

13.3 Ejercicios

- Hallar las dos derivadas parciales de primer orden.

$$(15) \quad z = x^2 - 4xy + 3y^2$$

$$\frac{\partial z}{\partial x} = 2x - 4y$$

$$\frac{\partial z}{\partial y} = -4x + 6y$$

$$(23) \quad z = \ln(x^2 + y^2)$$

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2} \quad \left| \quad \frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2}\right.$$

$$(27) \quad h(x, y) = e^{-(x^2 + y^2)}$$

$$h_x(x, y) = -2xe^{-(x^2 + y^2)}$$

$$h_y(x, y) = -2ye^{-(x^2 + y^2)}$$

$$(35) \quad z = e^y \sin xy.$$

$$\frac{\partial z}{\partial x} = ye^y \cos xy.$$

$$\begin{aligned} \frac{\partial z}{\partial y} &= e^y \sin(xy) + xe^y \cos x \\ &= e^y(x \cos xy + \sin xy) \end{aligned}$$

$$(37) \quad z = \sinh(2x+3y).$$

$$\frac{\partial z}{\partial x} = 2 \cosh(2x+3y)$$

$$\frac{\partial z}{\partial y} = 3 \cosh(2x+3y)$$

• Evaluar ∂_x y ∂_y en el punto dado.

$$(47) \quad F(x,y) = \cos(2x-y), \quad \left(\frac{\pi}{4}, \frac{\pi}{3}\right).$$

$$F_x(x,y) = -2 \sin(2x-y)$$

$$\text{At } \left(\frac{\pi}{4}, \frac{\pi}{3}\right), \quad F_x\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = -2 \sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = -2$$

$$F_y(x,y) = \sin(2x-y)$$

$$\text{At } \left(\frac{\pi}{4}, \frac{\pi}{3}\right), \quad F_y\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \frac{1}{2}$$

- Calcular las pendientes de la superficie en las direcciones x y y en el punto dado

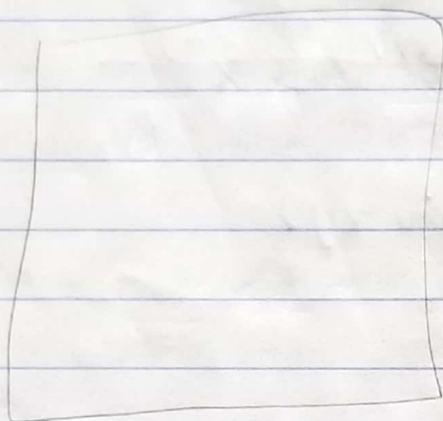
(53) $g(x, y) = 4 - x^2 - y^2$
 $(1, 1, 2)$

$$g_x(x, y) = -2x$$

$$(1, 1): g_x(1, 1) = -2$$

$$g_y(x, y) = -2y$$

$$(1, 1): g_y(1, 1) = -2$$



- Calcular las derivadas parciales de primer orden con respecto a x y z .

(59) $H(x, y, z) = \cos(x + 2y + 3z)$

$$H_x(x, y, z) = -\sin(x + 2y + 3z)$$

$$H_y(x, y, z) = -2\sin(x + 2y + 3z)$$

$$H_z(x, y, z) = -3\sin(x + 2y + 3z)$$

(61) $w = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

• Evaluar \int_x, \int_y y \int_z en el punto dado

(66) $F(x, y, z) = x^3 y z^2, (1, 1, 1)$

$$F_x(x, y, z) = 2xy^3 + 2yz$$

$$F_x(-2, 1, 2) = -4 + 4 = 0$$

$$F_y(x, y, z) = 3x^2 y^2 + 2xz - 3z$$

$$F_y(-2, 1, 2) = 12 - 8 - 6 = -2$$

$$F_z(x, y, z) = 2xy - 3y$$

$$F_z(-2, 1, 2) = -2 - 3 = -5$$

(70) $F(x, y, z) = \sqrt{3x^2 + y^2 - 2z^2}, (1, -2, 1)$

$$F_x(x, y, z) = \frac{6x}{2\sqrt{3x^2 + y^2 - 2z^2}} = \frac{3x}{\sqrt{3x^2 + y^2 - 2z^2}}$$

$$F_x(1, -2, 1) = \frac{6}{2\sqrt{3+4-2}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$F_y(x, y, z) = \frac{2y}{2\sqrt{3x^2 + y^2 - 2z^2}} = \frac{y}{\sqrt{3x^2 + y^2 - 2z^2}}$$

$$F_y(1, -2, 1) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$F_z(x, y, z) = \frac{-4z}{2\sqrt{3x^2 + y^2 - 2z^2}} = \frac{-2z}{\sqrt{3x^2 + y^2 - 2z^2}}$$

$$F_z(1, -2, 1) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$