Lesson 16: Common factors

Goals

- Comprehend (orally and in writing) the terms “factor,” “common factor,” and “highest common factor.”
- Explain (orally and in writing) how to determine the highest common factor of two whole numbers less than 100.
- List the factors of a number and identify common factors for two numbers in a real-world situation.

Learning Targets

- I can explain what a common factor is.
- I can explain what the highest common factor is.
- I can find the highest common factor of two whole numbers.

Lesson Narrative

In this lesson, students use contextual situations to learn about common factors and the highest common factor of two whole numbers. They develop strategies for finding common multiples and least common multiples. They develop a definition of the terms **common factor** and **highest common factor** for two whole numbers.

Addressing

- Find the highest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Three Reads

Required Materials

Graph paper
Multilink cubes

Required Preparation

For the first classroom activity, "Diego's Bake Sale," provide access to two different colours of multilink cubes (48 of one colour and 64 of the other) for students who would benefit from manipulatives. For students with visual impairment, provide access to manipulatives that are distinguished by their shape rather than colour.

In the second classroom activity, "Highest Common Factor," it may be helpful for some students to have access to graph paper to make rectangles that will help them find all possible factors of a whole number.

Student Learning Goals

Let's use factors to solve problems.

16.1 Shapes Made of Squares

Warm Up: 5 minutes

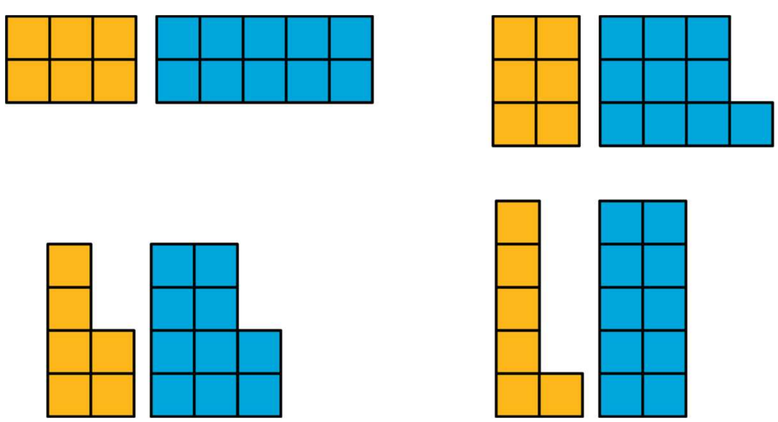
The purpose of this warm-up is for students to notice factors and common factors of 6 and 10 based on rectangles that can be made with the corresponding number of squares. Students may notice that the height of each pair of images changes, but they might not connect this to factors. The whole-class discussion should focus on how the images reflect factors of 6 and 10.

Launch

Tell students you will show them four pairs of images and their job is to find something that is similar and different about the pairs of images. Tell them to give a signal when they have at least one thing that is similar and one thing that is different. Give students 1 minute of quiet think time followed by a whole-class discussion.

Student Task Statement

How are the pairs of shapes alike? How are they different?



Student Response

Answers vary. Sample responses:

Similarities:

- Each pair has a blue shape and a yellow shape.
- Each shape is made of small squares.
- Each yellow shape is made up of 6 squares. Each blue shape is made up of 10 squares.

Differences:

- Some pairs have rectangles some do not.
- Some pairs have "L"-shaped shapes and some do not.
- The heights of each pair is going up by 1 every time: 2, 3, 4, 5
- The first pair with a height of 2 is the only one with two rectangles.
- The pair with a height of 4 is the only one without at least one rectangle.

Activity Synthesis

Ask students to share the things that are alike and different among the pairs of images. Record and display their responses for all to see. If possible, reference the images as the students share and record their responses on the images where appropriate.

If the following two ideas do not come up in the conversation, ask students these questions:

- “2 and 3 are both factors of 6. How is this reflected in the diagram?”
- “2 is a factor of both 6 and 10. How is this reflected in the diagram?”
- “4 is not a factor of either 6 or 10. How is this reflected in the diagram?”

End the discussion defining the term factor as one of two or more numbers, that when multiplied together result in a given product. In this particular case, a factor is the height that will make a rectangle have a given area.

