Lesson 12: What is surface area?

Goals

- Calculate the surface area of a cuboid and explain (orally and in writing) the solution method.
- Comprehend that the term “surface area” (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.

Learning Targets

- I know what the surface area of a three-dimensional object means.

Lesson Narrative

This lesson introduces students to the concept of **surface area**. They use what they learned about area of rectangles to find the surface area of prisms with rectangular **faces** (cuboids).

Students begin exploring surface area in concrete terms, by estimating and then calculating the number of square sticky notes it would take to cover a filing cabinet. Because students are not given specific techniques ahead of time, they need to make sense of the problem and persevere in solving it. The first activity is meant to be open and exploratory. In the second activity, they then learn that the surface area (in square units) is the number of unit squares it takes to cover all the surfaces of a three-dimensional shape without gaps or overlaps.

Later in the lesson, students use cubes to build cuboids and then determine their surface areas.

Addressing

- Represent three-dimensional shapes using nets made up of rectangles and triangles, and use the nets to find the surface area of these shapes. Apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- Represent three-dimensional shapes using nets made up of rectangles and triangles, and use the nets to find the surface area of these shapes. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Compare and Connect
- Discussion Supports
- Notice and Wonder
- Poll the Class

- Think Pair Share

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Multi-link cubes

Required Preparation

- Prepare 12 cubes per student and extra copies of isometric dot paper for Building with Multi-link Cubes activity.
- Build several cuboids that are each 2 cubes by 3 cubes by 5 cubes for the cool-down.

Student Learning Goals

Let's cover the surfaces of some three-dimensional objects.

12.1 Covering the Cabinet (Part 1)

Warm Up: 5 minutes

This activity prepares students to think about surface area, which they explore in this lesson and upcoming lessons. Students watch a video of a cabinet being gradually tiled with non-overlapping sticky notes. The cabinet was left only partially tiled, which raises the question of the number of sticky notes it takes to cover the entire cuboid. Students estimate the answer to this question.

This activity was inspired by Andrew Stadel. Media used with permission. http://www.estimation180.com/filecabinet.

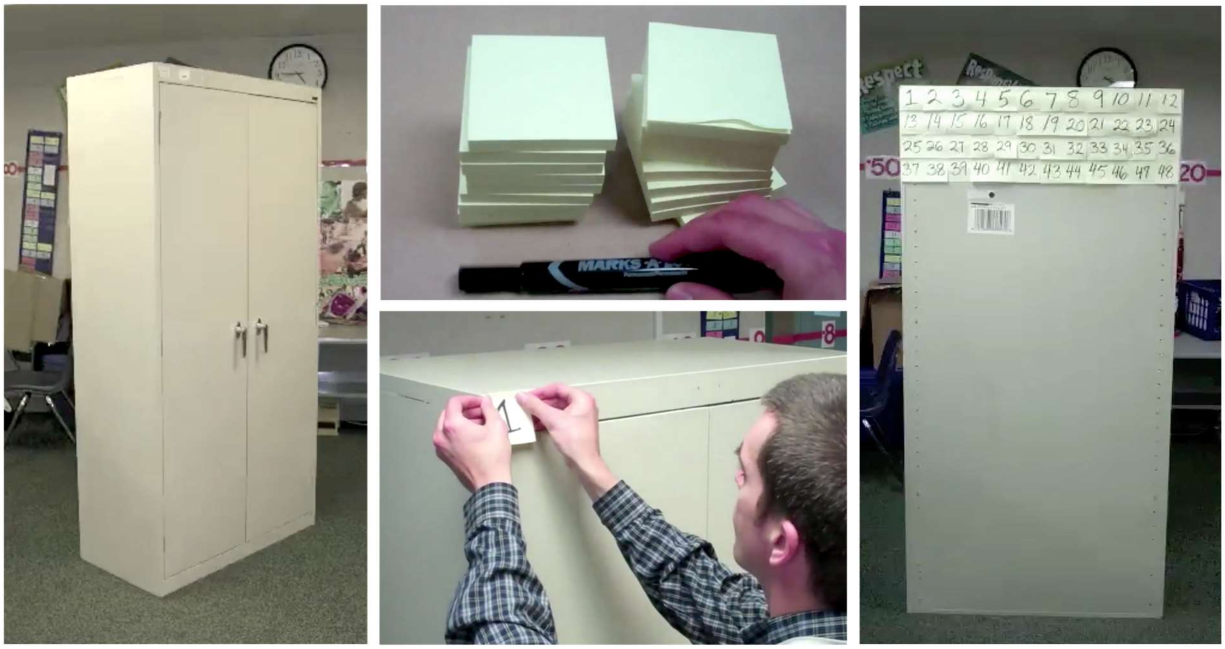
Instructional Routines

- Notice and Wonder
- Poll the Class

Launch

Arrange students in groups of 2. Show the video of a teacher beginning to cover a large cabinet with sticky notes or display the following still images for all to see. Before starting the video or displaying the image, ask students be prepared to share one thing they notice and one thing they wonder.

