Lesson 15: Shapes on the coordinate grid

Goals

- Determine the total length of multiple horizontal and vertical segments on the coordinate grid that are connected end-to-end.
- Draw a polygon on the coordinate grid given the coordinates for its vertices.
- Explain (orally) that coordinates can be a useful way of describing geometric shapes or modelling real-world locations.

Learning Targets

- I can find the lengths of horizontal and vertical segments on the coordinate grid.
- I can plot polygons on the coordinate grid when I have the coordinates for the vertices.

Lesson Narrative

In this lesson, students apply their understanding of rational coordinates and distance on the coordinate grid to construct polygons and navigate a maze. Students plot coordinates in all four quadrants and find horizontal and vertical distances.

Building On

- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,x-axis and x-coordinate, y-axis and y-coordinate).
- Draw polygons on the coordinate grid given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- Draw polygons on the coordinate grid given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate grid.

- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate grid. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards

- Draw polygons on the coordinate grid given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate grid. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions
- Discussion Supports
- Think Pair Share

Required Materials

Coloured pencils
Graph paper

Required Preparation

Provide students access to graph paper and coloured pencils for the lesson synthesis or upon request during the lesson.

Student Learning Goals

Let's use the coordinate grid to solve problems and puzzles.

15.1 Figuring Out The Coordinate Grid

Warm Up: 5 minutes (there is a digital version of this activity)

The purpose of this warm-up is for students to review properties of shapes and polygons within the context of graphing points on the coordinate grid.

As the students work, monitor and select students with different shapes, some that are polygons and some that are not, to share during the whole-class discussion. The focus of the whole-class discussion should be on the properties of a polygon.

Instructional Routines

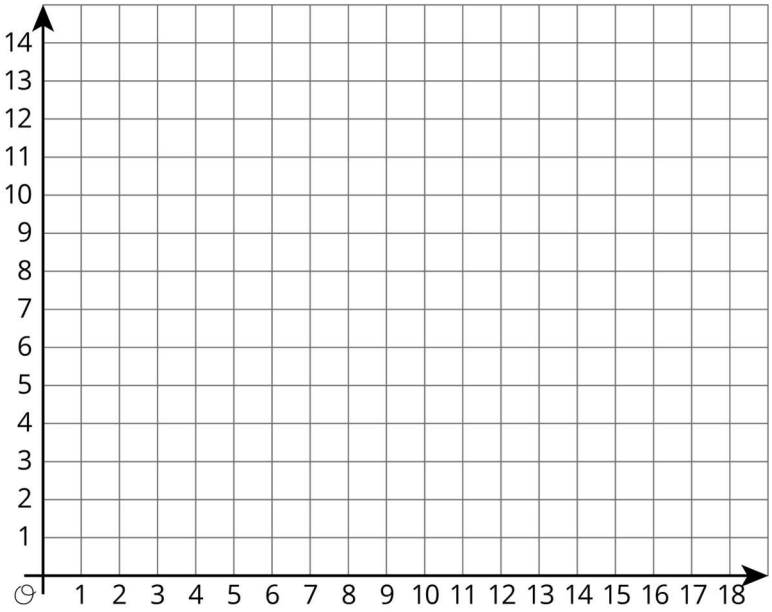
- Anticipate, Monitor, Select, Sequence, Connect

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time. After the 2 minutes, tell students to share their shape with their partner to check if it has at least three of the listed properties. Follow with a whole group discussion.

Student Task Statement



1. Draw a shape on the coordinate grid with at least three of following properties:
   - 6 vertices
   - Exactly 1 pair of parallel sides
   - At least 1 right angle
   - 2 sides with the same length
2. Is your shape a polygon? Explain how you know.

Student Response

1. Answers vary. Sample strategy: Start with two segments of equal lengths that meet at a right angle. This takes care of 2 requirements. To reach a third requirement, just choose 3 other points and connect them to the existing vertices so that there are 6 total vertices.
2. Answers vary.

