Lesson 13: Benchmark percentages

Goals

- Explain (orally and in writing) how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
- Generalise (orally) processes for calculating 10%, 25%, 50%, and 75% of a quantity.

Learning Targets

- When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to.

Lesson Narrative

The goal of this lesson is to help students understand the connection between benchmark percentages and common fractions. In these materials, we have identified 10%, 25%, 50%, and 75% as primary benchmark percentages and multiples of 10% as secondary benchmark percentages.

It is common to say that $25\% = \frac{1}{4}$ or $10\% = \frac{1}{10}$. In these materials we avoid this usage and say rather that 25% of a quantity is $\frac{1}{4}$ of that quantity, or that 10% of a quantity is $\frac{1}{10}$ of that quantity.

This lesson builds on understanding of equivalent fractions, multiplying fractions, and dividing by unit fractions from KS2.

Building On

- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Addressing

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Building Towards

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Stronger and Clearer Each Time

- Clarify, Critique, Correct
- Think Pair Share

Student Learning Goals

Let's contrast percentages and fractions.

13.1 What Percentage Is Shaded?

Warm Up: 5 minutes

In this warm-up, students are presented with bar models with a shaded portion, and they identify the percentage that is shaded.

Instructional Routines

- Think Pair Share

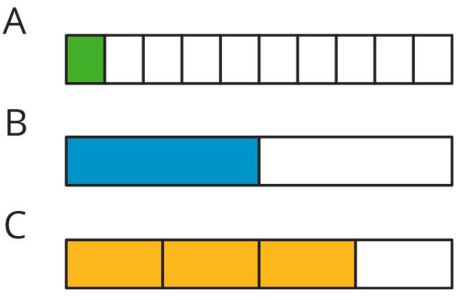
Launch

Display the image in the task statement for all to see, and ask students to think of at least one thing they notice. Ask a few students to share something they notice. It is likely that students notice there are three bar models of the same length, but the first is divided into 10 equal pieces, the second into 2 equal pieces, and the last into 4 equal pieces. Students may make claims about what fraction of each bar is shaded. Remind students that when we wonder, "What percentage of something is shaded?" it is understood that the whole thing is 100%.

Arrange students in groups of 2. Give 1–2 minutes of quiet think time, followed by partner- and whole-class discussions.

Student Task Statement

What percentage of each diagram is shaded?



Student Response

A. 10%

B. 50%

C. 75%

Activity Synthesis

Invite students to share how they reasoned about the percentage of each bar model that is shaded. Record and display their explanations for all to see. Highlight alternative ways of naming the size of the shaded portion, for example, "$\frac{3}{4}$ of the diagram" and "75% of the diagram."

13.2 Litres, Metres, and Hours

15 minutes

In this activity, students calculate three different benchmark percentages—50%, 10%, and 75%—given three different values that correspond to 100%. The repetition of the benchmark percentages allows students to notice regularity. They generalise the patterns in their calculations to determine how to find those percentages when the 100% value is x.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Ask students to complete the first three sub-questions of each problem mentally. If necessary, clarify that "using mental maths" means working out an answer without writing down their calculations and just recording the answer. For the last sub-question, ask them to write a sentence or two to explain their approach. Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

If students struggle to get started with mental math, offer some scaffolding. For example, ask, "How much of 100% is 50%?" Suggest to students that if they know that 50% of a number is the same as $\frac{1}{2}$ of that number, then they can think about what $\frac{1}{2}$ of the number is.

Student Task Statement

a. How much is 50% of 10 litres of milk?

b. How far is 50% of a 2 000-kilometre trip?

c. How long is 50% of a 24-hour day?

d. How can you find 50% of any number?

a. How far is 10% of a 2 000-kilometre trip?

b. How much is 10% of 10 litres of milk?

c. How long is 10% of a 24-hour day?