16.2 Diego’s Bake Sale

15 minutes

Students begin to think about common factors and the highest common factor in the context of finding ways to group equal amounts of baked goods into bags. Students find all common factors of 2 whole numbers, one representing the number of brownies and another representing cookies. They then compare these factors to determine the highest common factor.

Monitor for strategies and representations students use to make sure they account for all possible combinations. Some students may organise their work by number of bags, checking each time if the total number can be divided into those bags evenly without remainder. Other students may notice that combinations come in pairs. For example, 4 bags of 12 brownies can be paired with 12 bags of 4 brownies.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads

Launch

Arrange students in groups of 2. Give students 10 minutes work time followed by whole-class discussion. Encourage students to check in with their partner after each question to make sure they get every possible combination of bags.

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects. Provide students with multilink cubes and containers to model brownie and cookie combinations.
Supports accessibility for: Conceptual processing Reading: Three Reads. Only reveal the situation, hiding the questions below until the third read. In the first read, done as a whole class, the goal for students to understand the situation (e.g., Diego makes both brownies and cookies for his bake sale). Make sure students understand the meaning of "equal-size" in this context. In the second read, ask students to name the quantities in the problem, noting exact numbers are not necessary at this time for all quantities (e.g., the number of brownies, the number of cookies, the number of bags). In the third read, done in pairs, reveal the questions and instruct students to discuss a plan to solve each part before doing any calculations. This will help students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students might not find all combinations of factor pairs for each number. If this is the case, ask them to use multilink cubes and prompt them to find more combinations. For example, "Is there a way to place 64 multilink cubes into 4 groups with no multilink cubes left over? How many are in each group?"

Student Task Statement

Diego is preparing brownies and cookies for a bake sale. He would like to make equal-size bags for selling all of the 48 brownies and 64 cookies that he has. Organise your answer to each question so that it can be followed by others.

1. How can Diego package all the 48 brownies so that each bag has the same number of them? How many bags can he make, and how many brownies will be in each bag? Find all the possible ways to package the brownies.

2. How can Diego package all the 64 cookies so that each bag has the same number of them? How many bags can he make, and how many cookies will be in each bag? Find all the possible ways to package the cookies.

3. How can Diego package all the 48 brownies and 64 cookies so that each bag has the same combination of items? How many bags can he make, and how many of each will be in each bag? Find all the possible ways to package both items.

4. What is the largest number of combination bags that Diego can make with no left over? Explain to your partner how you know that it is the largest possible number of bags.

Student Response

1. Diego can make the following bags with brownies:
   - 1 bag of 48 brownies or 48 bags containing 1 brownie
   - 2 bags with 24 brownies or 24 bags with 2 brownies
   - 3 bags with 16 brownies or 16 bags with 3 brownies
   - 4 bags with 12 brownies or 12 bags with 4 brownies
   - 6 bags with 8 brownies or 8 bags with 6 brownies
2. Diego can make the following bags with cookies:
   - 1 bag of 64 cookies or 64 bags containing 1 cookie
   - 2 bags with 32 cookies or 32 bags with 2 cookies
   - 4 bags with 16 cookies or 16 bags with 4 cookies
   - 8 bags with 8 cookies
3. Diego can make the following bags with both brownies and cookies:
   - 1 bag with 48 brownies and 64 cookies
   - 2 bags with 24 brownies and 32 cookies
   - 4 bags with 12 brownies and 16 cookies
   - 8 bags with 6 brownies and 8 cookies
   - 16 bags with 3 brownies and 4 cookies
4. The largest amount of combination bags that Diego can make is 16 bags with 3 brownies and 4 cookies.

Activity Synthesis

For questions 1 and 2, invite students to share how they organised the different combinations of bags, and highlight their different strategies. Sequence responses by first selecting students who found all combinations using pictorial representations, then students who made an organised list or table, and finally students who were able to highlight the fact that factors come in pairs (i.e. the number of bags and the number of brownies can always be reversed). During this discussion, ask students how they know that they have found all possible bag combinations for each number. Confirm that there should be 10 different bag arrangements for the brownies and 7 different bag arrangements for the chocolate chip cookies.

For questions 3 and 4, ask students to compare answers with a partner. Did they find all the combinations? Confirm that there are 5 different bag arrangements, and the highest number of bags that can be made is 16. Select a group that used multilink cubes to share what this arrangement looks like when represented with the two different colours. If possible, create a visual representation of this arrangement that can be displayed for all to see throughout the rest of the unit.