Activity Synthesis

Ask selected students to share their shape and its properties. Display these shapes for all to see. After each student shares, ask the class if it is a polygon and how they know.

Defining characteristics of a polygon that should be emphasised during the discussion are:

- It is composed of line segments.
- Each line segment meets one and only one other line segment at each end.
- The line segments never intersect each other except at their endpoints.
- It lays flat on the coordinate grid.

15.2 Plotting Polygons

15 minutes (there is a digital version of this activity)

The purpose of this task is for students to practise plotting points on the coordinate grid to make polygons.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Give students 8 minutes quiet work time, 4 minutes for partner discussion, followed by whole-class discussion.

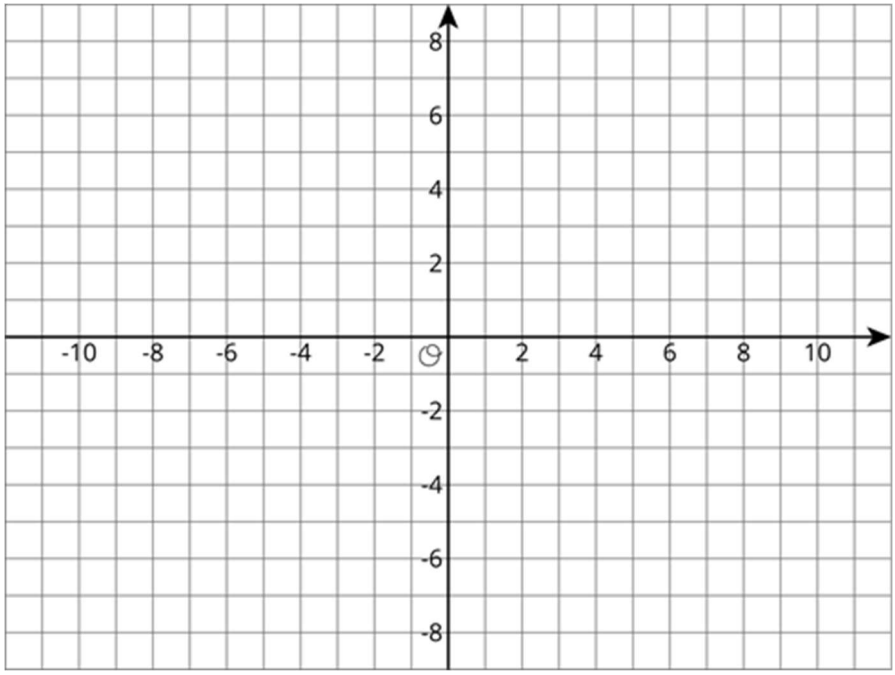
Students using digital materials will plot points and create polygons with a digital applet.

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select 2–3 of the polygons to plot on the coordinate grid. Chunking this task into more manageable parts may also benefit students who benefit from additional processing time.
Supports accessibility for: Organisation; Attention; Social-emotional skills