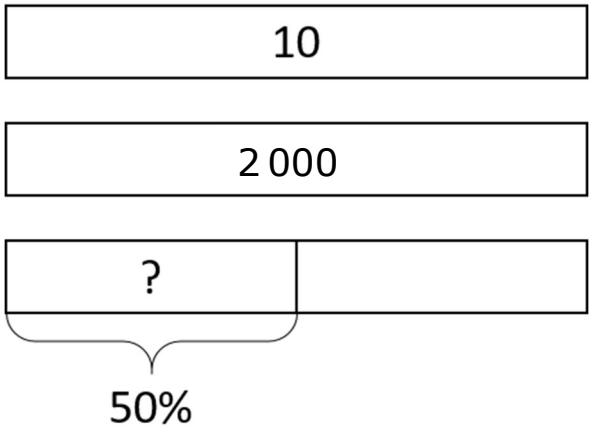
d. How can you find 10% of any number?

a. How long is 75% of a 24-hour day?

b. How far is 75% of a 2 000-kilometre trip?

c. How much is 75% of 10 litres of milk?

d. How can you find 75% of any number?



Student Response

a. 5 litres of milk

b. 1 000 kilometres

c. 12 hours

d. Divide by 2, or multiply by $\frac{1}{2}$.

a. 200 kilometres

b. 1 litre of milk

c. 2.4 hours

d. Divide by 10, or multiply by $\frac{1}{10}$.

a. 18 hours

b. 1500 kilometres

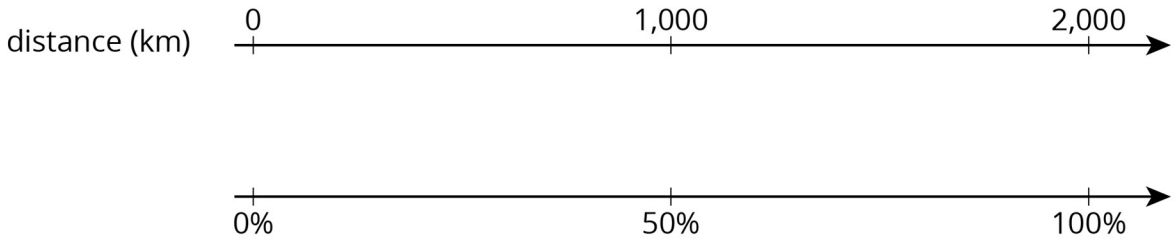
c. 7.5 litres of milk

d. Divide by 4, and multiply by 3 (or multiply by $\frac{3}{4}$).

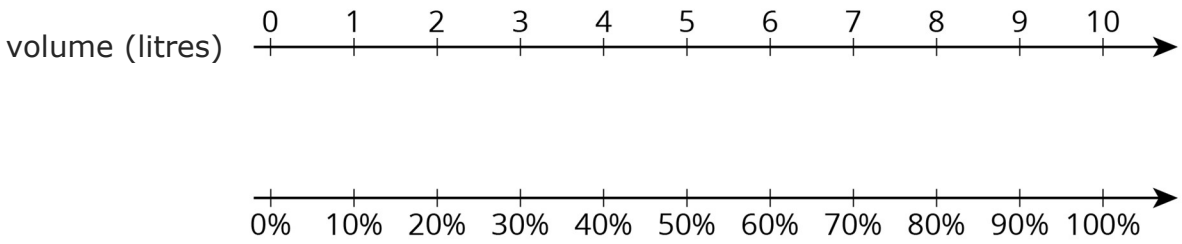
Activity Synthesis

Highlight the following:

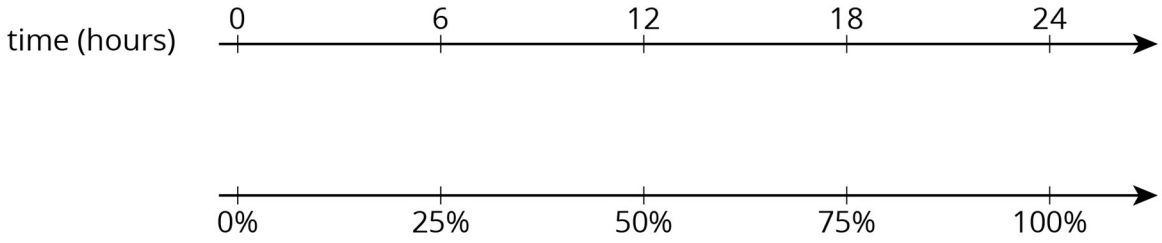
50% of a quantity is $\frac{1}{2}$ of that quantity. We can calculate it by dividing the quantity by 2 or multiplying the quantity by $\frac{1}{2}$. If no students bring up the multiplication method, ask what fraction 50% reminds them of or what number they could multiply by to get the same answer; either $\frac{1}{2}$ or 0.5 is fine.



