

Lesson 7: Scale drawings

Goals

- Describe (orally) what a “scale drawing” is.
- Explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances.
- Interpret the “scale” of a scale drawing.

Learning Targets

- I can explain what a scale drawing is, and I can explain what its scale means.
- I can use actual distances and a scale to find scaled distances.
- I can use a scale drawing and its scale to find actual distances.

Lesson Narrative

Up to this point, students have been exploring scaled copies, or two-dimensional images that have been recreated at certain scale factors. In this lesson, they begin to look at **scale drawings**, or scaled two-dimensional representations of actual objects or places. Students see that although scale drawings capture three-dimensional objects or places, they show scaled measurements in only two of the dimensions, and that all information is projected onto a plane.

In this and upcoming lessons, students see that the principles and strategies they used to reason about scaled copies are applicable to scale drawings. For example, previously they saw scale factor as a number that describes how lengths in a figure correspond to lengths in a copy of the figure (and vice versa). Now they see that **scale** serves a similar purpose: it describes how the lengths in an actual object are related to the lengths on a drawn representation of it. They learn that scale can be expressed in a number of ways, and use scale and scale drawings to find actual and scaled lengths.

Students begin by interpreting given scale drawings. In subsequent lessons, they will create or reproduce scale drawings at specified scales, as well as determine appropriate scales to use, given restrictions in the size of drawing.

Addressing

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

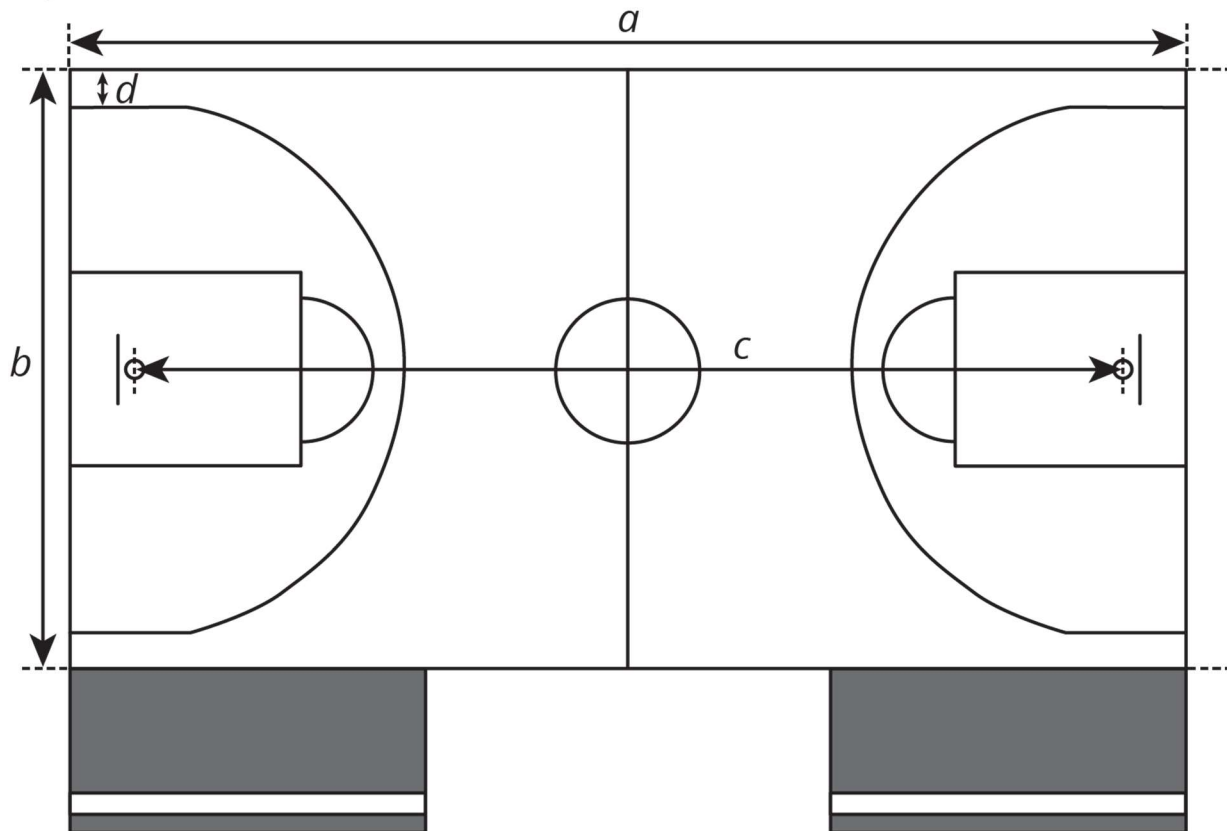
Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
 - Stronger and Clearer Each Time
-