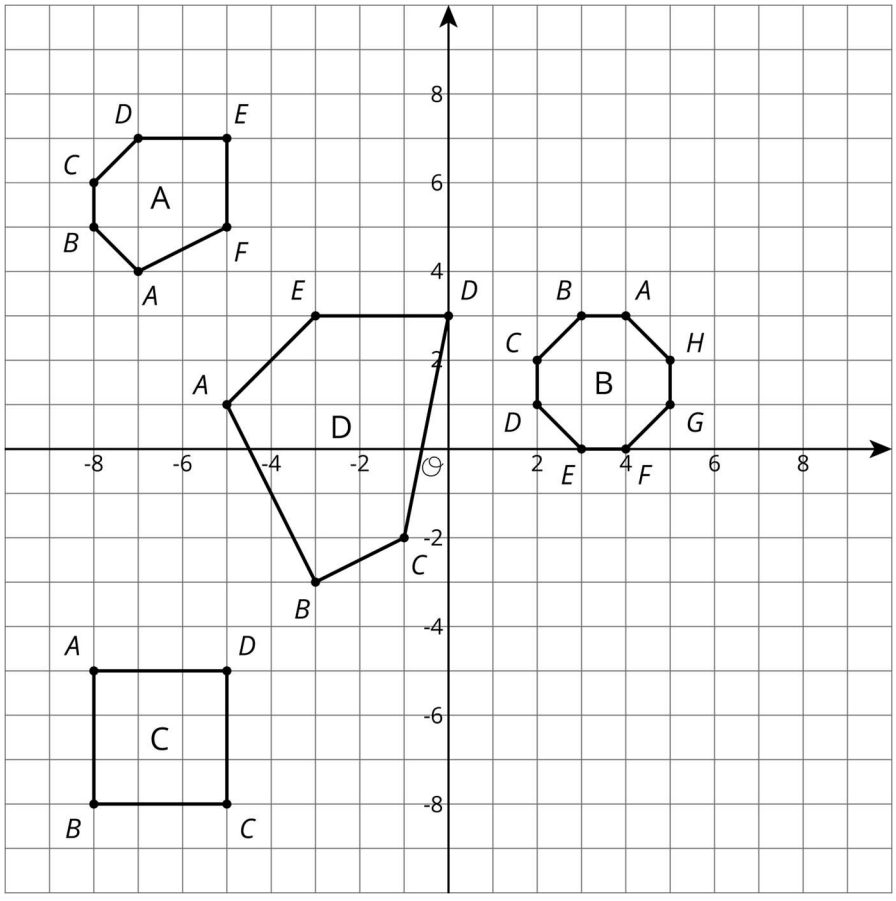
Student Task Statement

Here are the coordinates for four polygons. Plot them on the coordinate grid, connect the points in the order that they are listed, and label each polygon with its letter name.

1. Polygon A: (-7,4), (-8,5), (-8,6), (-7,7), (-5,7), (-5,5), (-7,4)
2. Polygon B: (4,3), (3,3), (2,2), (2,1), (3,0), (4,0), (5,1), (5,2), (4,3)
3. Polygon C: (-8,-5), (-8,-8), (-5,-8), (-5,-5), (-8,-5)
4. Polygon D: (-5,1), (-3,-3), (-1,-2), (0,3), (-3,3), (-5,1)



Student Response



Are You Ready for More?

Find the area of Polygon D in this activity.

Student Response

19.5 square units. There are many possible approaches to this problem: students can partition the polygon into squares and right triangles, or they can draw a rectangle around the polygon, then subtract the area outside the polygon from the area of the rectangle.

Activity Synthesis

The purpose of the discussion is to emphasise the connection between numbers, the coordinate grid, and geometry. To highlight these connections, ask:

- "How is the coordinate grid related to the number line?" (The coordinate grid has two axes that are both number lines.)
- "How are we able to make polygons on the coordinate grid?" (The vertices of a polygon are plotted as points on the coordinate grid.)

Complete the connection by explaining to students that the coordinate grid allows us to describe shapes and geometry in terms of numbers. This is how computers are able to create 2 and 3 dimensional images even though they can only interpret numbers.

Speaking: Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about the relationship between numbers, the coordinate grid, and geometry. As students share the connections they notice, revoice their statements using appropriate mathematical language, such as "points on the coordinate grid" or "the two axes of the coordinate grid."
Design Principle(s): Cultivate conversation

15.3 Four Quadrants of A-Maze-ing

15 minutes (there is a digital version of this activity)

The purpose of this task is for students to practise plotting coordinates in all four quadrants and find horizontal and vertical distances between coordinates in a puzzle. In past activities, students have been told the scale for the distance between grid lines, but here they must determine that each grid square has length 2 from the information given.