10% of a quantity is $\frac{1}{10}$ of that quantity. You can calculate it by dividing the quantity by 10, or multiplying the quantity by $\frac{1}{10}$.



75% of a quantity is $\frac{3}{4}$ of that quantity. You can calculate it by dividing the quantity by four and then multiplying by 3, or by multiplying the quantity by $\frac{3}{4}$.



Writing, Speaking, Listening: Stronger and Clearer Each Time. After providing some independent think time, use this routine with successive pair shares to give students a structured opportunity to revise and refine their explanations for how to find 75% of any number. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how…", "You should expand on...", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other strategies to find 75% of any number.
Design Principles(s): Optimise output (for explanation); Maximise meta-awareness

13.3 Nine is . . .

10 minutes

In this activity students find the values for 100% given different benchmark percentages. Students are likely to calculate the answers quickly. They are to spend the majority of the task time discussing how they reason about the questions.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

Launch

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

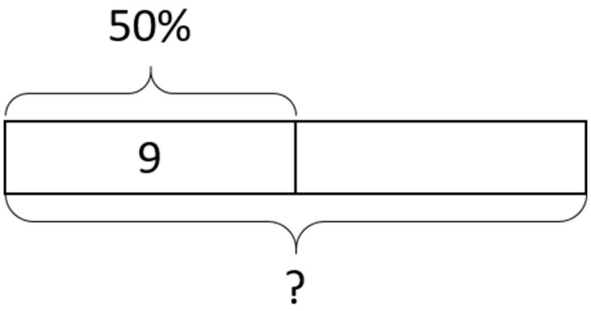
Students may think 9 is 100% and try to find the percentage of the number given. You can use the diagram in the task statement to help them make sense of the first question and

then encourage them to use mental maths, or draw additional diagrams, to solve the other three questions.

Student Task Statement

Explain how you can calculate each value mentally.

1. 9 is 50% of what number?
2. 9 is 25% of what number?
3. 9 is 10% of what number?
4. 9 is 75% of what number?
5. 9 is 150% of what number?



Student Response

Answers vary. Sample responses:

1. Because 50% of a quantity is $\frac{1}{2}$ of that quantity, I can multiply $9 \times 2 = 18$.
2. Because 25% of a quantity is $\frac{1}{4}$ of that quantity, I can multiply $9 \times 4 = 36$.
3. Because 10% of a quantity is $\frac{1}{10}$ of that quantity, I can multiply $9 \times 10 = 90$.
4. Because 75% of a quantity is $\frac{3}{4}$ of that quantity, I can divide $9 \div 3 = 3$ to find $\frac{1}{4}$ of the quantity and then multiply $3 \times 4 = 12$.
5. Because 150% of a quantity is $\frac{3}{2}$ of that quantity, I can divide $9 \div 3 = 3$ to find $\frac{1}{2}$ of the quantity and then multiply $3 \times 2 = 6$.

Activity Synthesis

Invite students to share different strategies for answering the questions, with or without using fractions. For example, some may multiply the given 9 by 2, 4, and 10, respectively, to

find values of 100% given the values of 50%, 25%, and 10% in the first three problems. Highlight any fractions students used to make sense of and solve the problems.

Reading, Writing, Speaking: Clarify, Critique, Correct. Before students share their answers for the first question, present an incorrect answer and explanation. For example, "9 is 50% of 4.5 because 9 times $\frac{1}{2}$ is 4.5." Ask students to identify the error, critique the reasoning, and write a correct explanation. Encourage students to use the bar model in the task statement to make sense of the question. As students discuss in partners, listen for students who clarify what it means for a quantity to be a percentage of another quantity. Also, listen for students who state that 50% of quantity is $\frac{1}{2}$ of that quantity. Prompt students to share their critiques and corrected explanations with the class. This routine will engage students in meta-awareness as they clarify the meaning of the statement "9 is 50% of a number" as "9 is $\frac{1}{2}$ of that number".
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

13.4 Matching the Percentage

Optional: 10 minutes

In this activity, students calculate benchmark percentages. Students are likely to calculate the answers quickly They are to spend the majority of the task time discussing how they reason about the questions.