16.3 Highest Common Factor

15 minutes

In the last activity, students worked with the concept of common factors in the context of distributing two kinds of baked goods equally. In this activity, students explore common factors of numbers more generally and are introduced to the term **highest common factor**. The final question connects the concept of highest common factor to geometry. They describe what the highest common factor is and how it applies to a geometric context.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Ask students to discuss what they think a **common factor** of two numbers is with a partner and select for 1-2 groups to share their thinking. Give 10 minutes of quiet work time followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. After students have solved the first two problems, check in with either select groups of students or the whole class. Invite students to share the strategies they have used so far, as well as any questions they have before continuing.
Supports accessibility for: Organisation; Attention Speaking, Listening, Writing: Stronger and Clearer Each Time. Use this routine to support students to respond to the question "What do you think the term 'highest common factor' means?" Give students time to meet with 2-3 partners, sharing their responses. Encouraging the listener can provide feedback that will help teams strengthen their ideas and clarify their language (e.g., "What other details about highest common factors are important?"). After 2–3 successive shares, individuals can

refine and revise their original draft. This helps students use mathematical language to strengthen their understanding of highest common factor.
Design Principle(s): Cultivate conversation; Support sense-making

Anticipated Misconceptions

Some students may not list all of the factors of a number. Prompt these students to try to find more factors. If additional support is needed, provide graph paper and ask the student if it's possible to make a rectangle area equal to the number with a height of 1, 2, 3, 4, etc. until they are convinced they have all the factors.

Student Task Statement

1. The **highest common factor** of 30 and 18 is 6. What do you think the term "highest common factor" means?

2. Find all of the **factors** of 21 and 6. Then, identify the highest common factor of 21 and 6.

3. Find all of the factors of 28 and 12. Then, identify the highest common factor of 28 and 12.

4. A rectangular notice board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of coloured paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

    a. What is the side length of the largest square that Elena could use to cover the notice board completely without gaps and overlaps? Explain or show your reasoning.

    b. How is the solution to this problem related to highest common factor?

Student Response

1. Answers vary. Possible response: The highest common factor is the largest factor that numbers share.

2. The factors of 21 are 1, 3, 7, 21. The factors of 6 are 1, 2, 3, and 6. The highest common factor is 3.

3. The factors of 28 are 1, 2, 4, 7, 14, 28. The factors of 12 are 1, 2, 3, 4, 6, and 12. The highest common factor is 4.

4.

    a. The square is 3 inches wide. You could fit 4 squares by 9 squares within the rectangle.

    b. Explanations vary. Sample explanation: The side length of the square must divide 12 since the squares must stack vertically to reach exactly 12 inches. The side length of the square must also divide 27 since the squares must stack horizontally to reach exactly 27 inches. So the side length of the square in inches is a common factor of 12 and 27. That means the side length (in inches) of the largest such square is the highest common factor of 12 and 27, which is 3.

Are You Ready for More?

A school has 1 000 lockers, all lined up in a hallway. Each locker is closed. Then . . .

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

Student Response

The lockers that are open are 1, 4, 9, 16, 25... all of the square numbers up to 1 000. This is because most numbers have factor pairs and therefore have an even number of factors. For example, $6 = 1 \times 6$ and 2×3. So the locker is opened twice and shut twice, meaning that it is closed at the end of the process. The exceptions are the square numbers, which have an odd number of factors. For example, $25 = 1 \times 25$ and 5×5, which means that it is only touched three times: opened, closed, and then opened again.

Activity Synthesis

Ask students to share their thinking to question 4. Record and display their responses for all to see. Consider asking how their responses would change if the notice board was 18 inches tall and 63 inches wide instead. Encourage students to use the terms “common factor” and “highest common factor” in their explanations.

Lesson Synthesis

In this lesson, students learned about common factors of 2 whole numbers, as well as the highest common factor. Discuss:

- “What are some situations when finding highest common factor is helpful?” (When forming the largest amount of equal mixed groups with no items left over, or when determining the largest side length of a square that can be used to tile a rectangle)
- “Explain what highest common factor means.” (It is the largest factor that numbers share.)

- "How can we can determine the highest common factor?" (List the factors of each number, circle the ones that are the same, and then find the largest number that is the same.)