Video 'File Cabinet - Act 1' available here: https://player.vimeo.com/video/304136534.



Give students a minute to share their observation and question with a partner. Invite a few students to share their questions with the class. If the question “How many sticky notes would it take to cover the entire cabinet?” is not mentioned, ask if anyone wondered how many sticky notes it would take to cover the entire cabinet.

Give students a minute to make an estimate.

Student Task Statement

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

Student Response

Estimates vary. The actual number of sticky notes is 935. Good estimates are in the 800–1 200 range.

Activity Synthesis

Poll the class for students' estimates, and record them for all to see. Invite a couple of students to share how they made their estimate. Explain to students that they will now think about how to answer this question.

12.2 Covering the Cabinet (Part 2)

20 minutes

After making an estimate of the number of sticky notes on a cabinet in the warm-up, students now brainstorm ways to find that number more accurately and then go about calculating an answer. The activity prompts students to transfer their understandings of the area of polygons to find the **surface area** of a three-dimensional object.

Students learn that the surface area of a three-dimensional shape is the total area of all its faces. Since the area of a region is the number of square units it takes to cover the region without gaps and overlaps, surface area can be thought of as the number of square units that needed to cover all sides of an object without gaps and overlaps. The square sticky notes illustrate this idea in a concrete way.

As students work, notice the varying approaches taken to determine the number of sticky notes needed to tile the faces of the cabinet (excluding the bottom). Identify students with different strategies to share later.

Instructional Routines

- Compare and Connect

Launch

Arrange students in groups of 2–4. Give students 1 minute of quiet time to think about the first question and another minute to share their responses with their group. Ask students to pause afterwards.

Select some students to share how they might go about finding out the number of sticky notes and what information they would need. Students may ask for some measurements:

- The measurements of the cabinet in terms of sticky notes: Tell students that the cabinet is 24 by 12 by 6.
- The measurements of the cabinet in inches or centimetres: Tell students that you don't have that information and prompt them to think of another piece of information they could use.
- The measurements of each sticky note: Share that it is 3 inches by 3 inches.

If no students mention needing the edge measurements of the cabinet in terms of sticky notes, let them begin working on the second question and provide the information when they realise that it is needed. Give students 8–10 minutes for the second question.

Anticipated Misconceptions

Students may treat all sides as if they were congruent rectangles. That is, they find the area of the front of the cabinet and then just multiply by 5, or act as if the top is the only side that is not congruent to the others. If there is a real cabinet (or any other large object in the shape of a cuboid) in the classroom, consider showing students that only the sides opposite each other can be presumed to be identical.

Students may neglect the fact that the bottom of the cabinet will not be covered. Point out that the bottom is inaccessible because of the floor.

Student Task Statement

Earlier, you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes to cover the cabinet. Show your reasoning.

Student Response

1. Find the area of each side of the cabinet, excluding the bottom, and add them together. Needed information: measurements of the cabinet edge lengths in sticky notes.

2. Answers vary. Strategies may be a combination of the following two strategies:

    - Multiply the number of sticky notes along each edge of each side. Add all of the products.

    - Multiply the edge lengths of each side of the cabinet to find the area of each side. Add all of the areas.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

Student Response

Two cabinets: 1 582 sticky notes. Three cabinets: 2 229 sticky notes. Twenty cabinets: 13 228 sticky notes.

Activity Synthesis

Invite previously identified students or groups to share their answer and strategy. On a visual display, record each answer and each distinct process for determining the surface area (i.e. multiplying the side lengths of each rectangular face and adding up the products). After each presentation, poll the class on whether others had the same answer or process.

Play the video that reveals the actual number of sticky notes needed to cover the cabinet. If students' answers vary from that shown on the video, discuss possible reasons for the differences. (For example, students may not have accounted for the cabinet's door handles. Some may have made a calculation error.)

