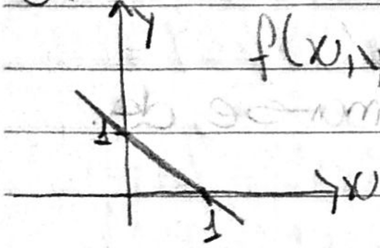
Resolução 15.2

/ /

2) Suponha que um arame tenha a equação $y = 1-x$ $(0 \leq x \leq 1)$ e que sua densidade de massa seja $\delta(x,y) = 2-x$. A massa do arame é.



$f(x,y) = 1-x \qquad \delta(x,y) = 2-x$

$(r(t)) = \langle 0, -t \rangle$

$(r'(t)) = \langle 0, -1 \rangle$

$|r'(t)| = \sqrt{0^2 + (-1)^2} = \boxed{1}$

$$M = \int_0^1 0 - t \cdot 1 dt = -\int_0^1 t\, dt = -\frac{t^2}{2}\Big|_0^1 = \frac{1}{2}_{//}$$

4) Se C for o círculo unitário $x^2+y^2=1$ orientado no sentido anti-horário e $F(x,y) = x\vec{i} + y\vec{j}$. Então

$\int_C F \cdot dr = \_\_\_\_\_ = \int_a^b F(r(t)) \cdot r'(t)\, dt$

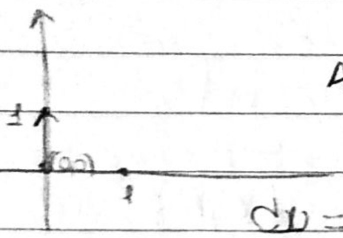
$F(x,y) = x + y$

$r(t) = \langle t, t \rangle$

$r'(t) = \langle 1, 1 \rangle$

1) Seja C o segmento de reta de $(0,0)$ até $(0,1)$. Determine, em cada parte, a integral de linha ao longo de C por inspeção.

a) $\int_C ds =$ o comprimento do segmento de linha $= 1$



$B(0,1)$

$A(0,0)$

$\vec{AB} = B - A$

$\vec{AB} = (0,1) - (0,0)$

$\vec{AB} = (0,1)$

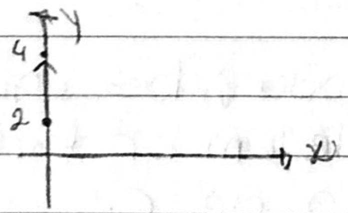
$\begin{cases} x = 0 + 0.t \\ y = 0 + 1.t \end{cases}$

$0 \leq t \leq 1$

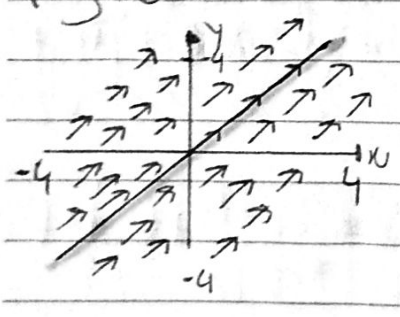
$\frac{dx}{dt} = 0 \qquad \frac{dy}{dt} = 1 \qquad \int_0^1 1\,dt = t\Big|_0^1 = \boxed{1}$

2) Seja C o segmento de reta de $(0,2)$ até $(0,4)$

a) $\int_C ds =$ o comp do seg. de reta/linha $= 2$.



3) Determine $\int_C F \cdot dr$ por inspeção para o campo de forças $F(x,y) = \vec{i} + \vec{j}$ e a curva C mostrada na figura.



Uma vez que F e r são paralelos, $F \cdot r = \|F\| . \|r\|$, e uma vez que F é constante, $\int_C F \cdot dr = \int_C d(F \cdot r) =$

$\int_C d(F \cdot r) = \int_C d(\|F\| . \|r\|) =$

C:

/ /

7-10) Calcule $\int_C F \cdot dr$ ao longo do segmento de reta C desde P até Q.