Instructional Routines

- Co-Craft Questions

Launch

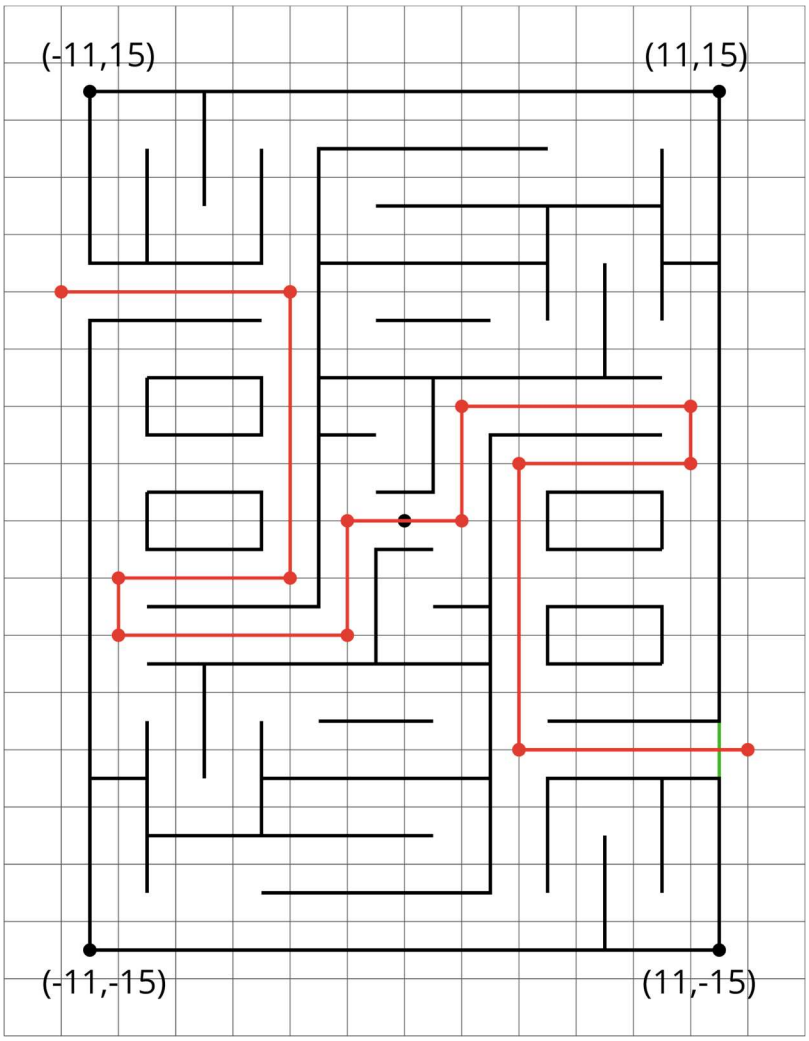
Arrange students in groups of 2. Tell students that they should not assume that each grid box is 1 unit. Give students 10 minutes quiet work time and 2 minutes for partner discussion. Follow with whole-class discussion.

Students using digital materials will be able to create a path through the maze by plotting points with an applet.

Representation: Internalise Comprehension. Check in with students after the first 2–3 minutes of work time. Check to make sure students have selected appropriate coordinates for the first points of Andre's route through the maze.
Supports accessibility for: Conceptual processing; Organisation Conversing: Co-craft Questions. Display only the picture of the maze and ask pairs of students to write possible mathematical questions about the situation. This is an opportunity for students to think about and relate to previous questions from previous lessons. Then, invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the coordinate grid and the points on the maze.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

1. The following diagram shows Andre's route through a maze. He started from the lower right entrance.



a. What are the coordinates of the first two and the last two points of his route?

b. How far did he walk from his starting point to his ending point? Show how you know.

2. Jada went into the maze and stopped at (-7,2).

a. Plot that point and other points that would lead her out of the maze (through the exit on the upper left side).

b. How far from (-7,2) must she walk to exit the maze? Show how you know.

Student Response

1.

a. The coordinates of the first two points are (12,-8) and (4,-8). The coordinates of the last two points are (-4,8) and (-12,8).

b. 80 units. Sample explanation: Counting grid squares as steps, Andre went 40 steps. Each step was 2 units, so the journey was a total of 80 units.