16.4 In Your Own Words

Cool Down: 5 minutes

Student Task Statement

1. What is the highest common factor of 24 and 64? Show your reasoning.

2. In your own words, what is the highest common factor of two whole numbers? How can you find it?

Student Response

1. 8. The common factors of 24 and 64 are 1, 2, 4, and 8, and 8 is the highest.

2. Answers vary. Sample response: The highest common factor of 2 whole numbers is the largest number that divides evenly into both numbers. You can find the highest common factor by listing the factors of each number and then finding the highest one that is the same for both numbers.

Student Lesson Summary

A factor of a whole number n is a whole number that divides n evenly without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly and without a remainder.

A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45; they are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **highest common factor** (sometimes written as HCF) of two whole numbers is the highest of all of the common factors. For example, 15 is the highest common factor for 45 and 60.

One way to find the highest common factor of two whole numbers is to list all of the factors for each, and then look for the highest factor they have in common. Let's try to find the highest common factor of 18 and 24. First, we list all the factors of each number.

- Factors of 18: **1**, **2**, **3**, **6**, 9,18

- Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the highest one, so 6 is the highest common factor of 18 and 24.

Glossary

- common factor

- highest common factor

Lesson 16 Practice Problems

1. **Problem 1 Statement**

A teacher is making gift bags. Each bag is to be filled with pencils and stickers. The teacher has 24 pencils and 36 stickers to use. Each bag will have the same number of each item, with no items left over. For example, she could make 2 bags with 12 pencils and 18 stickers each.

What are the other possibilities? Explain or show your reasoning.

Solution

3 bags with 8 pencils and 12 stickers ($3 \times 8 = 24$ and $3 \times 12 = 36$)

4 bags with 6 pencils and 9 stickers ($4 \times 6 = 24$ and $4 \times 9 = 36$)

6 bags with 4 pencils and 6 stickers ($6 \times 4 = 24$ and $6 \times 6 = 36$)

12 bags with 2 pencils and 3 stickers ($12 \times 2 = 24$ and $12 \times 3 = 36$)

2. **Problem 2 Statement**

a. List all the factors of 42.

b. What is the highest common factor of 42 and 15?

c. What is the highest common factor of 42 and 50?

Solution

a. 1, 2, 3, 6, 7, 14, 21, 42

b. 3

c. 2

3. **Problem 3 Statement**

A school chorus has 90 Year 7 students and 75 Year 8 students. The music director wants to make groups of performers, with the same combination of Year 7 and Year 8 students in each group. She wants to form as many groups as possible.

a. What is the largest number of groups that could be formed? Explain or show your reasoning.

b. If that many groups are formed, how many students of each year would be in each group?

Solution

a. 15 groups. The highest common factor of 75 and 90 is 15.

b. 6 Year 7 students and 5 Year 8 students ($6 \times 15 = 90$ and $5 \times 15 = 75$)

4. **Problem 4 Statement**

Here are some bank transactions from a bank account last week. Which transactions represent negative values?

Monday: £650 pay cheque deposited

Tuesday: £40 withdrawal from the ATM at the petrol pump

Wednesday: £20 credit for returned goods

Thursday: £125 deducted for mobile phone charges

Friday: £45 cheque written to pay for book order

Saturday: £80 withdrawal for weekend spending money

Sunday: £10 cash-back reward deposited from a credit card company

Solution

Tuesday, Thursday, Friday, and Saturday

5. **Problem 5 Statement**

Find the quotients.

a. $\frac{1}{7} \div \frac{1}{8}$

b. $\frac{12}{5} \div \frac{6}{5}$

c. $\frac{1}{10} \div 10$

d. $\frac{9}{10} \div \frac{10}{9}$

Solution

a. $\frac{8}{7}$

b. 2

c. $\frac{1}{100}$

d. $\frac{81}{100}$

6. **Problem 6 Statement**

An elephant can travel at a constant speed of 25 miles per hour, while a giraffe can travel at a constant speed of 16 miles in $\frac{1}{2}$ hour.

a. Which animal runs faster? Explain your reasoning.

b. How far can each animal run in 3 hours?

Solution

a. The giraffe is faster, because it covers more distance in the same amount of time.

b. The elephant can run 75 miles ($25 \times 3 = 75$), and the giraffe can run 96 miles ($16 \times 3 \times 2 = 96$).

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