7) $F(x,y) = 8i + 8j$; $P(-4,4), Q(-4,5)$

$r(t) = \langle 8t, 8t \rangle$

deslocamento

$\vec{PQ} = Q - P \Rightarrow (-4,5) - (-4,4) = (0,1)$

$4 \leq t \leq 5$

$$\int_C \vec{F} \cdot d\vec{r} = \int_4^5 (8 \cdot 0 + 8 \cdot 1)\, dt = 8$$

11) Seja C a curva representada pelas equações

11) Seja C a curva representada pelas equações $x=2t$, $y=t^2$ $(0 \leq t \leq 1)$. Calcule, em cada parte, a integral de linha ao longo de C.

a) $\int_C (x-\sqrt{y})\,ds = \int_C f(x,y)\,ds = \int_a^b f(r(t)).|\vec{r}'(t)|\,dt$

$f(r(t)) = (2t, -\sqrt{t^2})$ $\rightarrow \int_0^1 2t\sqrt{1+t^2}\,dt$

$r(t) = \langle 2t, t^2 \rangle$

$r'(t) = (2 + 2t)$

$|r'(t)| = \sqrt{2^2+(2t)^2} = \underbrace{\sqrt{4+4t^2}\,dt}_{ds=}$

$\int^1 (2t-\sqrt{t^2}).\sqrt{4+4t^2}\,dt$

$\int_0^1 2t\sqrt{1+t^2}\,dt = 2\int_0^1 t\sqrt{\underbrace{1+t^2}_{u}}\,dt = \frac{2}{2}\int_0^1 \sqrt{u}\,du$

$u = 1+t^2$

$du = 2t\,dt$

$\frac{du}{2} = t\,dt$

$\int_0^1 u^{1/2}\,dt = \frac{u^{3/2}}{3/2}\Big|_0^1$

$\Rightarrow \frac{2}{3}(1+t^2)^{3/2}\Big|_0^1 = \left(\frac{2}{3}\sqrt{(1+1^2)^3}\right) - \left(\frac{2}{3}\sqrt{(1+0^2)^3}\right) =$

$\frac{2}{3}\left(\sqrt{2^3} - \sqrt{1}\right) = \boxed{\frac{2}{3}(2\sqrt{2}-1)}$

$\downarrow$

$2^2 \cdot 2$

$\frac{2}{5/3}$

$f(r(t)) = (3t^2 \cdot t^2 \cdot \frac{2}{3}t^3)$

21) Calcule a integral de linha em relação à s ao longo da curva C.

$\int_C 3x^2yz\, ds \qquad C: x = t,\ y = t^2,\ z = \frac{2}{3}t^3\ (0 \leq t \leq 1)$

$r(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle \Rightarrow r'(t) = (1, 2t, 2t^2)$

$|r'(t)| = \sqrt{1^2 + (2t)^2 + (2t^2)^2} = \sqrt{1 + 4t^2 + 4t^4} = \boxed{2t^2 + 1\, dt}$

$\int_a^b f(r(t)) \cdot |\vec{r}'(t)|\, dt = \int_0^1 2t^7(2t^2+1)\, dt = \int_0^1 4t^9 + 2t^7\, dt$

$\frac{\cancel{4}^{2}t^{10}}{\cancel{10}_5} + \frac{\cancel{2}^{1}t^8}{\cancel{8}_4} = \frac{2}{5}t^{10} + \frac{1}{4}t^8 \Big|_0^1 = \frac{2}{5} + \frac{1}{4} = \boxed{\frac{13}{20}}$

37) Calcule $\int_C F \cdot dr$ ao longo da curva C.

$F(x,y) = x^2\vec{i} + xy\vec{j}. \quad C: r(t) = 2\cos t\,\vec{i} + 2\,\text{sen}\,t\,\vec{j}\ (0 \leq t \leq \pi).$

$\int_C \vec{F} \cdot \vec{dr} = \int_a^b \vec{F}(r(t)) \cdot \vec{r}'(t)\, ds$

$F(r(t)) = (2\cos t)^2 + 2(\cos t\, \text{sen}\, t)$

$r(t) = \langle \overset{x}{2\cos t}, \overset{y}{2\,\text{sen}\,t} \rangle$

$r'(t) = \langle -2\,\text{sen}\,t, 2\cos t \rangle$ $\qquad 2\pi$

$\int_0^\pi 4\cos^2 t + 2(\cos t \cdot \text{sen}\, t)\, dt = 4\int_0^\pi \underset{*}{\cos^2 t}\, dt + 2\int_0^\pi \cos t\, \text{sen}\, t\, dt$