- Discussion Supports

Required Materials

Copies of blackline master



Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Required Preparation

Prepare to display the examples and non-examples of scale drawings for all to see. Consider adding to the collection a local map showing the actual route of a train or bus line (example of scale drawing) and a diagrammatic transit map (non-example e.g. the underground map).

Ensure students have access to geometry toolkits, especially centimetre rulers and index cards or paper to use as a measuring tool.

You will need the Sizing Up a Basketball Court blackline master for this lesson. Prepare one copy per student.

Student Learning Goals

Let's explore scale drawings.

7.1 What is a Scale Drawing?

Warm Up: 5 minutes

This activity encourages students to notice characteristics of scale drawings by observing examples and counterexamples, and to articulate what a scale drawing is. Though students are not expected to come up with precise definitions, they are likely able to intuit that scale drawings are accurate two-dimensional depictions of what they represent, in the sense that all shapes, arrangements of parts, and relative sizes match those of the actual objects.

Expect student observations about scale drawings to be informal and not mathematical. For example, they might say that a scale drawing looks just like the object it is portraying, with the parts shown having the right size and being in the right places in the drawing. Or that in a scale drawing, a smaller part in the actual object does not end up being larger in the drawing.

Like any mathematical model of a real situation, a scale drawing captures some important aspects of the real object and ignores other aspects. It may not be apparent to students that scale drawings prioritise features of one plane of the object (and sometimes features of other planes parallel to it) and ignore other surfaces and dimensions. Notice students who show insights around this idea so they can share later.

Launch

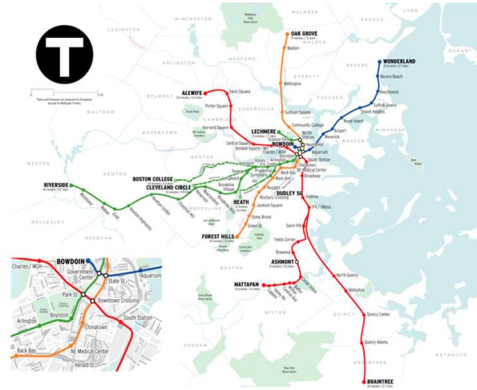
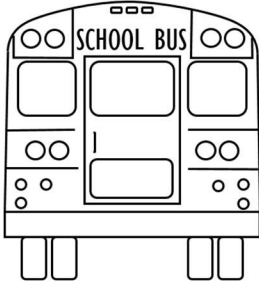
Arrange students in groups of 2. Before students look at the materials, poll the class to find out who has seen scale drawings. Ask a few students who are familiar with them to give a couple of examples of scale drawings they have seen. Then, give students 2 minutes to observe the examples and counterexamples of scale drawings and discuss in groups what they think a scale drawing is.

Anticipated Misconceptions

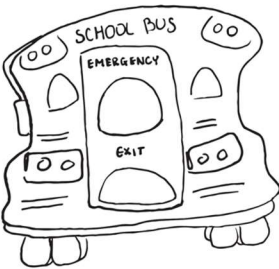
If students struggle to characterise scale drawings, offer prompts to encourage them to look closer. For example, ask: "How do the shapes and sizes of the objects in the drawings compare to those of the actual objects?" Students may say that sizes of the objects in the scale drawings are smaller than those of the actual objects. Ask them if any parts of the scale drawings are distorted, compared to the actual object—ask them to focus on the two images of the quarter, one of which is circular in shape while the other is not.

Student Task Statement

Here are some drawings of a school bus, a quarter, and the subway lines around Boston, Massachusetts. The first three drawings are **scale drawings** of these objects.



