Lesson 11: Interpreting graphs of proportional relationships

Goals

- Create the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and (0, 0).
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret (orally and in writing) points on the graph of a proportional relationship.

Learning Targets

- I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
- I can find the constant of proportionality from a graph.
- I understand the information given by graphs of proportional relationships that are made of points or a line.

Lesson Narrative

In the previous lesson students learned that the graph of a proportional relationship lies on a line through the origin. (Students should come to use and understand "the origin" to mean (0,0).) In this lesson, they start to make connections between the graph and the context modelled by the proportional relationship, and between the graph and the equation for the proportional relationship. Given a graph, they think about what situation it might represent and learn the importance of being precise about saying which quantities are represented on each axis. They interpret the meaning of the point $(1, k)$ on the graph both in terms of the constant of proportionality k in the equation $y = kx$ and in terms of a constant rate in the context.

Addressing

- Analyse proportional relationships and use them to solve real-world and mathematical problems.
- Recognise and represent proportional relationships between quantities.
- Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate.

Instructional Routines

- Stronger and Clearer Each Time
- Co-Craft Questions
- Think Pair Share

Required Materials

Rulers

Student Learning Goals

Let's read stories from the graphs of proportional relationships.

11.1 What Could the Graph Represent?

Warm Up: 5 minutes

This warm-up gives students an opportunity to think back to examples of proportional relationships they have encountered. Students are given a minute to think of some situations that could be represented by a graph. Several of their ideas should be shared with the class before students answer the remaining questions. During discussion, the characteristics of a graph of a proportional relationship should be reinforced.

Launch

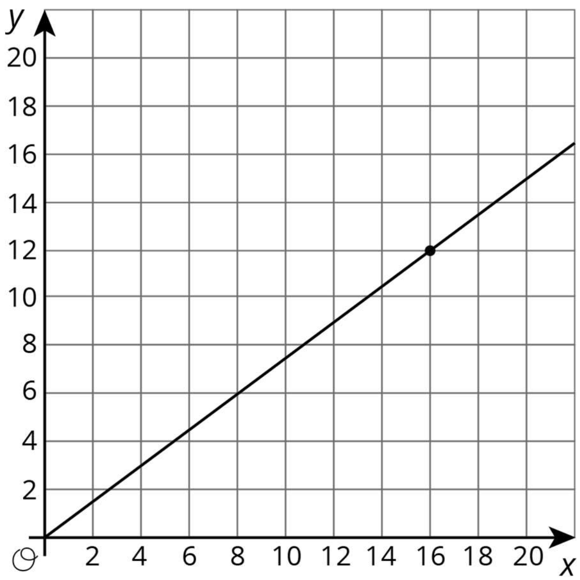
Tell students that they will look at an unlabelled graph, and their job is to think of a situation that the graph could represent. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have thought of a situation.

Invite some students to share their ideas and record the responses for all to see. (The purpose of this is to provide some inspiration to students who haven't come up with anything.) Ask students how they know all of the relationships are proportional. (Responses might include: when one value is 0 the other is 0, the situation involves equivalent ratios, or that any pair of values in the relationship has the same unit rate.)

Ask students to complete the rest of the questions.

Student Task Statement

Here is a graph that represents a proportional relationship.



1. Invent a situation that could be represented by this graph.
2. Label the axes with the quantities in your situation.
3. Give the graph a title.
4. There is a point on the graph. What are its coordinates? What does it represent in your situation?

Student Response

Answers vary. Possible response:

1. A car is moving at a constant speed. We could say that its speed is $\frac{3}{4}$ miles per minute or its pace is $\frac{4}{3}$ minutes per mile.
2. The horizontal axis is labelled time (minutes) and the vertical axis is labelled distance (miles).
3. Distance Travelled by a Car and How Much Time It Takes
4. The coordinates of the point are (16,12). In this situation, it means that the car travels 12 miles in 16 minutes.