$* \; 4\int_0^\pi \frac{1 - \cos(2t)}{2}\, dt = 2\int_0^\pi 1\, dt - \int_0^\pi \cos(2t)\, dt$

$2t\Big|_0^\pi = 2\pi - \int_0^\pi \cos(2t) = 2\pi\left[-\frac{1}{2}\,\text{sen}(2t)\right]\Big|_0^\pi = 2\pi$

credeal

$2\int_0^\pi \cos t\, \text{sen}\, t\, dt = 2\int_0^\pi u\, du = \cancel{2}\frac{u^2}{\cancel{2}} = (\cos t)^2\Big|_0^\pi$

$u = \cos t$

$du = \text{sen}\, t\, dt$

$\cos^2 t\Big|_0^\pi = \cos^2(\pi) - \cos^2(0) =$

$-1 - 1 = \boxed{0}$

/ /

41) Calcule a massa de um arame fino com o formato do arco circular $y=\sqrt{9-x^2}$ $(0 \leq x \leq 3)$ se a função de densidade for $\delta(x,y)=x\sqrt{y}$.

$$M=\int_c \delta(x,y)\,ds = \int_a^b \delta(\vec{r}(t)).|\vec{r}\,'(t)|\,dt$$

$r(t)=(x(t),y(t))$

$r(t)=\langle \underset{x}{3\cos t}, \underset{y}{3\operatorname{sen} t}\rangle,\ 0 \leq t \leq \pi$ $\qquad \frac{u'}{2\sqrt{u}} = \frac{3\cos t}{2\sqrt{3\operatorname{sen} t}}$

$r'(t)=\langle -3\operatorname{sen} t, 3\cos t\rangle$

$|r'(t)|=\sqrt{(-3\operatorname{sen} t)^2+(3\cos t^2)^2}$

$=\sqrt{9(\operatorname{sen}^2 t+\cos^2 t)} = \sqrt{9} = \boxed{3}$

↳ modulo do vetor velocidade

$$\int_0^\pi 3\cos t.\sqrt{3\operatorname{sen} t}.\ 3\,dt$$

$$\int_0^\pi 9\cos t.\sqrt{\underset{u}{3.\operatorname{sen} t}}\,dt = 9\int_0^\pi \cos t.\sqrt{3\operatorname{sen} t}\,dt$$

$u=\sqrt{3\operatorname{sen} t}$ $\qquad v=\cos t$

$du=$ $\qquad dv=-\operatorname{sen} t$

$$\int u\,dv = u.v-\int v\,du \Rightarrow 9\left(3\operatorname{sen} t.\cos t-\int_0^\pi \cos t.\right.$$

$(-1-\pi)$

/ /

23-30) Calcule a integral de linha em relação a curva C.

23- $\int_C (x+2y)\,dx + (x-y)\,dy$ C: $x=2\cos t$, $y=4\,\text{sen}\,t$
$0 < t < \pi/4$

$r(t) = \langle 2\cos t, 4\,\text{sen}\,t \rangle$

$r'(t) = \langle -2\,\text{sen}\,t, 4\cos t \rangle$

$|r'(t)| = \sqrt{(-2\,\text{sen}\,t)^2 + (4\cos t)^2}$

$|r'(t)| = \sqrt{4\,\text{sen}^2 t + 16\cos^2 t} = \sqrt{4(\text{sen}^2 t + 4\cos^2 t)}$

$= 2\sqrt{\text{sen}^2 t + 4\cos^2 t}$

$\int_a^b f(r(t))\,.\,|r'(t)|\,dt$ $f(r(t)) = (2\cos t + 2.4\,\text{sen}\,t) + (2\cos t - 4\,\text{sen}\,t)$

8

$f(r(t)) = 4\cos t + 4\,\text{sen}\,t$

$f(r(t)) = 4(\cos t + \text{sen}\,t)$

$\int_0^{\pi/4} 4(\cos t + \text{sen}\,t)\,dt = 4\int_0^{\pi/4} \cos t\,dt + \int_0^{\pi/4} \text{sen}\,t\,dt$

$4\left[-\text{sen}\,t\right]_0^{\pi/4} = 4\left(-\text{sen}\left(\frac{\pi}{4}\right) - \left(-\cancel{\text{sen}(0)}^{\,0}\right)\right) =$

$4\left(-\frac{\sqrt{2}}{2} - 0\right) = \boxed{-2\sqrt{2}}$