The next three drawings are *not* scale drawings of these objects.



Discuss with your partner what a scale drawing is.

Student Response

Answers vary. Sample descriptions:

- A scale drawing is a drawing that shows the object accurately and all parts in the drawing match the parts in the actual object.
- No parts in a scale drawing are distorted.
- A scale drawing is like a scaled copy of a real object, but it is a drawing that shows one flat surface of the object.

Activity Synthesis

Ask a few students to share what they noticed about characteristics of scale drawings and to compare and contrast scaled copies and scale drawings. Discuss questions such as the following. Record common themes and helpful descriptions.

- What do the examples have or show that the counterexamples do not?
- How are scale drawings like scaled copies you saw in earlier lessons? How are they different than scaled copies?

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- What aspects of the bus, coin, and the city of Boston do the scale drawings show? What aspects of the actual objects do scale drawings *not* show?

Notice misconceptions, but it is not necessary to address them right away, as students' understanding will be shaped in this and upcoming lessons. Tell students that they will continue to analyse scale drawings and revise their definitions in upcoming activities.

7.2 Sizing Up a Basketball Court

15 minutes

In this introductory activity, students explore the meaning of **scale**. They begin to see that a scale communicates the relationship between lengths on a drawing and corresponding lengths in the objects they represent, and they learn some ways to express this relationship:

- “ a units on the drawing represent b units of actual length”
- “at a scale of a units (on the drawing) to b units (actual)”
- “ a units (on the drawing) for every b units (actual)”

Students measure lengths on a scale drawing and use a given scale to find corresponding lengths on a basketball court. Because students are measuring to the nearest tenth of a centimetre, some of the actual measurements they calculate will not have the precision of the official measurements. For example, the official measurement for d is 0.9 m.

You will need the Sizing Up a Basketball Court blackline master for this activity.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Ask students if they have ever played basketball or seen a basketball court. If so, where? If some students have played basketball or seen a basketball court, ask them if they could throw a basketball across the width of a basketball court. What about across the full length of the court?

Arrange students in groups of 2. Distribute a copy of the blackline master and a ruler to each student. Give students 6–7 minutes of quiet work time to complete the first three questions. Ask them to share their responses with their partner before completing the remaining questions.

Writing, Speaking, Listening: Stronger and Clearer Each Time. After students have completed the first three questions, provide them an opportunity to refine their reasoning for the third question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language. For example, “How did you use the scale in your

calculations?”, “Why did you multiply each measurement from the drawing by 2?”, and “A detail (or word) you could add is ____, because . . .” Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to find actual measurements by using the measurements from a scale drawing.

Design Principles(s): Optimise output (for explanation); Maximise meta-awareness

Anticipated Misconceptions

Instead of using the scale to find actual measurements, students might try to convert distances in centimetres to metres (14 cm is 0.14 m). Explain that the distances they measured on paper could be converted to metres, but then the results are still lengths on paper, just expressed in metres, rather than the measurements of the actual basketball court. Draw students’ attention to the statement “1 cm represents 2 m” on the scale drawing and ask them to think about how to use it to find actual measurements.

Student Task Statement

Your teacher will give you a scale drawing of a basketball court. The drawing does not have any measurements labelled, but it says that 1 centimetre represents 2 metres.

1. Measure the distances on the scale drawing that are labelled a–d to the nearest tenth of a centimetre. Record your results in the first row of the table.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing				
actual court				

2. The statement “1 cm represents 2 m” is the **scale** of the drawing. It can also be expressed as “1 cm to 2 m,” or “1 cm for every 2 m.” What do you think the scale tells us?
3. How long would each measurement from the first question be on an actual basketball court? Explain or show your reasoning.
4. On an actual basketball court, the bench area is typically 9 metres long.
 - a. Without measuring, determine how long the bench area should be on the scale drawing.
 - b. Check your answer by measuring the bench area on the scale drawing. Did your prediction match your measurement?