Tell students that the question they have been trying to answer is one about the surface area of the cabinet. Explain that the **surface area** of a three-dimensional shape is the total area of all its surfaces. We call the flat surfaces on a three-dimensional shape its **faces**.

The surface area of a cuboid would then be the combined area of all six of its faces. In the context of this problem, we excluded the bottom face, since it is sitting on the ground and will not be tiled with sticky notes. Discuss:

- "What unit of measurement are we using to represent the surface area of the cabinet?" (Square sticky notes)
- "Would the surface area change if we used larger or smaller sticky notes? How?" (Yes, if we use larger sticky notes, we would need fewer. If we use smaller ones, we would need more.)

Speaking, Listening: Compare and Connect. As students share their strategies for determining the number of sticky notes that cover the cabinet, ask students to make connections between the various strategies. Some students will calculate the number of sticky notes that will cover each of the five faces of the cabinet and add them together. Other students may realise that opposite faces of the cabinet are congruent so it is only necessary to calculate the area of three faces of the cabinet. Encourage students to explain why both methods result in the same answer. This will promote students' use of mathematical language as they make sense of the various methods for finding the surface area of a cuboid.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

12.3 Building with Multi-link Cubes

20 minutes (there is a digital version of this activity)

This activity encourages students to apply strategies for finding the area of polygons to finding the *surface area* of cuboids. Students use 12 cubes to build a cuboid, think about its surface area, and use isometric dot paper to draw their cuboid.

As students build their cuboids, notice those with different designs and those with the same design but different approaches to finding surface area (e.g. by counting individual square, by multiplying the edge lengths of rectangular faces, etc.).

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Read the first two lines of the task statement together. Remind students that we refer to the flat surfaces of a three-dimensional shape as **faces**.

Give students a minute to think about how we know the surface area of the shown cuboid is 32 square units. Ask 1–2 students to explain their reasoning to the class. Use students' explanations to highlight the meaning of surface area, i.e., that the area of all the faces need to be accounted for, including those we cannot see when looking at a two-dimensional drawing.

Tell students they will use 12 cubes to build a different cuboid, draw it, and find its surface area. Consider doing a quick demonstration on how to draw a simple cuboid on isometric dot paper. (Start with one cube and then add a cube in each dimension.) Tell students that in this activity, we call each face of a single cube, "1 square unit."

Give each student 12 cubes to build a cuboid and 6–8 minutes of quiet work time. If students are using multi-link cubes, say that we will pretend all of the faces are completely smooth, so they do not need to worry about the “innies and outies” of the multi-link cubes.

As students work, consider arranging two students with contrasting designs or strategies as partners. Ask partners to share their answers, explanations, and drawings. Stress that each partner should focus their explanation on how they went about finding surface area. The listener should think about whether the explanation makes sense or if anything is amiss in the reasoning.

For students in digital classrooms, an applet can be used to build and draw cuboids. Physical cubes are still recommended and preferred for the building of the shapes, however.

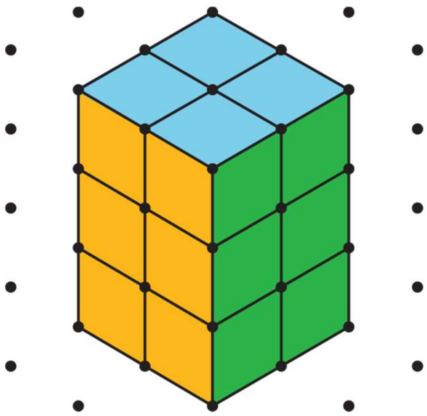
Action and Expression: Develop Expression and Communication. Support multiple forms of communication. Some students may benefit from an explicit demonstration and additional practice to learn how to draw cubes using isometric dot paper. Invite students who are unable to represent their shapes using isometric dot paper to explain their reasoning orally, using virtual or concrete manipulatives.
Supports accessibility for: Visual-spatial processing; Conceptual processing; Fine-motor skills

Anticipated Misconceptions

Students may count the faces of the individual multi-link cubes rather than faces of the completed cuboid. Help them understand that the faces are the visible ones on the outside of the shape.

