Lesson 2: Truth and equations

Goals

- Comprehend the word “variable” to refer to a letter standing in for a number and recognise that a coefficient next to a variable indicates multiplication (in spoken and written language).
- Generate values that make an equation true or false and justify (orally and in writing) whether they are “solutions” to the equation.
- Use substitution to determine whether a given number makes an equation true.

Learning Targets

- I can match equations to real life situations they could represent.
- I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a solution to the equation.

Lesson Narrative

Students begin the lesson by digging into what it means for an equation to be true or not true. They expand previously-held understandings of equations by thinking about the assumption that equations are always true. Students learn that a letter standing in for a number is called a variable. Students learn that, for an equation with a variable, a value of the variable that makes the equation true is called a solution of the equation. They find solutions to equations by using bar models or reasoning about the meaning of "solution" once an equation is written.

This lesson is where "next to" notation is introduced (for example, $10m$ means $10 \times m$).

Alignments

Addressing

- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Building Towards

- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Instructional Routines

- Collect and Display

- Compare and Connect
- Think Pair Share

Student Learning Goals

Let's use equations to represent stories and see what it means to solve equations.

2.1 Three Letters

Warm Up: 10 minutes

In this warm-up, students consider what it means for an equation to be true or false. They also practice substituting values for a letter and evaluating expressions with addition and multiplication. The term **variable** is introduced.

Launch

Allow students 2 minutes of quiet work time on the first part of the first question and ask them to pause their work. Ask a student to explain how they decided whether the equation was true or false given the values of a, b, and c. As the student explains, demonstrate this process by writing:

$a + b = c$

$3 + 4 = 5$

Since the sum of 3 and 4 is not 5, the equation is false for these values. Explain that a letter used to stand in for a number is called a **variable**. Throughout this unit, students will have many chances to understand and use this term.

Give students 2 minutes to complete the rest of the task.

Student Task Statement

1. The equation $a + b = c$ could be true or false.
   a. If a is 3, b is 4, and c is 5, is the equation true or false?
   b. Find new values of a, b, and c that make the equation true.
   c. Find new values of a, b, and c that make the equation false.
2. The equation $x \times y = z$ could be true or false.
   a. If x is 3, y is 4, and z is 12, is the equation true or false?
   b. Find new values of x, y, and z that make the equation true.
   c. Find new values of x, y, and z that make the equation false.

Student Response

Answers vary. Sample responses:

a. False

b. a is 1, b is 2, c is 3

c. a is 4, b is 5, c is 10

a. True

b. x is 3, y is 5, z is 15

c. x is 1, y is 2, z is 3

Activity Synthesis

The discussion should focus on the idea that an equation can be either true or false, and that the truth of an equation with variables depends on the values of the variables. This is an important idea to highlight, since throughout their work in previous years, the assumption that equations were true was likely unstated. Additionally, the equals sign may have been used previously to signal that a calculation should be done rather than communicate that two expressions were equal to each other. Share that we can write $2 + 3 = 7$, which students and teachers likely would have previously said was “wrong” or “a mistake,” but can now be understood as a mathematical statement that is not true.

Invite students to share their values for true and false equations. If students struggle with expressing their ideas about equations, some guiding questions might help:

- “What does it mean when an equation contains a letter?” (The letter is called a variable; it is standing in for a number.)
- “What makes an equation true? What makes an equation false?” (If the expressions on each side have the same value, the equation is true. If the expressions on each side have different values, the equation is false.)
- “How can we determine whether an equation is true or false?” (Evaluate both sides and check whether the values on each side of the equation are equal. If they are, then the equation is true. If they are not, the equation is false.)

2.2 Storytime

15 minutes

In this activity, students see how an equation can represent a situation with an unknown amount. Students are presented with three stories. Each story involves the same three quantities: 5, 20, and an unknown amount x. Students think about the actions (running a number of miles, splitting up cups of cat food) and relationships (five times as many clubs) in the situations and consider the operations needed to describe them. They also make sense of what the unknown quantity represents in each story and how to show its relationship to the other two numbers and the quantities they represent. Monitor for one student for each situation who chooses a correct equation and has a reasonable way to explain their reasoning, either verbally or by using a diagram.