Student Response

- 14 cm, 7.5 cm, 12.4 cm, 0.5 cm, although students may round measurements differently.
- Answers vary. Sample responses:
 - The scale tells us how the lengths on the drawing compare to actual lengths.
 - The scale tells us how to use the measurements on the drawing to find actual measurements.
- Sample reasoning: Since every centimetre represents 2 metres, I multiplied each measurement from the drawing by 2 to find the actual measurement in metres.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing	14 cm	7.5 cm	12.4 cm	0.5 cm
actual court	28 m	15 m	24.8 m	1 m

- 4.5 cm. Answers vary.

Activity Synthesis

Before debriefing as a class, display the table showing only the scaled distances so students can do a quick check of their measurements (which they may round differently). Explain to students that the distances on a scale drawing are often referred to as “scaled distances.” The distances on the basketball court, in this case, are called *actual* distances.

Focus the discussion on the meaning of scale and how students used the given scale to find actual distances. Invite a few students to share their response to the second and third questions. To further students’ understanding of scale, discuss:

- Does “1 cm for every 2 m” mean that the actual distance is twice that on the drawing? Why or why not?
- Which parts of the court should be drawn by using “1 cm for every 2 m” rule?
- Can we reverse the order in which we list the scaled and actual distances? For example, can we say “2 m of actual distance to 1 cm on the drawing” or “2 m to 1 cm”?

Note that the scaled distance is conventionally stated first, but the actual distance represented could also come first as long as it is clear from the context.

If needed, a short discussion about accuracy of measurements on the scale drawing might highlight some possible sources of measurement error including:

- Not measuring in a straight line
- The lines on the scale drawing have width, and this could contribute a small error depending on whether the measurement is from the inside or the outside of the lines

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: scale, scaled distance, scale drawing. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.
Supports accessibility for: Memory; Language

7.3 Tall Structures

15 minutes

This activity introduces students to graphic scales. Students interpret them, use them to find actual measurements (heights of tall buildings), and express them non-graphically. In earlier lessons, students used markings on an index card or sheet of paper to measure a drawing and create scaled copies. Here, they use non-standard measuring tools again to solve problems. Students make markings on an unmarked straightedge to measure scaled lengths on a drawing, and then use the given scale to determine actual lengths. This measuring strategy builds on students' work with measurement in Years 2 and 3 (i.e., placing multiple copies of a shorter object end to end and expressing lengths in terms of the number of objects).

As students work, encourage them to be as precise as possible in making their marks and in estimating lengths that are less than 1 scale-segment long. Most distances students are measuring here are not line segments but rather distances between a point (the tip of a building) and a line (the ground).

Monitor the different ways students reason about scaled and actual distances. Here are two likely approaches for finding the difference between the Burj Khalifa and the Eiffel Tower (the second question):

- Find the actual height of each tower and then find their difference.
- Find the difference in scaled heights, and then use the scale to find the difference in actual heights.

Select students who use these approaches to share during the whole-class discussion in this sequence.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Display the scale drawing of the structures. Before beginning the work of the task, students may be interested in or eager to share the locations of the structures. Consider taking a few minutes to elicit what they know, or display a world map showing the locations of the structures.

Ask students what the segment labelled with “0 m” and “100 m” might mean. Some students are likely to say that it also conveys a scale. Verify that a scale can indeed be communicated graphically; an actual distance is not represented by a numerical measurement, but rather, by the length of the segment.

Provide access to index cards or sheets of paper students can use to measure. Tell students to check their answers to the first question with a partner. Tell them to discuss as necessary until they reach an agreement before proceeding to work on the rest of the problems. Give students 4–5 minutes of quiet work time and partner discussion.