Activity Synthesis

Ask a few students to share their situations and other responses. After each, ask the class if they need more information to understand the situation. After a few students have shared, ask the class to think about how all the situations were different and what they had in common. What sorts of things are always true about proportional relationships? Some possible responses might be:

- When one quantity is 0, the other is also 0.
- There is always the same amount of one quantity for every 1 of the other quantity.
- Context-specific considerations like constant speed, the same taste, or the same colour.

Remind students that a coordinate point, (x, y) is made up of the "x-coordinate" and the "y-coordinate."

11.2 Tyler's Walk

15 minutes

This activity is intended to further students' understanding of the graphs of proportional relationships in the following respects:

- points on the graph of a proportional relationship can be interpreted in the context represented
- for these points, the quotient of the coordinates is—excepting (0,0)—the constant of proportionality
- if the first coordinate is 1, then the corresponding coordinate is k, the constant of proportionality.

Students explain correspondences between parts of the table and parts of the graph. The graph is simple so that students can focus on what a point means in the situation represented. Students need to realise, however, that the axes are marked in 10-unit intervals. The discussion questions are opportunities for students to construct viable arguments and critique the reasoning of others.

Instructional Routines

- Co-Craft Questions
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organisation; Attention Speaking, Reading: Co-Craft Questions. To help students make sense of graphs of proportional relationships, start by displaying only the first line of this task ("Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.") and the graph. Ask students to write down

possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the remainder of the questions. Listen for and amplify any questions involving correspondences between parts of the table and parts of the graph. This helps students produce the language of mathematical questions and talk about the relationship between distance and time.
Design Principle(s): Maximise meta-awareness; Support sense-making

Anticipated Misconceptions

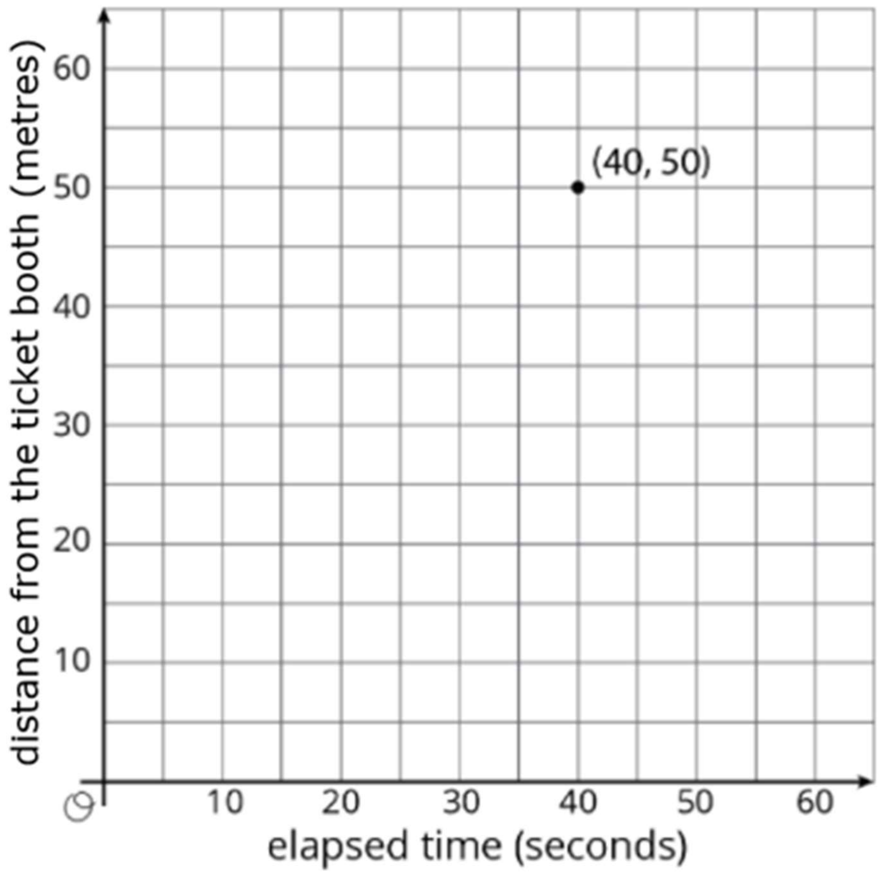
These questions can be used for discussion or for students who need scaffolding.

