13.3 Ejercicios

- Hallar las dos derivadas parciales de primer orden.

(15) $z = x^2 - 4xy + 3y^2$

$$\frac{\partial z}{\partial x} = 2x - 4y$$

$$\frac{\partial z}{\partial y} = -4x + 6y$$

(23) $z = \ln(x^2 + y^2)$

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2+y^2} \quad \Bigg| \quad \frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$$

(27) $h(x,y) = e^{-(x^2+y^2)}$

$$h_x(x,y) = -2xe^{-(x^2+y^2)}$$

$$h_y(x,y) = -2ye^{-(x^2+y^2)}$$

(35) $z = e^y \operatorname{sen} xy$.

$$\frac{\partial z}{\partial x} = ye^y \cos xy$$

$$\frac{\partial z}{\partial y} = e^y \sin(xy) + xe^y \cos x$$

$$= e^y(x \cos xy + \sin xy)$$

(37) $z = \operatorname{senh}(2x + 3y)$.

$$\frac{\partial z}{\partial x} = 2 \cosh(2x+3y)$$

$$\frac{\partial z}{\partial y} = 3 \cosh(2x+3y)$$

- Evaluar $f_x \wedge f_y$ en el punto dado.

(47) $F(x,y) = \cos(2x - y)$, $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$.

$F_x(x,y) = -2 \sin(2x - y)$

At $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$, $F_x\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = -2 \sin\left(\frac{\pi}{2}, \frac{\pi}{3}\right) = -1$

$F_y(x,y) = \sin(2x - y)$

At $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$, $F_y\left(\frac{\pi}{4}, \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \frac{1}{2}$

- Calcular las pendiente de la superficie en las direcciones y x ∧ de y en el punto dado

(53) $g(x,y) = 4 - x^2 - y^2$

$(1,1,2)$

$g_x(x,y) = -2x$

$(1,1): g_x(1,1) = -2$

$g_y(x,y) = -2y$

$(1,1): g_y(1,1) = -2$

- Calcular las derivadas parciales de primer orden. Con respecto a x ∧ y ∧ z

(59) $H(x,y,z) = \text{sen}(x + 2y + 3z)$

$H_x(x,y,z) = \cos(x + 2y + 3z)$

$H_y(x,y,z) = 2\cos(x + 2y + 3z)$

$H_z(x,y,z) = 3\cos(x + 2y + 3z)$

(61) $w = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}} \qquad \frac{\partial w}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

• Evaluar f_x, f_y ∧ f_z en el punto dado

(66) $F(x,y,z) = x^3yz^2$, $(1,1,1)$

$F_x(x,y,z) = 2xy^3 + 2yz$

$F_x(-2,1,2) = -4 + 4 = 0$

$F_y(x,y,z) = 3x^2y^2 + 2xz - 3z$

$F_y(-2,1,2) = 12 - 8 - 6 = -2$

$F_z(x,y,z) = 2xy - 3y$

$F_z(-2,1,2) = -21 - 3 = -7$

(70) $F(x,y,z) = \sqrt{3x^2+y^2-2z^2}$, $(1,-2,1)$.

$$F_x(x,y,z) = \frac{6x}{2\sqrt{3x^2+y^2-2z^2}} = \frac{3x}{\sqrt{3x^2+y^2-2z^2}}$$

$$F_x(1,-2,1) = \frac{6}{2\sqrt{3+4-2}} = \frac{3}{\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{5}}$$

$$F_y(x,y,z) = \frac{2y}{2\sqrt{3x^2+y^2-2z^2}} = \frac{y}{\sqrt{3x^2+y^2-2z^2}}$$

$$F_y(1,-2,1) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$F_z(x,y,z) = \frac{-4z}{2\sqrt{3x^2+y^2-2z^2}} = \frac{-2z}{\sqrt{3x^2+y^2-2z^2}}$$

$$F_z(1,-2,1) = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$