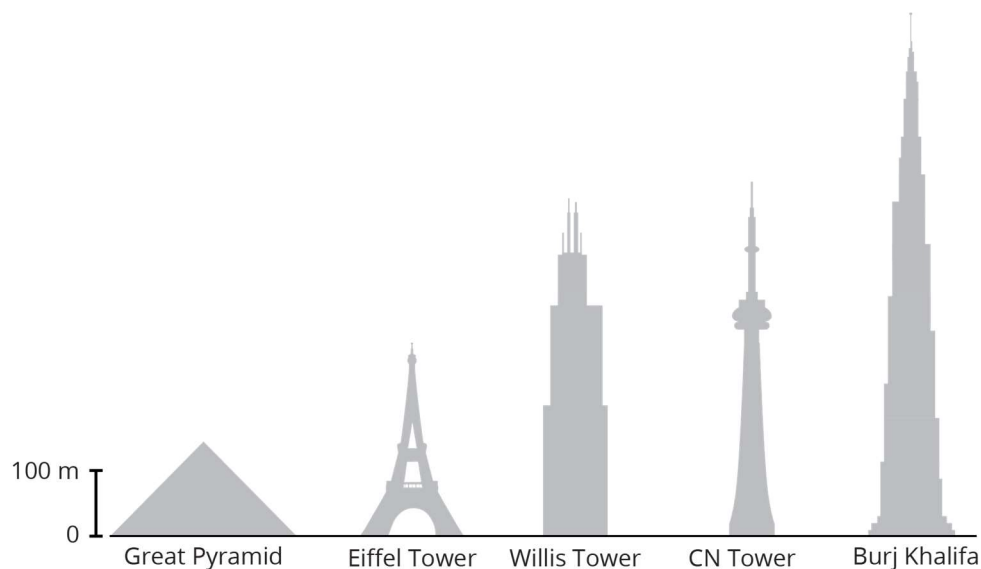
Anticipated Misconceptions

Students may not measure heights of the buildings at a right angle from the ground line. Remind students that heights are to be measured perpendicular to the ground or base line.

If needed, demonstrate how to use the edge of a sheet of paper or an index card to measure lengths on a scale drawing with a graphic scale.

Student Task Statement

Here is a scale drawing of some of the world’s tallest structures.



1. About how tall is the actual Willis Tower? About how tall is the actual Great Pyramid? Be prepared to explain your reasoning.
2. About how much taller is the Burj Khalifa than the Eiffel Tower? Explain or show your reasoning.
3. Measure the line segment that shows the scale to the nearest tenth of a centimetre. Express the scale of the drawing using numbers and words.

Student Response

1. The Willis Tower is a bit more than 500 m tall. It takes about 5 of the segment lengths to measure the height. The Great Pyramid is approximately 150 m. Its height is about $1\frac{1}{2}$ segments long.
2. The Burj Khalifa is approximately 550 m taller than the Eiffel Tower. Sample explanations:
 - It takes about 3 of the 100-m segments to measure the Eiffel Tower, so it is about 300 m tall. It takes about $8\frac{1}{2}$ of the 100-m segments to measure the Burj Khalifa, so it is about 850 metres tall. $850 - 300 = 550$.
 - It takes about 3 of the 100-m segments to measure the Eiffel Tower and about $8\frac{1}{2}$ segments to measure the Burj Khalifa. This is a difference of $5\frac{1}{2}$ segments on the scale drawing, which would be an actual difference of about 550 m.
 - The Burj Khalifa looks almost 3 times as tall as the Eiffel Tower, which would be a difference of between 500 and 600 m.
3. Answers vary depending on the printed size of the scale. Sample responses:
 - 0.7 cm on the drawing represents 100 m.
 - 0.7 cm to 100 m.
 - 0.7 cm for every 100 m in actual height.

Are You Ready for More?

The tallest mountain in the United States, Mount Denali in Alaska, is about 6 190 m tall. If this mountain were shown on the scale drawing, how would its height compare to the heights of the structures? Explain or show your reasoning.

Student Response

Mount Denali will be far taller (about 7.5 times taller) than the Burj Khalifa. Sample explanations:

- I found out that the Burj Khalifa is about 830 m. Mount Denali is 6 190 m, and 6 190 divided by 830 is about 7.5, so the mountain will be about 7.5 times taller than the Burj Khalifa.
- It takes about 62 of the 100-m segments to measure the mountain ($6\,190 \div 100 = 61.9$). It takes about $8\frac{1}{4}$ of the 100-m segments to measure the Burj Khalifa. This means the mountain is about 7.5 times taller than the Burj, since 62 divided by $8\frac{1}{4}$ is about 7.5.