2.

a. Jada's path is (-7,2), (-4,2), (-4,8), (-12,8).

b. Jada's path is 17 units. Sample explanation: Counting grid squares as steps, Jada went 8.5 steps. Each step was 2 units, so the journey was a total of 17 units.

Activity Synthesis

The key idea is that it is possible to find distances and describe situations involving movement using the coordinate grid. This abstraction is important to appreciate because it means we can use numbers (in this case, *pairs* of numbers on the coordinate grid) to model situations that involve distance or movement. This will play a key role in later studies. To highlight these ideas, consider asking:

- How were you or your partner able to find the coordinates in the maze? Did you come up with any strategies or shortcuts?
- How did you find the distances that Andre and Jada travelled?
- What other situations involving movement could be represented with a coordinate grid?

Students may come up with examples like board games, maps, and perhaps even 3-dimensional examples.

Lesson Synthesis

Challenge students to create a drawing with a perimeter of 30 units using a continuous path of horizontal and vertical line segments. Ask students to identify the coordinates of vertices and justify that the perimeter is the given length. If time allows, arrange students in groups of 2 and ask them to draw their partner's shape on a coordinate grid with only verbal information and no coordinates. Students can check their drawing by asking for the exact coordinates. Ask students to explain why coordinates are useful for communicating information about flat space. Consider displaying student work for all to see throughout the rest of the unit. It may be interesting for students to see the variety of shapes that all have a perimeter of 30 units.