Instructional Routines

- Compare and Connect

Launch

It is likely that "next to" notation is new for students. Explain that $20x$ means the same thing as $20 \times x$, and we will frequently use this notation from now on. Explain that in the term $20x$, 20 is called the **coefficient**. Use this term throughout the lesson when the need naturally arises to name it.

Explain that for each situation, the task is to find *one* equation that represents it. Some of the equations will go unused.

Give students 5 minutes of quiet work time to answer the questions, followed by a whole-class discussion.

Representation: Develop Language and Symbols. Activate or supply background knowledge. Invite students to create bar models to represent each situation, before selecting the equations that match.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Anticipated Misconceptions

Students who focus on key words might be misled in each situation. For the first situation, students might see the word "total" and decide they need to add 5 and 20. In the second situation, the words "five times as many" might prompt them to multiply 5 by 20. The third story poses some additional challenges: students see the word "divided" but there is no equation with division. Additionally, students might think that division always means divide the larger number by the smaller. Here are some ways to help students make sense of the situations and how equations can represent them:

- Suggest that students act out the situation or draw a picture. Focus on what quantity in the story each number or variable represents, and on the relationships among them.

- Use bar models to represent quantities and think about where a situation describes a total and where it describes parts of the total.

- Ask students about the relationships between operations. For the third situation, ask what operation is related to dividing and might help describe the situation.

Student Task Statement

Here are three situations and six equations. Which equation best represents each situation? If you get stuck, consider drawing a diagram.

$x + 5 = 20$

$x + 20 = 5$

$x = 20 + 5$

$5 \times 20 = x$

$5x = 20$

$20x = 5$

1. After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. She ran x miles before Friday.
2. Andre's school has 20 clubs, which is five times as many as his cousin's school. His cousin's school has x clubs.
3. Jada volunteers at the animal shelter. She divided 5 cups of cat food equally to feed 20 cats. Each cat received x cups of food.

Student Response

1. $x + 5 = 20$
2. $5x = 20$
3. $20x = 5$

Activity Synthesis

For each situation, ask a student to share which equation they selected and the reason they chose it. If any students drew a diagram to help them reason about the situation, ask them to present their diagram and draw connections between the situation, equation, and diagram. Consider these questions for discussion:

- "All of the equations and situations had the same numbers. Describe how you decided which equations represented which situations."
- "Did any of the words in the stories confuse or mislead you? How did you move past the confusion?"
- "Did you rule out any equations right from the start? Which ones? Why?" (Students might notice that $x + 20 = 5$ has no solutions if they have not yet learned about negative numbers. They might also incorrectly claim that $20x = 5$ is impossible if they are focused only on whole numbers.)

Speaking, Representing, Writing: Compare and Connect. When students share the equation they selected and the reason they chose it, ask the class questions to draw that attention to the connections between the situation, equation and diagram (e.g., “I see addition in the equation, where do you see addition in the situation?”). These exchanges help strengthen students’ mathematical language use and reasoning with multiple representations.
Design Principle(s): Maximise meta-awareness

2.3 Using Structure to Find Solutions

15 minutes

Having described situations with equations, students now solve equations by noticing and thinking about their structure and figuring out which value from a given set makes the equation true.

Instructional Routines

- Collect and Display
- Think Pair Share

Launch

Remind students that a letter used to represent an unknown value is called a variable. An equation with a variable can be true or false. A value that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In this task, students will look for solutions to equations. That means they will look for a value that can be used in place of a variable in an equation that makes the equation true.

Arrange students in groups of 2. Allow students 10 minutes to work quietly and share their responses with a partner, followed by a whole-class discussion.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: variable, solution. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.
Supports accessibility for: Memory; Language

Anticipated Misconceptions

Instead of solving, students might follow the operation symbol and combine the numbers in that way (for example, adding 12.6 and 4.1 to get 16.7 for the equation $12.6 = b + 4.1$). Encourage students to express the relationships of the equation in words and to draw diagrams that describe those statement. $12.6 = b + 4.1$ can be stated as “When a number is added to 4.1, the sum is 12.6.” The bar model then shows the parts 4.1 and an unknown quantity b, and a total of 12.6.