Student Task Statement

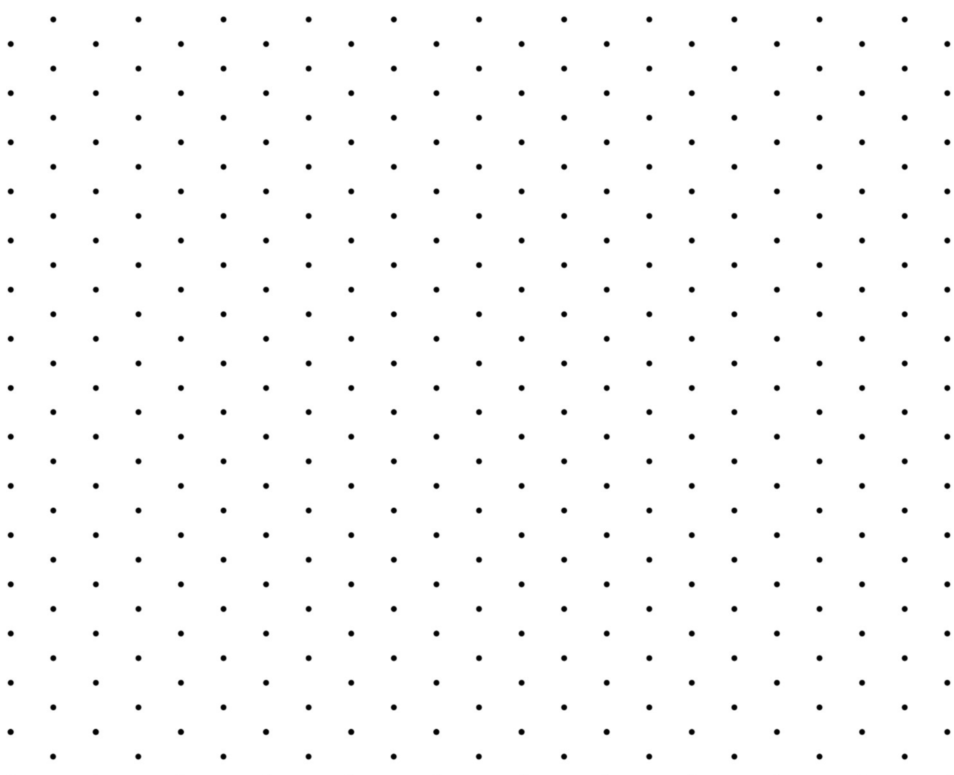
Here is a sketch of a cuboid built from 12 cubes:



It has six **faces**, but you can only see three of them in the sketch. It has a **surface area** of 32 square units.

Your teacher will give you 12 multi-link cubes. Use all of your multi-link cubes to build a different cuboid (with different edge lengths than the cuboid shown here).

1. How many faces does your shape have?
2. What is the surface area of your shape in square units?
3. Draw your shape on isometric dot paper. Colour each face a different colour.



Student Response

1. There are 6 faces—front, back, left, right, top, and bottom.

2. Answers vary based on design. Sample responses:

   - For a cuboid that is 12 unit by 1 unit by 1 unit, the surface area is 50 square units. $(4 \times 12) + (2 \times 1) = 50$

   - For a cuboid that is 6 units by 2 units by 1 unit, the surface area is 40 square units. $(2 \times 12) + (2 \times 6) + (2 \times 2) = 40$

   - For a cuboid that is 4 units by 3 units by 1 unit, the surface area is 38 square units. $(2 \times 12) + (2 \times 4) + (2 \times 3) = 38$

3. Drawings vary, but all cuboids should have one edge length that is 1 unit.

Activity Synthesis

After partner discussions, select a student to highlight for the class the strategy (or strategies) for finding surface area methodically. Point out that in this activity each face of their cuboid is a rectangle, and that we can find the area of each rectangle by multiplying its side lengths and then add the areas of all the faces.

Explain that later, when we encounter non-cuboids, we can likewise reason about the area of each face the way we reasoned about the area of a polygon.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language (e.g., cubes, faces, surface area, square units, etc.), to restate and/or revoice the strategy (or strategies) presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. This will provide more students with an opportunity to produce language that describes strategies for finding surface area.
Design Principle(s): Support sense-making; Maximise meta-awareness

Lesson Synthesis

In this lesson, we found the surface areas of a cabinet and of cuboids built out of cubes.

- "What does it mean to find the **surface area** of a three-dimensional shape?" (It means finding the number of unit squares that cover the entire surface of the object without gaps or overlaps.)