Activity Synthesis

Poll the class for their answers to the first question. Ask what might be some sources of discrepancies. The main issue here is measurement error, but there are also different methods to make the measurements and estimate the heights of the buildings. Two methods to estimate the heights of the buildings (and their limitations) are:

- Estimate how many times “taller” each building is compared to the line segment giving the scale. Then multiply this number by 100 m. The accuracy here is not very good unless, for example, the height of the building is very close to being a whole number times the length of the scale.
- Measure the segment giving the scale (in centimetres, for example), and then express the scale using centimetres (for example, 0.7 cm represents 100 m). Then measure each building and use the scale to find the actual height. Estimating and rounding will be necessary when measuring the scale and when measuring the buildings.

Ask previously selected and sequenced students to highlight the different approaches for comparing the heights of the Burj Khalifa and the Eiffel Tower. The main difference between the two strategies is the order of arithmetic. Taking the difference of the actual building heights means multiplying scaled heights first and then subtracting. Taking the difference of the scaled heights and then applying the scale factor means subtracting scaled heights first and then multiplying by the scale factor.

If time permits, discuss the scale drawing of the towers more broadly. Consider asking:

- Besides height information, what other information about the towers does the drawing show? (Widths of the buildings and their overall shapes.)
- What information does it *not* show? (The depth of each building, any projections or protrusions, shapes of different parts of the buildings, etc.)
- How is this scale drawing the same as that of the basketball court? (They both show information on a single plane and are drawn using a scale.) How are they different? (The basketball court is a flat surface, like the drawing of the court. The drawing of the towers is a side view or front view; the actual objects represented are not actually flat objects.)

Speaking: Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Lesson Synthesis

- What is a **scale drawing**?
- How can we describe the **scale** for a scale drawing?
- How do we find distances using a scale drawing?

A scale drawing is a scaled representation of an object. The scale tells us how lengths on the drawing relate to lengths on the actual object. For example, in the basketball court activity, we saw that 1 centimetre on the drawing represented 2 metres of actual distance on the actual court.

If we have a scale drawing, we can use the scale to find lengths on the actual object. For example, if a line segment in the scale drawing of the basketball court is 5 cm, then it represents a 10 m line segment on a real court, because $2 \times 5 = 10$. It is important to remember that a scale drawing shows scaled measurements in only two dimensions, i.e., measurements of a particular surface of an object and those that have been projected onto a particular plane. For example, the drawing of the basketball court did not show the height of the basketball hoops.

7.4 Length of a Bus and Width of a Lake

Cool Down: 5 minutes

Launch

If desired, provide access to four-function calculators.

Student Task Statement

1. A scale drawing of a school bus has a scale of $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is $4\frac{1}{2}$ inches on the scale drawing, what is the actual length of the bus? Explain or show your reasoning.
2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1 000 m, what is the width of the lake on the scale drawing? Explain or show your reasoning.

Student Response

1. 45 ft. Sample explanation: There are 9 groups of $\frac{1}{2}$ in $4\frac{1}{2}$. If $\frac{1}{2}$ inch represents 5 feet, then $4\frac{1}{2}$ inches represents 9×5 or 45 feet.
2. 12.5 cm. Sample reasoning: Since every 80 m is represented by 1 cm, 1 000 m is represented by $1\,000 \div 80$ or 12.5 cm.

Student Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings. On a scale drawing:

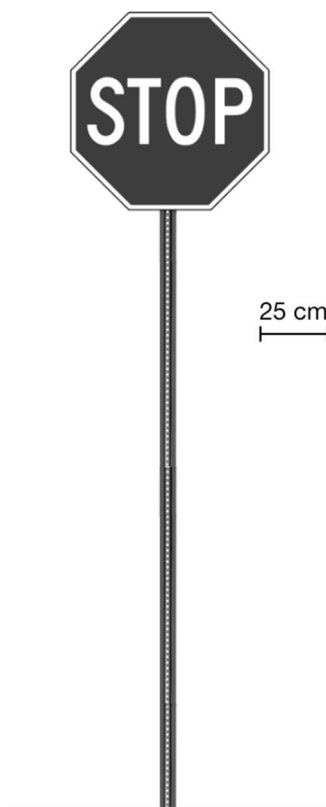
- Every part corresponds to something in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A **scale** tells us how actual measurements are represented on the drawing. For example, if a map has a scale of “1 inch to 5 miles” then a $\frac{1}{2}$ inch line segment on that map would represent an actual distance of 2.5 miles

Sometimes the scale is shown as a segment on the drawing itself. For example, here is a scale drawing of a stop sign with a line segment that represents 25 cm of actual length.