- "What quantities are shown in the graph?" (Distance in metres that Tyler is from the ticket booth and time elapsed in seconds since he started walking.)
- "How far is the ticket booth from the bumper cars?" (50 metres, assuming that Tyler walked in a straight line.) This is an opportunity for attention to precision and making explicit assumptions about a situation.
- "Do the values in your table show a proportional relationship? How do you know?" (Based on prior lessons in this unit, students should identify the relationship as proportional because for every point the unit rate is the same.)
- "What do the coordinates of the points on the graph show?" (The first coordinate gives amount of time in seconds that elapsed since Tyler started walking. Its corresponding second coordinate shows how many metres away from the ticket booth Tyler was at the corresponding time, assuming that Tyler walked in a straight line.)

Student Task Statement

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.

1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?
2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.
3. What does the point (0,0) mean in this situation?
4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.
5. What is the constant of proportionality for the relationship between time and distance? What does it tell you about Tyler's walk? Where do you see it in the graph?



time (seconds)	distance (metres)
0	0
20	25
30	37.5
40	50
1	

Student Response

1. 40 seconds after Tyler started walking, he was 50 metres from the ticket booth. The 40 represents the elapsed time in seconds since Tyler started walking away from the ticket booth; the 50 represents Tyler's distance in metres from the ticket booth at that time.

2. Students should write 1.25 in the empty cell of the table and plot (0,0), (1,1.25), (20,25), and (30,37.5).

3. Before any time passed, there was no distance between Tyler and the ticket booth.

4. Tyler was 1.25 metres from the ticket booth after 1 second. The corresponding point is $(1,1.25)$.

5. The constant of proportionality is 1.25. It tells us that Tyler is walking at a speed of 1.25 metres per second. It appears as the second coordinate in $(1,1.25)$.

Are You Ready for More?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

Student Response

The graph would be steeper. For the same first coordinate, the second coordinate would be twice as big as in the original situation. The table would include $(0,0)$, $(20,50)$, and $(1,2.5)$. Tyler would already arrive at the bumper cars after 20 seconds and his speed would be 2.5 metres per second.

Activity Synthesis

After students work on the task, it is important to discuss how the axis labels and the description in the task statement help us interpret points on the graph.

Consider asking these questions:

- "What quantities are shown in the graph?" (Distance in metres that Tyler is from the ticket booth and time elapsed in seconds since he started walking.)
- "How far is the ticket booth from the bumper cars?" (50 metres, assuming that Tyler walked in a straight line.) This is an opportunity for attention to precision and making explicit assumptions about a situation.
- "Do the values in your table show a proportional relationship? How do you know?" (Based on prior lessons in this unit, students should identify the relationship as proportional because for every point the unit rate is the same.)
- "What do the coordinates of the points on the graph show?" (The first coordinate gives amount of time in seconds that elapsed since Tyler started walking. Its corresponding second coordinate shows how many metres away from the ticket booth Tyler was at the corresponding time, assuming that Tyler walked in a straight line.)

Ask students for the equation of this proportional relationship. Finally ask where students see k, the constant of proportionality, in each representation, the equation, the graph, the table, and the verbal description of the situation.

11.3 Seagulls Eat What?

15 minutes (there is a digital version of this activity)

In this activity, make sure students understand what it means when we draw a solid line instead of just points in a straight line to represent the proportional relationship.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time followed by partner and whole-class discussion.