- "How can we find the number of unit squares that cover the entire surface of an object?" (We can count them, or we can find the area of each **face** of the object and add the areas of all faces.)

- "How are finding surface area and finding area alike? How are they different?" (They both involve finding the number of unit squares that cover a region entirely without gaps and overlaps. Both have to do with two-dimensional regions. Finding area involves a single polygon. Finding surface area means finding the sum of the areas of multiple polygons (faces) of which a three-dimensional shape is composed.)

12.4 A Multi-link Cube Cuboid

Cool Down: 5 minutes

Launch

Prepare several cuboids that are each 2 cubes by 3 cubes by 5 cubes. Display one for all to see and pass the rest around for students to examine, if needed.

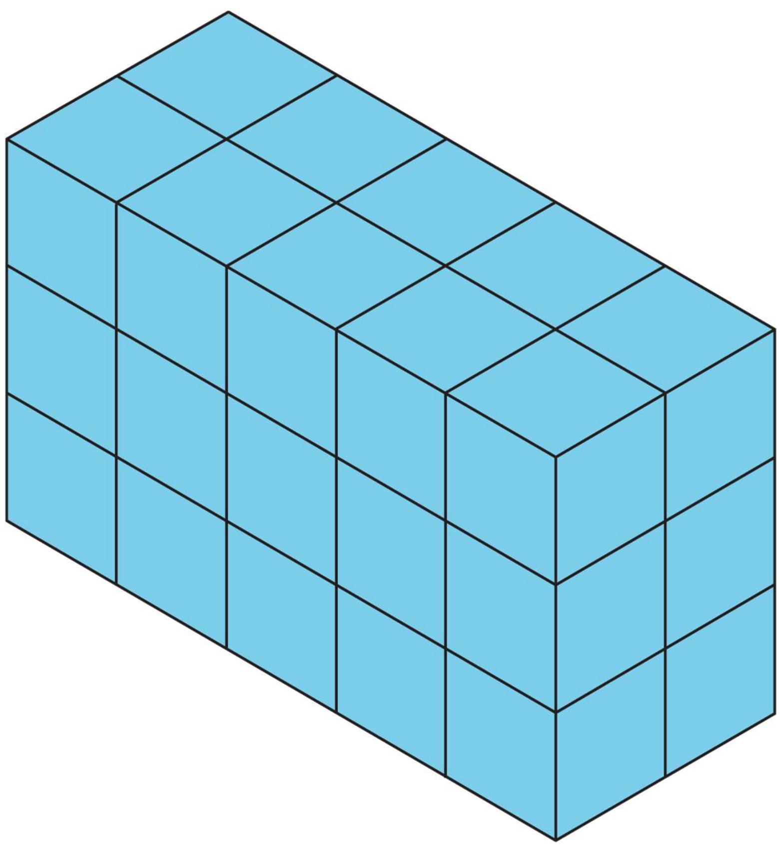
Anticipated Misconceptions

Students may not include the bottom face as it is not visible when the cuboid is sitting on a table.

Students may find the number of cubes instead of the surface area due to their previous volume work with cuboids in KS2.

Student Task Statement

A cuboid made is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units? Explain or show your reasoning.



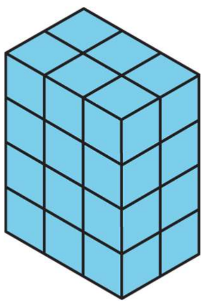
Student Response

The surface area is $2 \times [(3 \times 5) + (2 \times 5) + (2 \times 3)] = 62$ (or 62 square units).

Student Lesson Summary

- The **surface area** of a shape (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional shape has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a cuboid has six faces. The surface area of the cuboid is the total of the areas of the six rectangular faces.



So the surface area of a cuboid that has edge-lengths 2 cm, 3 cm, and 4 cm has a surface area of $(2 \times 3) + (2 \times 3) + (2 \times 4) + (2 \times 4) + (3 \times 4) + (3 \times 4)$ or 52 square centimetres.

