

Investigation on congruency of triangles

Two triangles are congruent if they are identical in size and shape. This means that, the corresponding sides and angles of the two congruent triangles are equal.

Whenever two triangles are congruent, we can state the following: $\triangle ABC \equiv \triangle FGD$

Note: An included angle in a triangle is formed by two sides of a triangle.



Angle \hat{B} is included between sides BC and AB. Angle ___ is included between sides ___ and ___



use to move the diagrams



use to drag labels (eg length and angles)



use to measure length

a = 3



Slider a

b = 4.5



Slider b



use to measure angles



Use to get the original triangles

Activity 1i.

a. Use the applet "Discover SSS" to complete the table below:

Triangle	Slider a = 3 and b = 4.5					
$\triangle ABC$ (Green)	AB = 3	AC = 4.5	BC = 4.36	$\hat{A} =$	$\hat{B} =$	$\hat{C} =$
$\triangle FGD$ (Blue)	DF = 3	DG = 4.5	FG = 4.36	$\hat{D} =$	$\hat{F} =$	$\hat{G} =$
Set Slider a = 3.8 and b = 5						
$\triangle ABC$ (Green)	AB = 3.8	BC = 5.01	AC = 5	$\hat{A} =$	$\hat{B} =$	$\hat{C} =$
$\triangle FGD$ (Blue)	DF = 3.8	FG = 5.01	DG = 5	$\hat{D} =$	$\hat{F} =$	$\hat{G} =$
Set Slider a = 4.6 and b = 5.5						
$\triangle ABC$ (Green)	AB = 4.6	BC = 5.68	AC = 5.5	$\hat{A} =$	$\hat{B} =$	$\hat{C} =$
$\triangle FGD$ (Blue)	DF = 4.6	FG = 5.68	DG = 5.5	$\hat{D} =$	$\hat{F} =$	$\hat{G} =$

b. Using the measurements in the table above and the applet, fill in the gaps:

AB _____ DF; AB is opposite angle _____ and DF is opposite angle _____; and \hat{C} _____ \hat{G}

BC _____ FG; BC is opposite angle _____ and FG is opposite angle _____ and \hat{A} _____ \hat{D}

AC _____ DG; AC is opposite angle _____ and DG is opposite angle _____; and \hat{B} _____ \hat{F}

c. What can you say about the corresponding angles in each case? _____

d. What can you conclude about $\triangle ABC$ and $\triangle FGD$ in each case? _____

e. If the corresponding sides of two triangles are _____; then the triangles are _____. This is written as . _____

Activity 1ii.

a. Use applet “Does AAA work” to complete the table below:

Triangle	Slider a =2 and b = 3.5					
ΔABC	AB =	BC=	AC=	\hat{A} =	\hat{B} =	\hat{C} =
ΔFGD	DF =	FG =	DG =	\hat{D} =	\hat{F} =	\hat{G} =
Slider a = 3 and b = 4						
ΔABC	AB =	BC=	AC=	\hat{A} =	\hat{B} =	\hat{C} =
ΔFGD	DF =	FG =	DG =	\hat{D} =	\hat{F} =	\hat{G} =

b. Using the measurements in the table above, complete the gaps

\hat{C} _____ \hat{G} but AB _____ DF

\hat{A} _____ \hat{D} but BC _____ FG

\hat{B} _____ \hat{F} but AC _____ DG and

c. What can you conclude about ΔABC and ΔFGD in each case? _____

d. In two triangles, three corresponding angles may be equal but _____

Activity 2i

a. Use applet “Discover SAS” to complete the table below:

Triangle	Slider a = 3.5 and $\alpha = 45^\circ$					
ΔABC (Green)	AB = 3.5	BC=	AC=4.87	$\hat{A} = 45^\circ$	\hat{B} =	\hat{C} =
ΔPQR (Pink)	PQ = 3.5	QR =	PR = 4.87	$\hat{P} = 45^\circ$	\hat{Q} =	\hat{R} =
Set Slider a = 4.5 and $\alpha = 60^\circ$						
ΔABC (Green)	AB = 4.5	BC=	AC=6.26	$\hat{A} = 60^\circ$	\hat{B} =	\hat{C} =
ΔPQR (Pink)	PQ = 4.5	QR =	PR = 6.26	$\hat{P} = 60^\circ$	\hat{Q} =	\hat{R} =
Set Slider a = 5.5 and $\alpha = 75^\circ$						
ΔABC (Green)	AB =	BC=	AC=	\hat{A} =	\hat{B} =	\hat{C} =
ΔPQR (Pink)	PQ =	QR =	PR =	\hat{P} =	\hat{Q} =	\hat{R} =

b. What can you conclude about the corresponding sides and angles of $\triangle ABC$ and $\triangle PQR$ in each case.