Writing, Conversing: Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. At the appropriate time, give students time to meet with 2–3 partners to share their response to the final question. Students should first check to see if they agree with each other about what the value of k means in the given context. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, “How did you calculate k?”, “Why did you do…?”, or “What does k mean in this context?” Then provide students with 3–4 minutes to revise their initial draft based on feedback from their peers.
Design Principle(s): Optimise output (for justification); Maximise meta-awareness

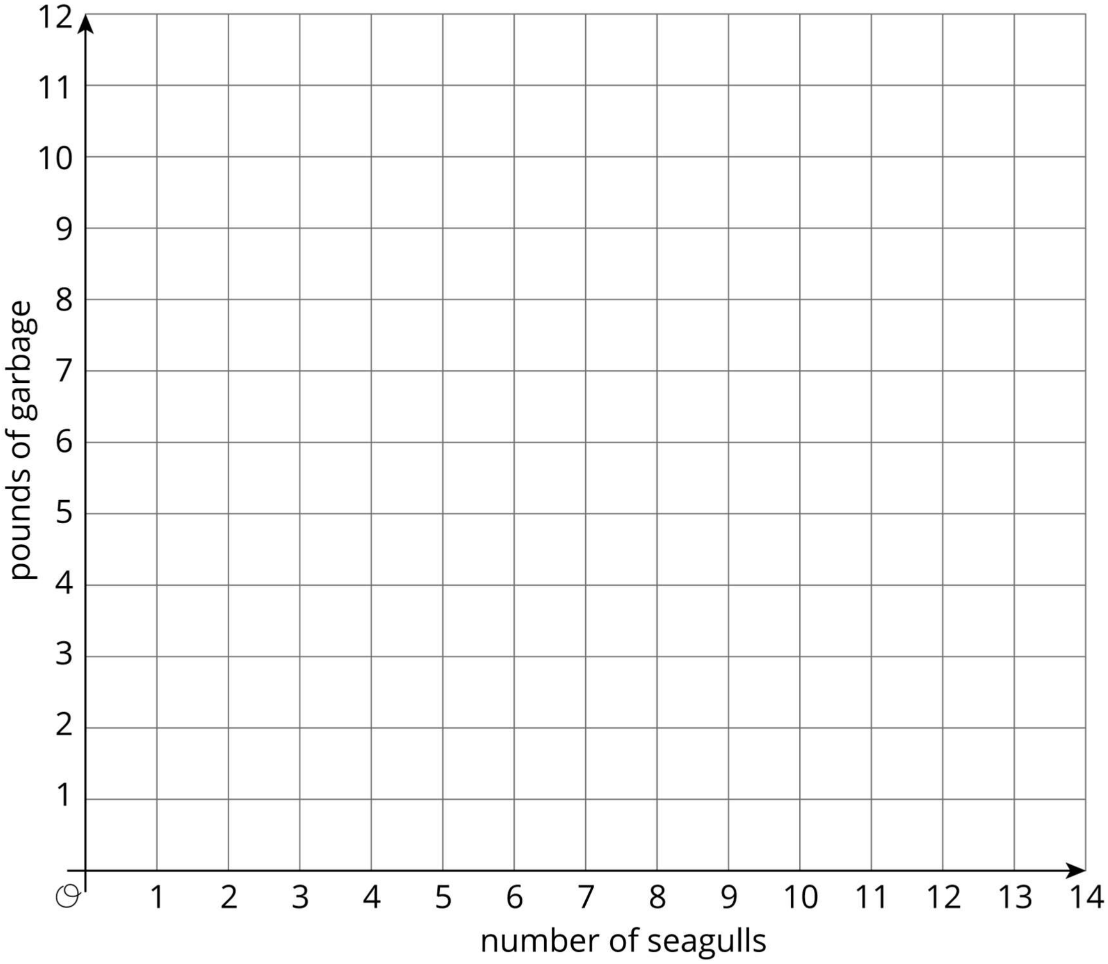
Anticipated Misconceptions

If students struggle to find k, encourage them to create a table with a few rows in it and ask them how they can use the table to find k.

