4 Statement**

One night, it is 24°C warmer in Tucson than it was in Minneapolis. If the temperatures in Tucson and Minneapolis are opposites, what is the temperature in Tucson?

a. -24°C

b. -12°C

c. 12°C

d. 24°C

Solution C

5. **Problem 5 Statement**

Lin ran 29 metres in 10 seconds. She ran at a constant speed.

a. How far did Lin run every second?

b. At this rate, how far can she run in 1 minute?

Solution

a. 2.9 metres every second, because $29 \div 10 = 2.9$.

b. 174 metres, because $(2.9) \times 60 = 174$.

6. **Problem 6 Statement**

Noah is helping his band sell boxes of chocolate to fund a field trip. Each box contains 20 bars and each bar sells for £1.50.

a. Complete the table for values of m.

boxes sold (b)	money collected (m)
1	
2	
3	
4	
5	
6	
7	
8	

b. Write an equation for the amount of money, m, that will be collected if b boxes of chocolate bars are sold. Which is the independent variable and which is the dependent variable in your equation?

c. Write an equation for the number of boxes, b, that were sold if m dollars were collected. Which is the independent variable and which is the dependent variable in your equation?

Solution

a. Values for m: 30, 60, 90, 120, 150, 180, 210, 240

b. $m = 30b$, b is independent, m is dependent

c. $b = \frac{m}{30}$, m is independent, b is dependent