Student Task Statement

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, consider drawing a diagram. Be prepared to explain why your solution is correct.

1. $1\,000 - a = 400$
2. $12.6 = b + 4.1$
3. $8c = 8$
4. $\frac{2}{3} \times d = \frac{10}{9}$
5. $10e = 1$
6. $10 = 0.5f$
7. $0.99 = 1 - g$
8. $h + \frac{3}{7} = 1$

List:

$\frac{3}{7}$

$\frac{4}{7}$

$\frac{3}{5}$

$\frac{5}{3}$

$\frac{7}{3}$

0.01

0.1

0.5

1

2

8.5

9.5

16.7

20

400

600

1 400

Student Response

1. 600
2. 8.5
3. 1
4. $\frac{5}{3}$
5. 0.1
6. 20
7. 0.01
8. $\frac{4}{7}$

Are You Ready for More?

One solution to the equation $a + b + c = 10$ is $a = 2$, $b = 5$, $c = 3$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

Student Response

If a, b, and c are *positive*, there are 36 solutions. If $a = 1$, the possibilities are that $b = 1$ and $c = 8$, $b = 2$ and $c = 7$, and so on, giving 8 solutions. If $a = 2$, then b and c could respectively be 1 and 7, 2 and 6, 3 and 5, etc. This gives 7 solutions for $a = 2$. If $a = 9$ and all numbers are positive, there are no possible numbers for both b and c. The total number of solutions (for a value of 1 through 8) is $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ or 36.

If a, b, and c *non-negative* and includes 0, there are 66 solutions. If $a = 0$, there are 11 combinations of b and c. If $a = 1$, there are 10 combinations, and so on. The total number of solutions (for a value of 0 through 10) is $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, which is 66.

If a, b, and c are *any* integers (including negative), then there is an unlimited number of solutions.

Activity Synthesis

The goal of the discussion is to ensure that students understand what it means to be a solution to an equation. To guide the discussion, consider the following:

- Solicit or display correct solutions.
- Choose a few equations and ask students to explain how they know a solution is correct.
- Highlight correct uses of the new terms variable, solution, and coefficient.

Conversing, Representing, Writing: Collect and Display. Prepare students for the discussion by asking them to first share their responses to "Why are your solutions correct?" with a partner. Listen for, collect, and display vocabulary (e.g., coefficient, variable, true, false, solution) and include any diagrams that students use to represent each equation. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language during their paired and whole-group discussions.
Design Principles(s): Support sense-making; Cultivate conversation

Lesson Synthesis

The purpose of the discussion is to review appropriate use and understanding of new vocabulary and the concepts they represent. Consider asking some of the following questions:

- "What does it mean for an equation to be true? False? What does the equals sign have to do with whether the equation is true or false?" (A true equation has expressions with equal value on each side of the equals sign.)
- "Is an equation with a variable always true?" (It is only true for values of the variable that make the sides equal.)
- "What do we call a number written next to a variable? What does it mean?" (It is called the coefficient. Multiply the number by the value of the variable.)
- "Why might it be helpful to eliminate symbols that show multiplication?" (Answers vary. Sample response: It makes equations easier to read.)
- "What do we call a number that can be used in place of the variable that makes the equation true?" (It is called a solution of the equation.)

2.4 How Do You Know a Solution is a Solution?

Cool Down: 5 minutes

Student Task Statement

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing the sentences:

The word "solution" means . . .

88 is a solution to $\frac{1}{8}x = 11$ because . . .

Student Response

The word "solution" means a value that makes the equation true.

88 is a solution to $\frac{1}{8}x = 11$, because if x is 88, the equation is $\frac{1}{8} \times 88 = 11$, which is true.

Student Lesson Summary

An equation can be true or false. An example of a true equation is $7 + 1 = 4 \times 2$. An example of a false equation is $7 + 1 = 9$.

An equation can have a letter in it, for example, $u + 1 = 8$. This equation is false if u is 3, because $3 + 1$ does not equal 8. This equation is true if u is 7, because $7 + 1 = 8$.