Student Task Statement

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straight edge to draw a line through this point and (0,0).
3. Plot the point $(1, k)$ on the line. What is the value of k? What does the value of k tell you about this context?

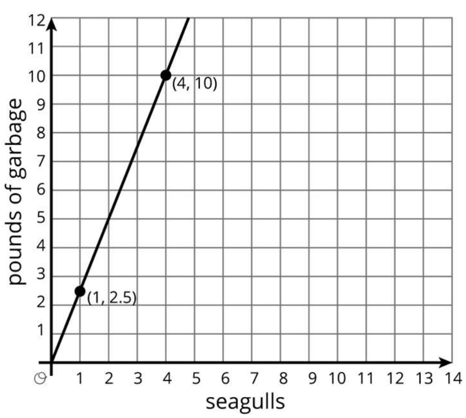


Student Response

1. Point (4,10) is plotted.

2. Line is drawn.

3. Point (1,2.5) is plotted. The value of k, 2.5, tells you the number of pounds of garbage consumed per seagull.



Activity Synthesis

Invite students to share their value and interpretation of k. Ask them for different ways to express this information. (*Each seagull eats 2.5 pounds of garbage* or *The rate of garbage consumption is 2.5 pounds per seagull*.)

Ask students if it is possible to interpret the meaning of each point on the solid line. (No, only whole numbers of seagulls make sense.) Ask, why it is still useful to draw in the line. "How can it help us to learn more about the situation?" (It helps us to easily find out how much garbage different numbers of seagulls eat. It also helps us to estimate the value of k.)

Lesson Synthesis

Revisit the key insights from this lesson:

1. We can interpret points on a graph in terms of the context it represents.

2. The y-value that goes with the x-value of 1 is special because it shows us the value of the constant of proportionality. It can be seen using a table or a graph.

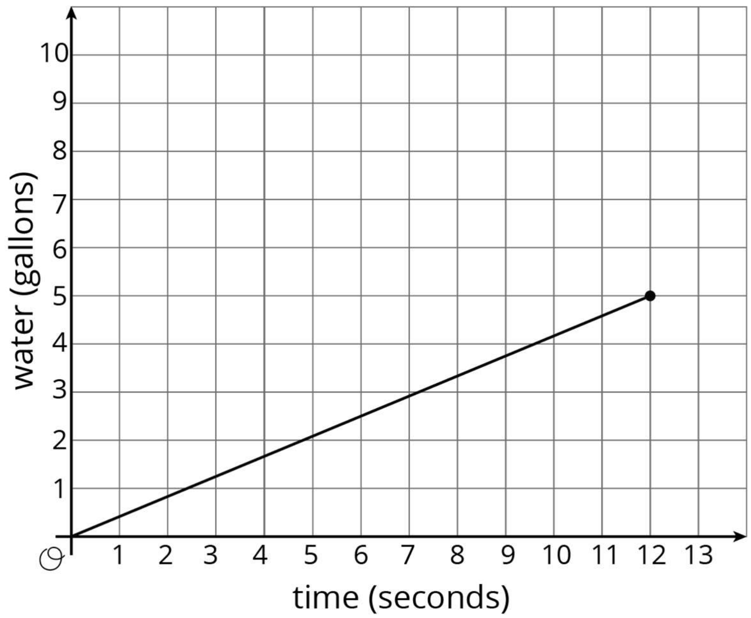
Display the completed graph from one of the activities. Choose a point on the graph and ask students to interpret its coordinates in the situation. Then choose the point with x-coordinate 1 and ask about the significance of its y-coordinate.

11.4 Filling a Bucket

Cool Down: 5 minutes

Student Task Statement

Water runs from a hose into a bucket at a steady rate. The amount of water in the bucket for the time it is being filled is shown in the graph.



1. The point (12,5) is on the graph. What do the coordinates tell you about the water in the bucket?

2. How many gallons of water were in the bucket after 1 second? Label the point on the graph that shows this information.

