Lesson 1: Relationships between quantities

Goals

- Determine unknown values in a relationship that is not proportional, and explain (orally and in writing) the solution method.
- Interpret and describe (orally and in writing) relationships that are predictable, but not proportional.
- Justify (orally) that a given relationship is not proportional.

Learning Targets

- I can think of ways to solve some more complicated word problems.

Lesson Narrative

In this introductory lesson, students encounter some engaging contexts characterised by relationships that are not proportional. The goal is simply to see that we need some new strategies—it is the work of the upcoming unit to develop strategies for efficiently solving problems about contexts like some of the ones in this lesson. In this lesson, is not expected that students write expressions or equations, or use any specific representation. Students may choose to make diagrams or tables or reason in some other way.

Alignments

Building On

- Recognise and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate grid and observing whether the graph is a straight line through the origin.

Building Towards

- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Three Reads
- Compare and Connect
- Think Pair Share

Student Learning Goals

Let's try to solve some new kinds of problems.

1.1 Pricing Theatre Popcorn

Warm Up: 10 minutes

A context is described, and students generate two sets of values. The purpose of this warm-up is to remind students of some characteristics that make a relationship proportional or not proportional, so that later in the lesson, they are better equipped to recognise that a relationship is not proportional and explain why.

The numbers were deliberately chosen to encourage different ways of viewing a proportional relationship. For 20 ounces and 35 ounces, students might move from row to row and think in terms of scale factors. This approach is less straightforward for 48 ounces, and some students may shift to thinking in terms of unit rates.

There are many possible rationales for choosing numbers so that size is not proportional to price. As long as the numbers are different from those in the "proportional" column, the relationship between size and price is guaranteed to be not proportional. Look for students who have a reasonable way to explain why their set of numbers is not proportional, like "the unit price is different for each size," or "each size costs a different amount per ounce."

Instructional Routines

- Think Pair Share

Launch

Ask students to remember the last time they went to the movies. What do they know about the popcorn for sale? What sizes does it come in? About how much does it cost? Tell students that in this activity, they will come up with prices for different sizes of popcorn—one set of prices in which the price is in proportion to the size, and another set of prices in which the price is not in proportion to the size, but is still reasonable. Ask students to be ready to explain the reasons they chose the numbers they did.

Arrange students in groups of 2. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

Student Task Statement

A movie theatre sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

Complete one column of the table with prices where popcorn is priced at a constant rate. That is, the amount of popcorn is proportional to the price of the bag. Then complete the other column with realistic example prices where the amount of popcorn and price of the bag are not in proportion.

volume of popcorn (ounces)	price of bag, proportional ($)	price of bag, not proportional ($)
10	6	6
20		
35		
48		

Student Response

Answers vary for the rightmost column. Sample response:

volume of popcorn (ounces)	price of bag, proportional ($)	price of bag, not proportional ($)
10	6	6
20	12	11
35	21	20
48	28.8	25

Activity Synthesis

Invite a student to share their prices for the proportional relationship and how they decided on those numbers. Ask if any students thought of it in a different way.

Then, invite a student to share their prices for the relationship that is not proportional and record these for all to see. Ask students to explain ways you can tell that the relationship is not proportional.

1.2 Entrance Fees

10 minutes

This context was used in an earlier unit about proportional relationships as an example of a relationship that is not proportional. However, a different rule for determining the entrance fee is used here.

Watch for students who organise the given information in a table or another visual representation, and for unique, correct approaches to the first two questions.

Instructional Routines

- Three Reads
- Think Pair Share

Launch

Tell students that unlike in the previous activity where they could make up any numbers, this activity has a relationship where there *is* a pattern, and part of the work is to figure out the pattern. This activity has to do with an entrance fee to a park, where the fee is based on the number of people in the vehicle.

Keep students in the same groups. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use the same colour to highlight the number of people with its corresponding entrance fee, making annotations of how the fee was calculated.
Supports accessibility for: Visual-spatial processing Reading, Representing: Three Reads. Use this routine to help students understand the question and to represent the relationships between quantities. Use the first read to orient students to the situation. Ask students to describe what the situation is about without using numbers (the park charges an entrance fee that includes the number of people in the car). After the second read, ask students what can be counted or measured in this situation. Listen for, and amplify, the quantities that vary in relation to each other: number of people in a vehicle; entrance fee amount, in pounds. After the third read, ask students to organise the information (using a list, table, or diagram) and brainstorm ideas for how much the park charges for each person in the car.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may misunderstand that the first two questions require noticing and extending a pattern, and (because of the warm-up) think that any reasonable number is acceptable. Encourage them to organise the given information and think about what rule the park might use to determine the entrance fee based on the number of people in the vehicle.