The width of the octagon in the drawing is about three times the length of this segment, so the actual width of the sign is about 3×25 , or 75 cm.

Because a scale drawing is two-dimensional, some aspects of the three-dimensional object are not represented. For example, this scale drawing does not show the thickness of the stop sign.

A scale drawing may not show every detail of the actual object; however, the features that are shown correspond to the actual object and follow the specified scale.



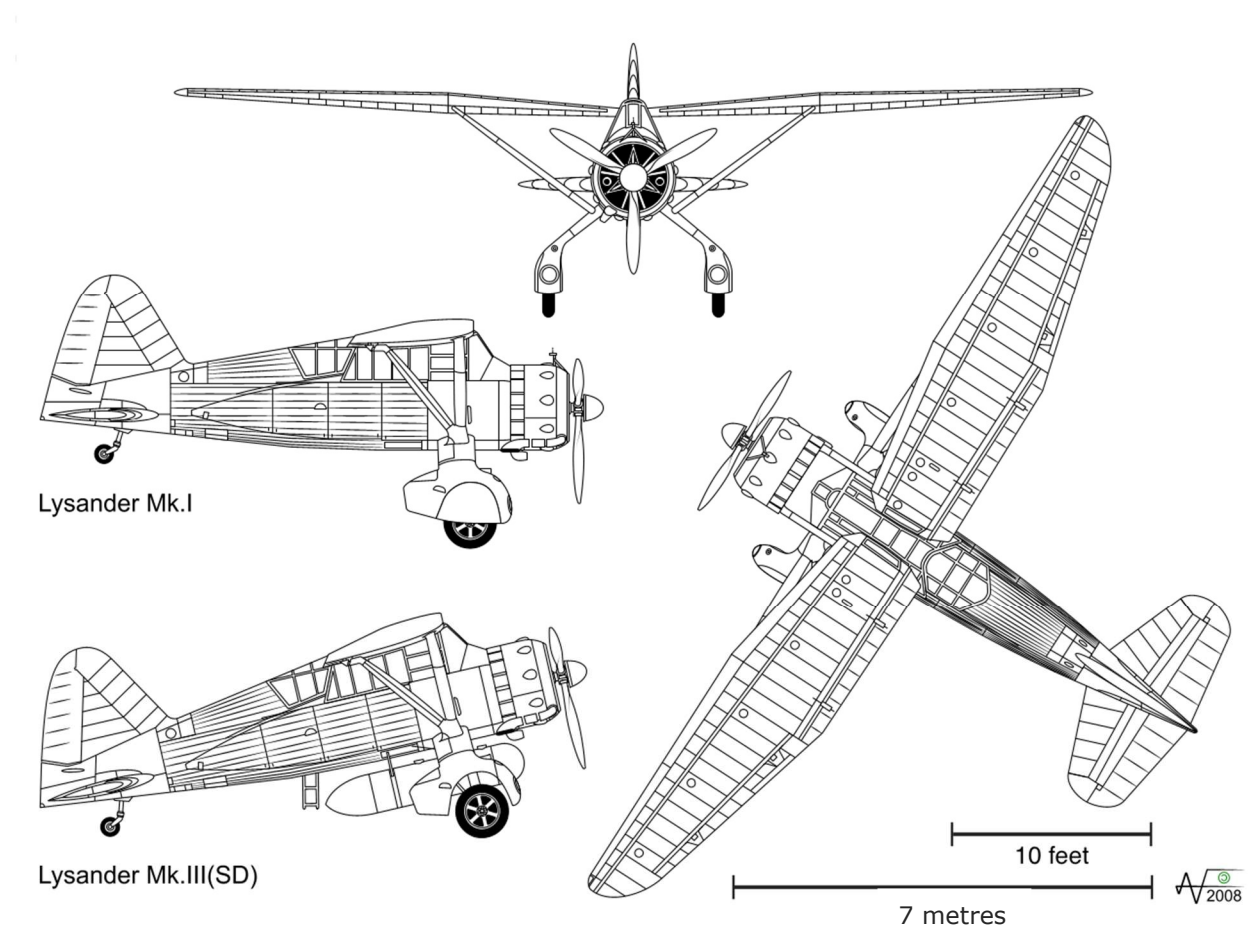
Glossary

- scale
- scale drawing

Lesson 7 Practice Problems

Problem 1 Statement

The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.