Student Response

1. After 12 seconds, there were 5 gallons of water in the bucket.

2. $\frac{5}{12}$ or equivalent. The point $\left(1, \frac{5}{12}\right)$ should be labelled.

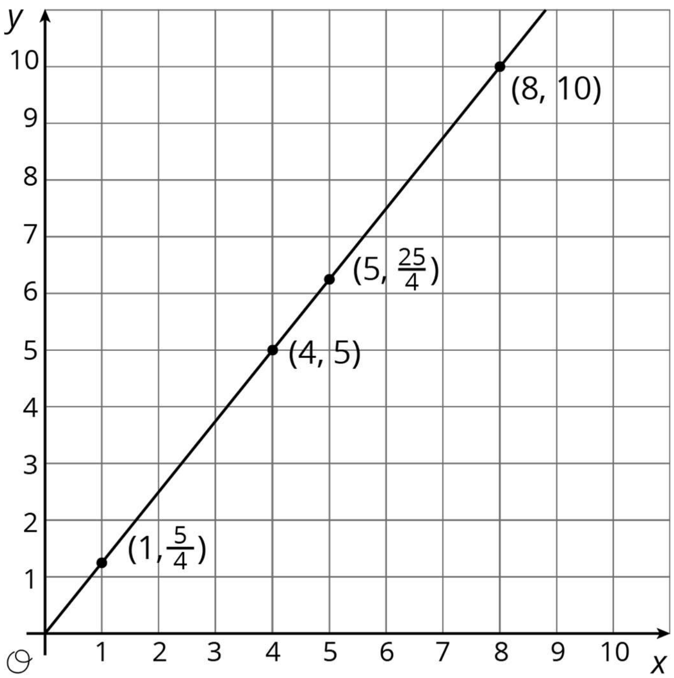
Student Lesson Summary

For the relationship represented in this table, y is proportional to x. We can see in the table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.

The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.



If y represents the distance in feet that a snail crawls in x minutes, then the point (4,5) tells us that the snail can crawl 5 feet in 4 minutes.

If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point (4,5) tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