Students may come up with “rules” that aren’t supported by the context or the given information. For example, they may notice that each additional person costs £3, but then reason that 30 people must cost £90. Whatever their rule, ask them to check whether it works for all of the information given. For example, since 2 people cost £14, we can tell that “£3 per person” is not the rule.

Student Task Statement

A state park charges an entrance fee based on the number of people in a vehicle. A car containing 2 people is charged £14, a car containing 4 people is charged £20, and a van containing 8 people is charged £32.

1. How much do you think a bus containing 30 people would be charged?

2. If a bus is charged £122, how many people do you think it contains?

3. What rule do you think the state park uses to decide the entrance fee for a vehicle?

Student Response

1. £98. Sample reasoning: From 2 people to 4 people, there are 2 additional people that cost 6 additional pounds. From 4 people to 8 people, there are 4 additional people that cost 12 additional pounds. It seems like each additional person costs 3 additional pounds. From 8 people to 30 people is 22 additional people, so they should cost 66 additional pounds, and $32 + 66 = 98$.

2. 38 people. Sample reasoning: £122 is £24 more than £98. An additional £24 is 8 additional people, and $30 + 8 = 38$.

3. £8 for the vehicle plus £3 for each passenger. Sample reasoning: 2 people cost £14, so if each person is charged £3, that leaves £8 for the vehicle.

Activity Synthesis

Invite a student who organised the given information in a table to share. If no students did this, display this table for all to see:

number of people	entrance fee in pounds
2	14
4	20
8	32
30	
	122

Ask: "What are some ways that you can tell that this relationship is not proportional?" Possible responses:

- 2 people to 4 people is double, but 14 to 20 is not double.
- $14 \div 2 = 7$, but $20 \div 4 = 5$. If the entrance fee were in proportion to the number of people, each quotient would be equal.
- You can't describe the situation with an equation like $px = q$.

Invite students who had different strategies for answering the first two questions to share their responses. Ask them to share as many unique strategies as time allows. Ask each student who responds to state their rule that the park uses to decide the entrance fee. Record all unique, correct rules for all to see so students can see different ways of expressing the same idea. For example, the rule might be expressed:

- 8 pounds for the vehicle plus 3 pounds per person
- 3 pounds for every person and an additional £8
- 3 times the number of people plus 8
- $8 + 3 \times$ people

Note: We have the entire rest of the unit to systematically develop relationships like these. There is no need to formalise or generalise anything yet!

1.3 Making Toast

Optional: 10 minutes

In this activity, students are presented with a different relationship that is not proportional and also doesn't fit a pattern that can be characterised by an equation in the form $y = px + q$ (like the previous activity could be). This optional activity is a good opportunity for students to interpret another context and describe a relationship, but it can be safely skipped if the previous activity takes too much time.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Keep students in the same groups. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Provide students with a two column table for processing and organising information. Invite students to share their column labels (for example, number of slices and number of seconds) and how they organised the given information.
Supports accessibility for: Language; Organisation

Student Task Statement

A toaster has 4 slots for bread. Once the toaster is warmed up, it takes 35 seconds to make 4 slices of toast, 70 seconds to make 8 slices, and 105 seconds to make 12 slices.

1. How long do you think it will take to make 20 slices?

2. If someone makes as many slices of toast as possible in 4 minutes and 40 seconds, how many slices do think they can make?

Student Response

1. 175 seconds

2. 32 slices

Sample table:

number of slices	seconds it would take to make that number of slices
1	35
2	35
3	35
4	35
5	70
6	70
7	70
8	70
9	105
10	105
11	105
12	105

Are You Ready for More?

What is the smallest number that has a remainder of 1, 2, and 3 when divided by 2, 3, and 4, respectively? Are there more numbers that have this property?

Student Response

11; yes

Activity Synthesis

Invite students to share their responses and their reasoning. Select as many unique approaches as time allows.

Reading, Speaking: Compare and Connect. During the whole-class discussion, invite students to look for what is the same and what is different between the various approaches to solving the problem. Display and discuss differences in the tables and diagrams. Invite students to make connections by looking for the same quantity (e.g., 20 slices) in each representation. These exchanges strengthen students' mathematical language use and

reasoning based on ratios.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

The goal of this lesson is to recognise that there are situations in the world that are more complicated than what we have studied until this point, and to let students know this unit is about developing tools to solve some more sophisticated problems. Questions for discussion:

- "Describe some rules we encountered in this lesson for how one quantity was related to another quantity."
- "What made these situations more complicated than relationships we have seen in the past?"
- "What were some tools or strategies we used that were particularly helpful?"