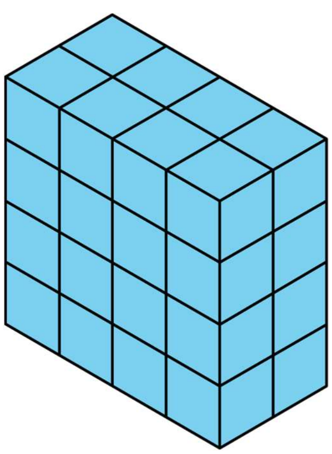
Glossary

- face
- surface area

Lesson 12 Practice Problems

1. **Problem 1 Statement**

   What is the surface area of this cuboid?



   a. 16 square units

b. 32 square units

c. 48 square units

d. 64 square units

Solution D

2. **Problem 2 Statement**

Which description can represent the surface area of this trunk?

a. The number of square inches that cover the top of the trunk.

b. The number of square feet that cover all the outside faces of the trunk.

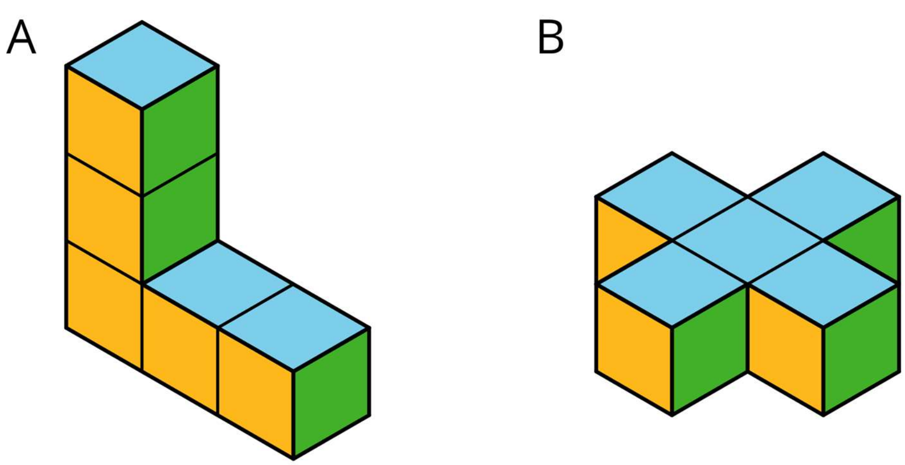
c. The number of square inches of horizontal surface inside the trunk.

d. The number of cubic feet that can be packed inside the trunk.

Solution B

3. **Problem 3 Statement**

Which shape has a greater surface area?



Solution

Shape A and shape B have the same surface area of 22 square units.

4. **Problem 4 Statement**

A cuboid is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

Solution

88 square units. Two faces are 4 units by 2 units, amounting to 16 square units. Two faces are 4 units by 6 units, amounting to 48 square units. Two faces are 2 units by 6 units, amounting to 24 square units. $16 + 48 + 24 = 88$.

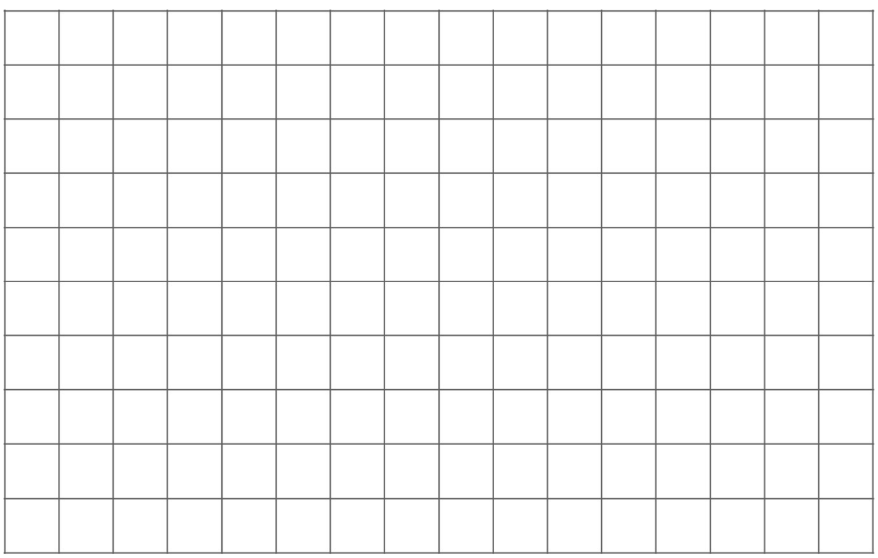
5. **Problem 5 Statement**

Draw an example of each of these triangles on the grid.

a. A right-angled triangle with an area of 6 square units.