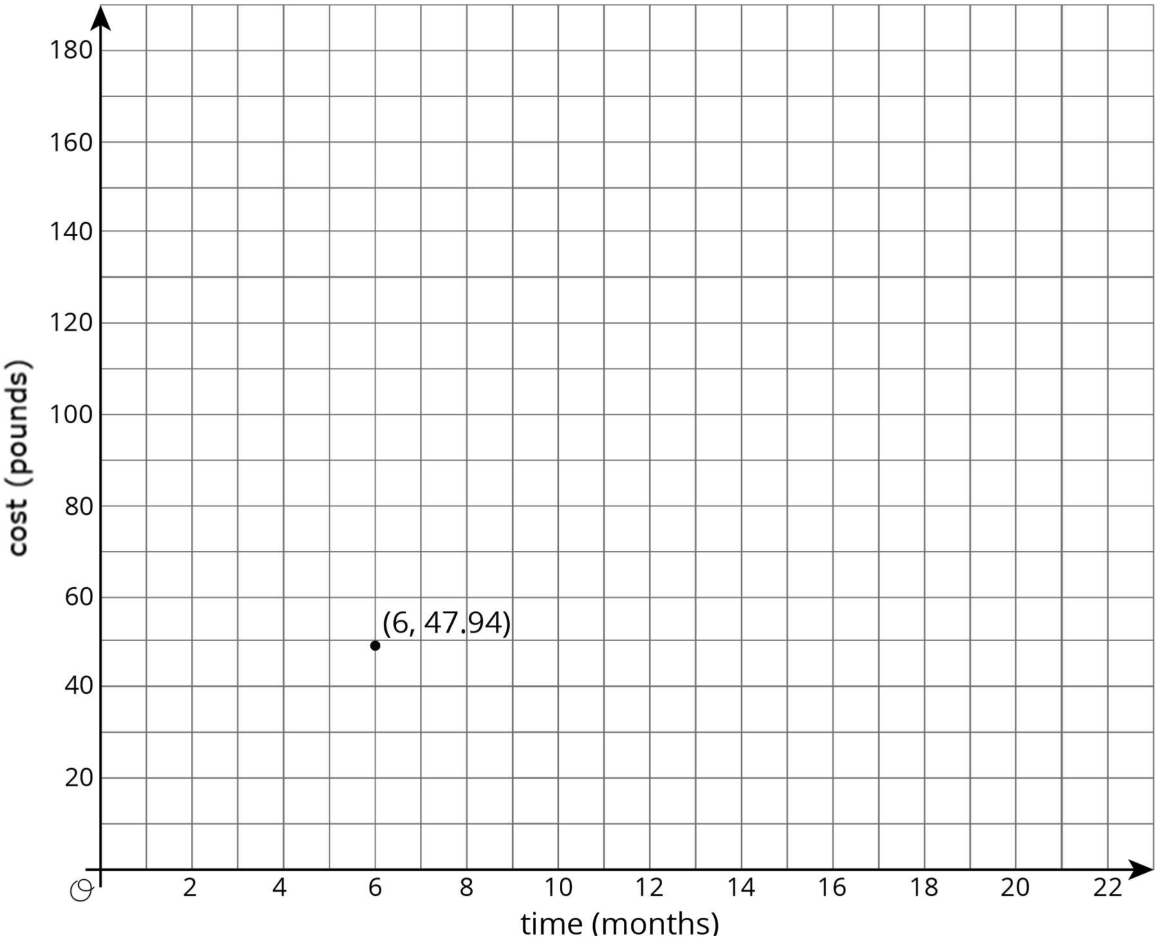
We can find the constant of proportionality by looking at the graph, because $\frac{5}{4}$ is the y-coordinate of the point on the graph where the x-coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x, the corresponding constant of proportionality is the y-value when $x = 1$.

Lesson 11 Practice Problems

1. **Problem 1 Statement**

   There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is £47.94. The point (6,47.94) is shown on the graph below.



a. What is the constant of proportionality in this relationship?

b. What does the constant of proportionality tell us about the situation?

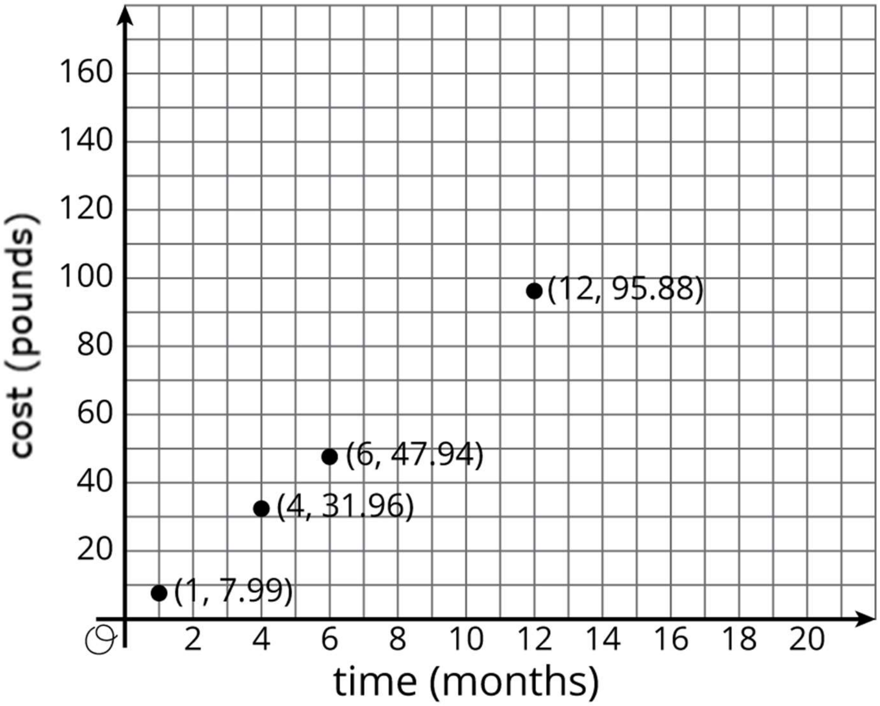
c. Add at least three more points to the graph and label them with their coordinates.

d. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

Solution

a. £7.99

b. The movie streaming service costs £7.99 for one month of service.