1.4 Movie Theatre Popcorn, Revisited

Cool Down: 5 minutes

Student Task Statement

A movie theatre sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

volume of popcorn (ounces)	price of bag (£)
10	6
20	8
35	11
48	13.60

If the theatre wanted to offer a 60-ounce bag of popcorn, what would be a good price? Explain your reasoning.

Student Response

Answers vary. Sample responses:

- £16, because there is a pattern of £4 plus £0.20 per ounce.
- £15, because there should be a discount for buying a larger bag of popcorn.

Student Lesson Summary

In much of our previous work that involved relationships between two quantities, we were often able to describe amounts as being so much more than another, or so many times as much as another. We wrote equations like $x + 3 = 8$ and $4x = 20$ and solved for unknown amounts.

In this unit, we will see situations where relationships between amounts involve more operations. For example, a pizza store might charge the amounts shown in the table for delivering pizzas.

number of pizzas	total cost in pounds
1	13
2	23
3	33
5	53

We can see that each additional pizza adds £10 to the total cost, and that each total includes a £3 additional cost, maybe representing a delivery fee. In this situation, 8 pizzas will cost $8 \times 10 + 3$ and a total cost of £63 means 6 pizzas were ordered.

In this unit, we will see many situations like this one, and will learn how to use diagrams and equations to answer questions about unknown amounts.

Lesson 1 Practice Problems

1. **Problem 1 Statement**

   Lin and Tyler are drawing circles. Tyler's circle has twice the diameter of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler?

   Solution

   No, radius and area are not proportional. The area of Tyler's circle will be 4 times as big as the area of Lin's circle.

2. **Problem 2 Statement**

   Jada and Priya are trying to solve the equation $\frac{2}{3} + x = 4$.

   – Jada says, "I think we should multiply each side by $\frac{3}{2}$ because that is the reciprocal of $\frac{2}{3}$."

   – Priya says, "I think we should add -$\frac{2}{3}$ to each side because that is the opposite of $\frac{2}{3}$."

   a. Which person's strategy should they use? Why?

   b. Write an equation that can be solved using the other person's strategy.

Solution

a. Priya is correct. The operation in the expression $\frac{2}{3}+x$ is addition. Adding the additive inverse of $\frac{2}{3}$ to both sides of the equation will change the equation to the form "$x =$. ..."

b. Answers vary. Sample response: $\frac{2}{3}x = 4$.

3. **Problem 3 Statement**

What are the missing operations?

a. 48 ? (-8) = (-6)

b. (-40) ? 8 =(-5)

c. 12 ? (-2) = 14

d. 18 ? (-12) = 6

e. 18 ? (-20) = -2

f. 22 ? (-0.5) = -11

Solution

a. Divide

b. Divide

c. Subtract

d. Add

e. Add

f. Multiply

4. **Problem 4 Statement**

In American football, the team that has the ball has four chances to gain at least ten yards. If they don't gain at least ten yards, the other team gets the ball. Positive numbers represent a gain and negative numbers represent a loss. Select **all** of the sequences of four plays that result in the team getting to keep the ball.

a. 8, -3, 4, 21

b. 30, -7, -8, -12

c. 2, 16, -5, -3

d. 5, -2, 20, -1

e. 20, -3, -13, 2

Solution ["A", "C", "D"]

5. **Problem 5 Statement**

A sandwich store charges £20 to have 3 turkey subs delivered and £26 to have 4 delivered.

a. Is the relationship between number of turkey subs delivered and amount charged proportional? Explain how you know.

b. How much does the store charge for 1 additional turkey sub?

c. Describe a rule for determining how much the store charges based on the number of turkey subs delivered.

Solution

a. No. Sample reasoning: If they deliver 3 turkey subs, they charge £6.67 per sub, but for 4 subs, they charge £6.50 per sub.

b. £6

c. The rule could be £6 per sub plus a £2 delivery fee. 6 times 3 is 18, but they charged £2 more than that for 3 subs. 6 times 4 is 24, but they charged £2 more than that for 4 subs.

6. **Problem 6 Statement**

Which question cannot be answered by the solution to the equation $3x = 27$?

a. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?

b. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?

c. Diego paid £27 to have 3 pizzas delivered and £35 to have 4 pizzas delivered. What is the price of one pizza?

d. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

Solution C