c. If, in two triangles, two pairs of corresponding sides are _____ and the corresponding pair of included angles are _____. Then the two triangles are _____. This is written as _____.

Activity 2ii

a. Use applet “Does SSA work” to complete the table below:

Triangle						
$\triangle PQS$	PQ =	QS =	PS =	$\hat{P} =$	$P\hat{Q}S =$	$P\hat{S}Q =$
$\triangle PQR$	PQ =	QR =	PR =	$\hat{P} =$	$P\hat{Q}R =$	$P\hat{R}Q =$

b. What sides and angles are equal in both triangles? _____

c. Are the triangles congruent?

d. What can you conclude if, in two triangles, two pairs of corresponding sides are equal but the pair of corresponding equal angles are non-included angles: _____.

Activity 3

a. Use applet “SAA” to complete the table below:

Triangle	Slider a = 5 ; $\alpha = 45^\circ$ and $\gamma = 30^\circ$					
$\triangle ABC$ (Green)	AB = 5	BC =	AC =	$\hat{A} = 45^\circ$	$\hat{B} = 30^\circ$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 5	QR =	PR =	$\hat{P} = 45^\circ$	$\hat{Q} = 30^\circ$	$\hat{R} =$
	Slider a = 6 ; $\alpha = 50^\circ$ and $\gamma = 35^\circ$					
$\triangle ABC$ (Green)	AB = 6	BC =	AC =	$\hat{A} = 50^\circ$	$\hat{B} = 35^\circ$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 6	QR =	PR =	$\hat{P} = 50^\circ$	$\hat{Q} = 35^\circ$	$\hat{R} =$
	Slider a = 7 ; $\alpha = 55^\circ$ and $\gamma = 40^\circ$					
$\triangle ABC$ (Green)	AB = 7	BC =	AC =	$\hat{A} = 55^\circ$	$\hat{B} = 40^\circ$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 7	QR =	PR =	$\hat{P} = 55^\circ$	$\hat{Q} = 40^\circ$	$\hat{R} =$

b. What can you conclude about the corresponding sides and angles of the triangles?

c. What can you conclude about $\triangle ABC$ and $\triangle PQR$ in each case. _____

d. If, in two triangles, one pair of corresponding sides are _____ and two pairs of corresponding angles are _____, then the triangles are _____. This is written as _____.

Activity 4

a. Use applet “RHS” to complete the table below:

Triangle	Slider a = 3,5 ; b = 4					
$\triangle ABC$ (Green)	AB = 3.5	BC = 4	AC =	$\hat{A} = 90^\circ$	$\hat{B} =$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 3.5	QR = 4	PR =	$\hat{P} = 90^\circ$	$\hat{Q} =$	$\hat{R} =$
	Slider a = 4 ; b = 5					
$\triangle ABC$ (Green)	AB = 4	BC = 5	AC =	$\hat{A} = 90^\circ$	$\hat{B} =$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 4	QR = 5	PR =	$\hat{P} = 90^\circ$	$\hat{Q} =$	$\hat{R} =$
	Slider a = 4.5 ; b = 6					
$\triangle ABC$ (Green)	AB = 4.5	BC = 6	AC =	$\hat{A} = 90^\circ$	$\hat{B} =$	$\hat{C} =$
$\triangle PQR$ (Pink)	PQ = 4.5	QR = 6	PR =	$\hat{P} = 90^\circ$	$\hat{Q} =$	$\hat{R} =$

b. What can you conclude about the corresponding sides and angles of the triangles?

c. What can you conclude about $\triangle ABC$ and $\triangle PQR$ in each case. _____

d. If, in two right-angled triangles, the hypotenuse of each triangle is equal and a pair of corresponding sides are _____, then the triangles are _____. This is written as _____