Use the scales and scale drawings to approximate the actual lengths of:

- the wingspan of the plane, to the nearest foot
- the height of the plane, to the nearest foot
- the length of the Lysander Mk. I, to the nearest metre

Solution

- a. 46 feet
- b. 12 feet
- c. 9 metres

Problem 2 Statement

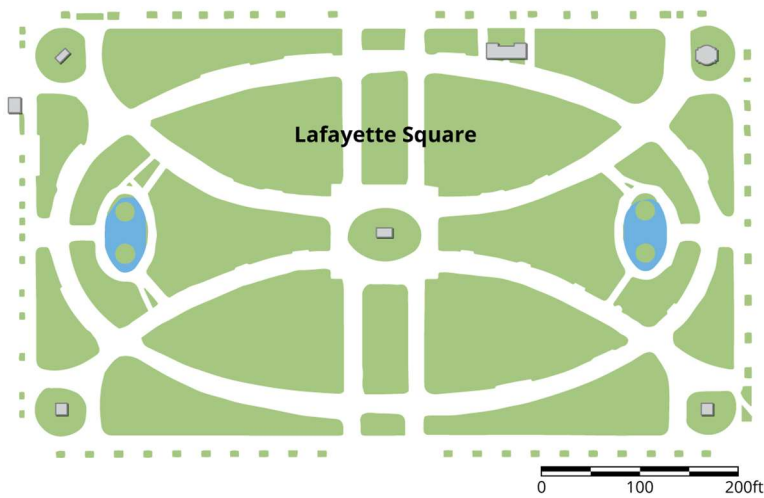
A blueprint for a building includes a rectangular room that measures 3 inches long and 5.5 inches wide. The scale for the blueprint says that 1 inch on the blueprint is equivalent to 10 feet in the actual building. What are the dimensions of this rectangular room in the actual building?

Solution

30 feet long and 55 feet wide

Problem 3 Statement

Here is a scale map of Lafayette Square, a rectangular garden north of the White House.



- a. The scale is shown in the lower right corner. Find the actual side lengths of Lafayette Square in feet.
- b. Use an inch ruler to measure the line segment of the graphic scale. About how many feet does one inch represent on this map?

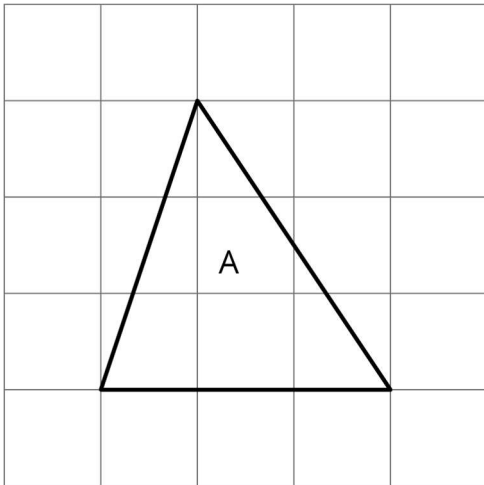
Solution

- a. About 800 ft by 500 ft
- b. Answers vary depending on the size of printed scale. Sample response: 1 in represents 300 feet.

Problem 4 Statement

Here is triangle A. Lin created a scaled copy of triangle A with an area of 72 square units.

- How many times larger is the area of the scaled copy compared to that of triangle A?
- What scale factor did Lin apply to the triangle A to create the copy?
- What is the length of the bottom side of the scaled copy?



Solution

- 16 times larger ($72 \div 4.5 = 16$)
- 4
- 12 units



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