A letter in an equation is called a **variable**. In $u + 1 = 8$, the variable is u. A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In $u + 1 = 8$, the solution is 7.

When a number is written next to a variable, the number and the variable are being multiplied. For example, $7x = 21$ means the same thing as $7 \times x = 21$. A number written next to a variable is called a **coefficient**. If no coefficient is written, the coefficient is 1. For example, in the equation $p + 3 = 5$, the coefficient of p is 1.

Glossary

- coefficient
- solution to an equation
- variable

Lesson 2 Practice Problems

1. **Problem 1 Statement**

   Select **all** the true equations.

a. $5 + 0 = 0$

b. $15 \times 0 = 0$

c. $1.4 + 2.7 = 4.1$

d. $\frac{2}{3} \times \frac{5}{9} = \frac{7}{12}$

e. $4\frac{2}{3} = 5 - \frac{1}{3}$

Solution ["B", "C", "E"]

2. **Problem 2 Statement**

Mai's water bottle had 24 ounces in it. After she drank x ounces of water, there were 10 ounces left. Select **all** the equations that represent this situation.

a. $24 \div 10 = x$

b. $24 + 10 = x$

c. $24 - 10 = x$

d. $x + 10 = 24$

e. $10x = 24$

Solution ["C", "D"]

3. **Problem 3 Statement**

Priya has 5 pencils, each x inches in length. When she lines up the pencils end to end, they measure 34.5 inches. Select **all** the equations that represent this situation.

a. $5 + x = 34.5$

b. $5x = 34.5$

c. $34.5 \div 5 = x$

d. $34.5 - 5 = x$

e. $x = (34.5) \times 5$

Solution ["B", "C"]

4. **Problem 4 Statement**

Match each equation with a solution from the list of values.

a. $2a = 4.6$

b. $b + 2 = 4.6$

c. $c \div 2 = 4.6$

d. $d - 2 = 4.6$

e. $e + \frac{3}{8} = 2$

f. $\frac{1}{8}f = 3$

g. $g \div \frac{8}{5} = 1$

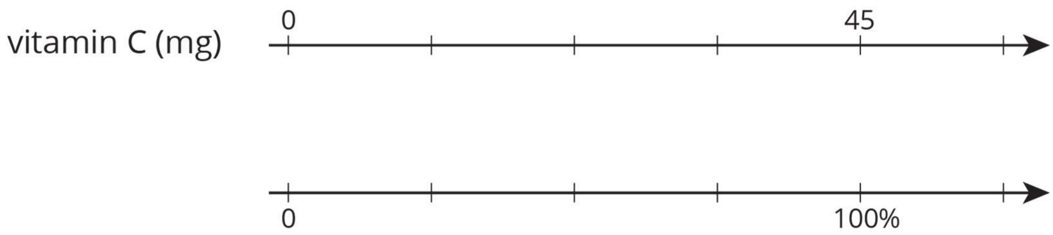
1. $\frac{8}{5}$
2. $1\frac{5}{8}$
3. 2.3
4. 2.6
5. 6.6
6. 9.2
7. 24

Solution

- A: 3
- B: 4
- C: 6
- D: 5
- E: 2
- F: 7
- G: 1

5. **Problem 5 Statement**

The daily recommended allowance of vitamin C for a Year 7 student is 45 mg. 1 orange has about 75% of the recommended daily allowance of vitamin C. How many milligrams are in 1 orange? If you get stuck, consider using the double number line.



Solution

33.75 mg. Sample reasoning using double number line diagram:

6. **Problem 6 Statement**

There are 90 kids in the band. 20% of the kids own their own instruments, and the rest rent them.

a. How many kids own their own instruments?

b. How many kids rent instruments?

c. What percentage of kids rent their instruments?

Solution

a. 18 kids ($90 \times 0.2 = 18$)

b. 72 kids ($90 - 18 = 72$)

c. 80% ($100 - 20 = 80$)

7. **Problem 7 Statement**

Find each product.

a. $(0.25) \times (1.4)$

b. $(0.061) \times (0.43)$

c. $(1.017) \times (0.072)$

d. $(5.226) \times (0.037)$

Solution

a. 0.35

b. 0.02623

c. 0.073224

d. 0.193362