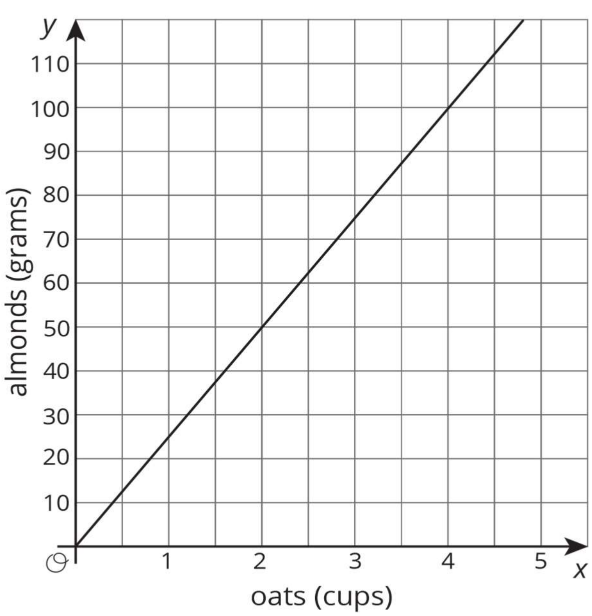


c.

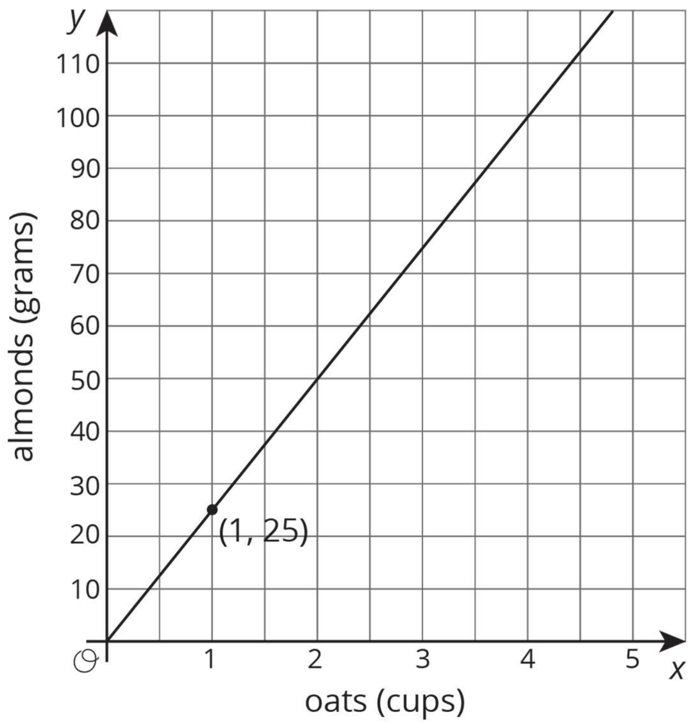
d. $C = 7.99m$

2. **Problem 2 Statement**

The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of k, and explain its meaning.



Solution



The point (1,25) is on the graph. It means that for each cup of oats there are 25 grams of almonds in the granola mix.

3. **Problem 3 Statement**

To make a friendship bracelet, some long strings are lined up then taking one string and tying it in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with the all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around your friend's wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

Solution

Yes, since each row will have the same number of knots in it, the number of knots will always be a multiple of the number of rows.

4. **Problem 4 Statement**

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

Solution

A constant of proportionality and variables for the quantities.