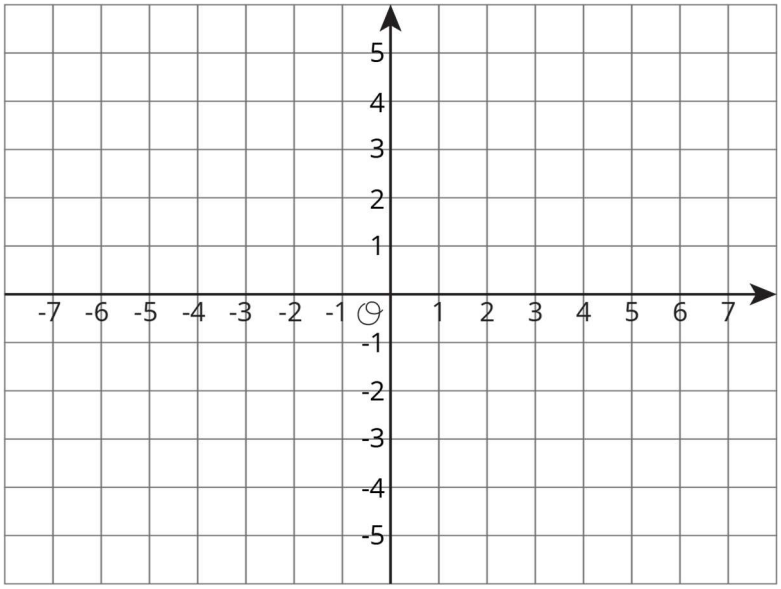
15.4 Perimeter of A Polygon

Cool Down: 5 minutes

Student Task Statement

1. Plot the following points on the coordinate grid and connect them to create a polygon.

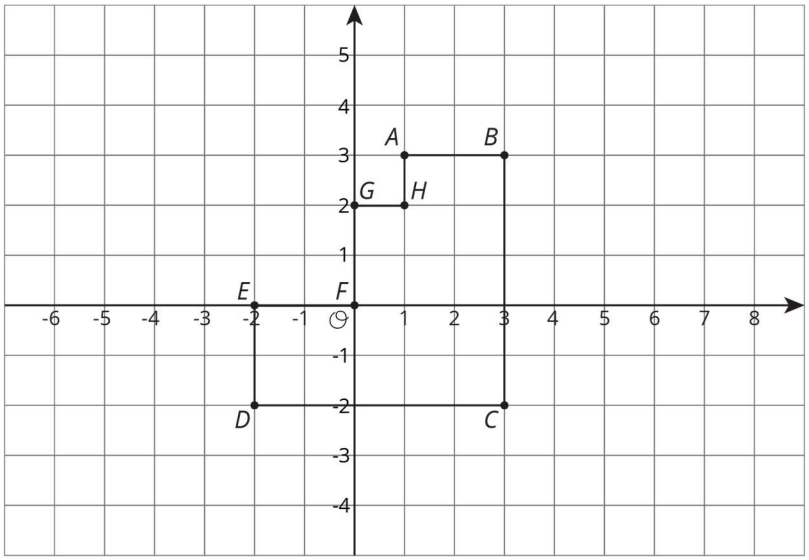
   $A = (1,3)$
   $B = (3,3)$
   $C = (3,-2)$
   $D = (-2,-2)$
   $E = (-2,0)$
   $F = (0,0)$
   $G = (0,2)$
   $H = (1,2)$
   $I = (1,3)$



2. Find the perimeter of the polygon.

Student Response

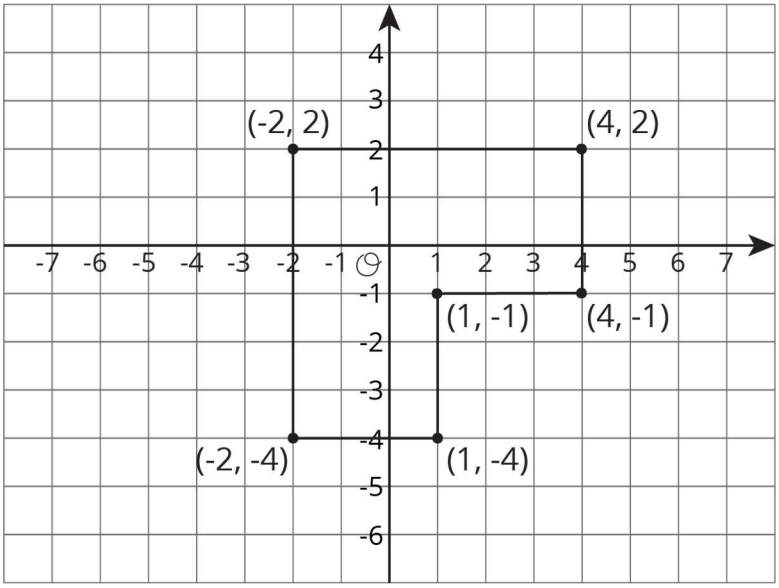
1.



2. The perimeter is 20 units.

Student Lesson Summary

We can use coordinates to find lengths of segments on the coordinate grid.



For example, we can find the perimeter of this polygon by finding the sum of its side lengths. Starting from (-2,2) and moving clockwise, we can see that the lengths of the segments are 6, 3, 3, 3, 3, and 6 units. The perimeter is therefore 24 units.

In general:

- If two points have the same x-coordinate, they will be on the same vertical line, and we can find the distance between them.
- If two points have the same y-coordinate, they will be on the same horizontal line, and we can find the distance between them.

Lesson 15 Practice Problems

1. **Problem 1 Statement**

   The coordinates of a rectangle are (3,0), (3,-5), (-4,0) and (-4,-5)

   a. What is the length and width of this rectangle?

   b. What is the perimeter of the rectangle?

   c. What is the area of the rectangle?

   Solution

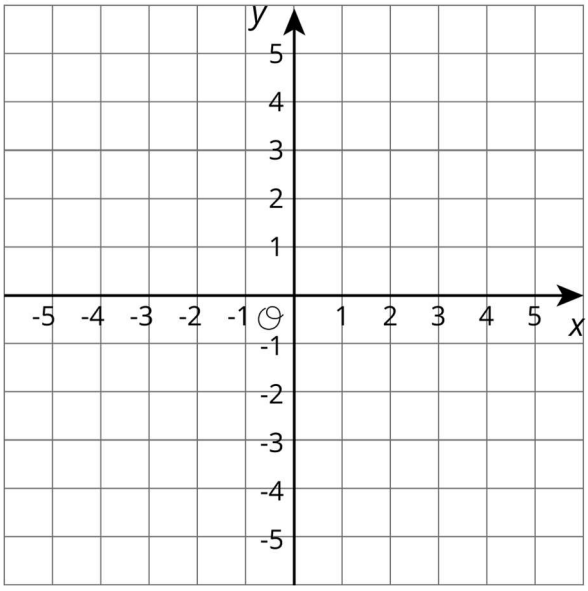
   a. The length is 7 units and the width is 5 units.