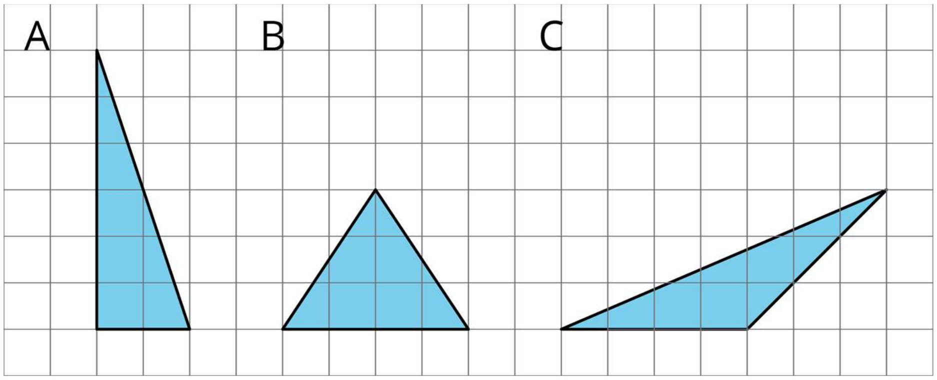
b. An acute-angled triangle with an area of 6 square units.

c. An obtuse-angled triangle with an area of 6 square units.



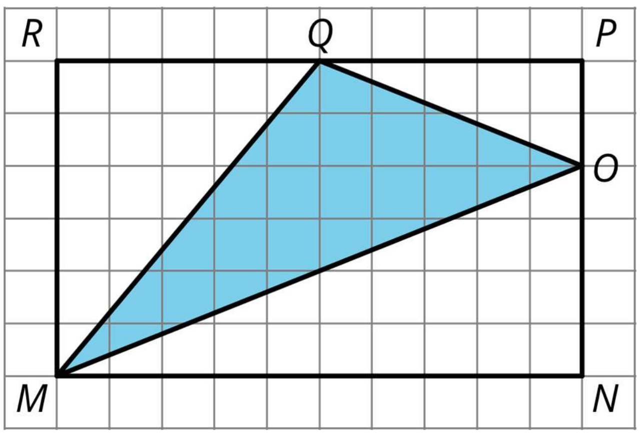
Solution

Answers vary. Sample response:



6. **Problem 6 Statement**

Find the area of triangle MOQ in square units. Show your reasoning.



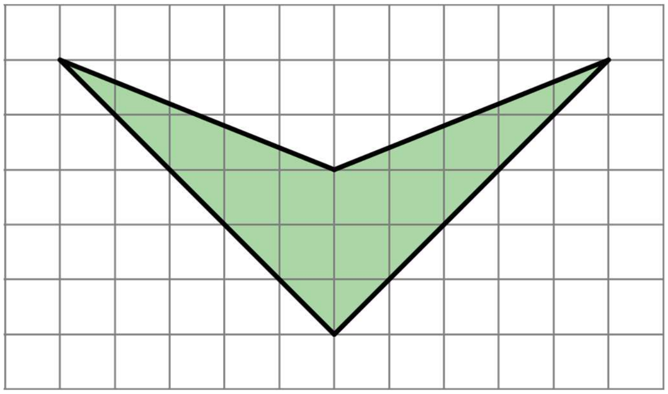
Solution

20 square units. Reasoning varies. Sample reasoning: The area of triangle MOQ can be found by subtracting the areas of the three right-angled triangles from the area of rectangle $MNPR$.

- The area of rectangle $MNPR$ is 10×6 or 60 square units.
- The area of triangle QRM is $\frac{1}{2} \times 6 \times 5$ or 15 square units.
- The area of triangle MNO is $\frac{1}{2} \times 10 \times 4$ or 20 square units.
- The area of triangle OPQ is $\frac{1}{2} \times 2 \times 5$ or 5 square units. $60 - (15 + 20 + 5) = 20$.

7. **Problem 7 Statement**

Find the area of this shape. Show your reasoning.



Solution

15 square units. Reasoning varies. Sample reasoning:

- The shape can be decomposed into two identical triangles with a vertical cut down the middle. Each triangle has base 3 units and height 5 units, so its area is $\frac{1}{2} \times 3 \times 5$ or 7.5 square units. $2 \times (7.5) = 15$.
- The shape can be decomposed into two identical triangles and rearranged into a parallelogram with base 3 units and height 5 units. $3 \times 5 = 15$.