   b. The perimeter is 24 units, because $7 + 7 + 5 + 5 = 24$.

   c. The area is 35 square units, because $7 \times 5 = 35$.

2. **Problem 2 Statement**

   Draw a square with one vertex on the point (-3,5) and a perimeter of 20 units. Write the coordinates of each other vertex.



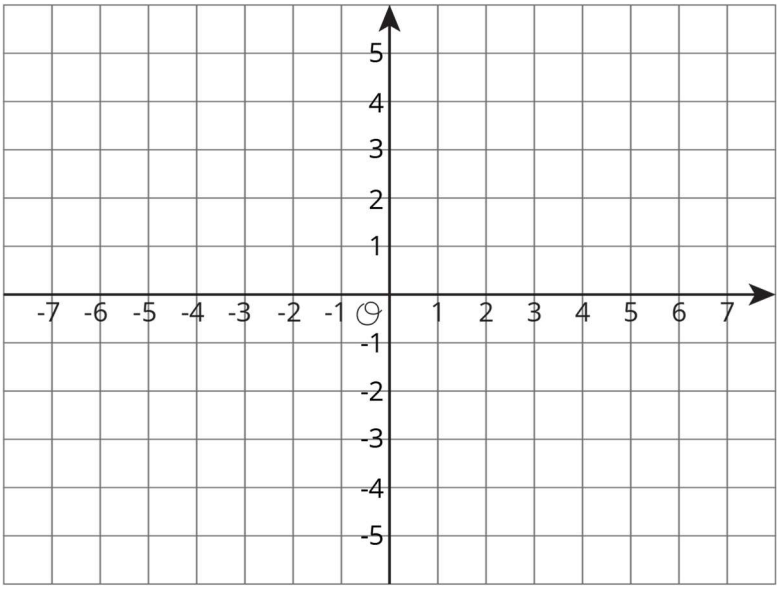
Solution

Answers vary. Sample response: The coordinates of each point are (-3,5), (-3,0), (2,5), (2,0).

3. **Problem 3 Statement**

   a. Plot and connect the following points to form a polygon.

   (-3,2), (2,2), (2,-4), (-1,-4), (-1,-2), (-3,-2), (-3,2)



   b. Find the perimeter of the polygon.

Solution

   a. The plotted polygon is a hexagon.

   b. 22 units. (Going in the same order as the points listed, the sides of the polygon have lengths 5, 6, 3, 2, 2, and 4 units.)

4. **Problem 4 Statement**

For each situation, select **all** the equations that represent it. Choose one equation and solve it.

   a. Jada's cat weighs 3.45 kg. Andre's cat weighs 1.2 kg more than Jada's cat. How much does Andre's cat weigh?

   $x = 3.45 + 1.2$

   $x = 3.45 - 1.2$

   $x + 1.2 = 3.45$

$x - 1.2 = 3.45$

b. Apples cost £1.60 per pound at the farmer's market. They cost 1.5 times as much at the grocery store. How much do the apples cost per pound at the grocery store?

$y = (1.5) \times (1.60)$

$y = 1.60 \div 1.5$

$(1.5)y = 1.60$

$\frac{y}{1.5} = 1.60$

Solution

a. $x = 3.45 + 1.2$
$x - 1.2 = 3.45$
$x = 4.65$

b. $y = (1.5) \times (1.60)$
$\frac{y}{1.5} = 1.60$
$y = 2.40$