Instructional Routines

- Think Pair Share

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Representation: Internalise Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Because 5 goes into 20 four times, students may answer that 5 is 4% of 20. If this happens, explain that 5 is really $\frac{1}{4}$ of 20 and ask them what percentage represents one quarter.

Student Task Statement

Match the percentage that describes the relationship between each pair of numbers. One percentage will be left over. Be prepared to explain your reasoning.

1. 7 is what percentage of 14?

2. 5 is what percentage of 20?
3. 3 is what percentage of 30?
4. 6 is what percentage of 8?
5. 20 is what percentage of 5?

- 4%
- 10%
- 25%
- 50%
- 75%
- 400%

Student Response

1. 50%. 7 is half of 14, so 7 is 50% of 14.
2. 25%. 5 times 4 is 20, so 5 is $\frac{1}{4}$ (or 25%) of 20.
3. 10%. 3 times 10 is 30, so 3 is $\frac{1}{10}$ (or 10%) of 30.
4. 75%. 2 is $\frac{1}{4}$ of 8, so 6 is $\frac{3}{4}$ (or 75%) of 8.
5. 400%. 5 is 100% of 5, and 20 is 4 times that, so 20 is 400% of 5.

Are You Ready for More?

1. What percentage of the world's current population is under the age of 14?
2. How many people is that?
3. How many people are 14 or older?

Student Response

Answers may vary depending on the year.

1. 25% (as of 2017)
2. 1.875 billion people are under 14 years old (as of 2017)
3. 5.625 billion people are over 14 years old (as of 2017)

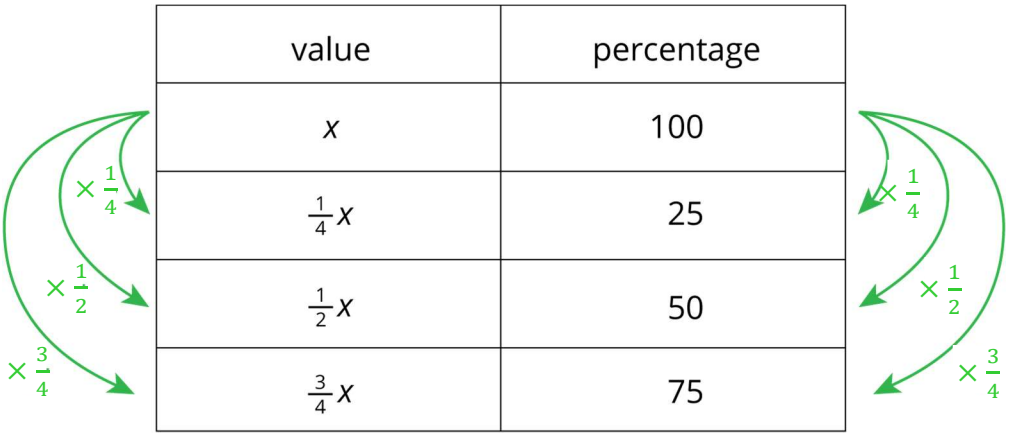
Activity Synthesis

Invite students to share their mental maths strategies. If needed, consider illustrating the relationship between of B and C (where A% of B is C) using bar models or double number line diagrams to help students visualise the association between the benchmark percentages and fractions.

Lesson Synthesis

Certain percentages are easy to think about in terms of fractions. Ask students how they can think about each benchmark percentage by using a fraction. Demonstrate the correspondences using a double number line, bar model, or a table (as shown).

- 25% of a number is always $\frac{1}{4}$ of that number.
- 50% of a number is always $\frac{1}{2}$ of that number.
- 75% of a number is always $\frac{3}{4}$ of that number
- 10% of a number is always $\frac{1}{10}$ of that number.



value	percentage
x	100
$\frac{1}{4}x$	25
$\frac{1}{2}x$	50
$\frac{3}{4}x$	75

13.5 Around the Clock

Cool Down: 5 minutes

In this activity, students find C (where A% of B is C), given benchmark percentages and a single value for B in the context of telling time.

Student Task Statement

Answer each question and explain your reasoning.

1. How long is 50% of 60 minutes?
2. How long is 10% of 60 minutes?

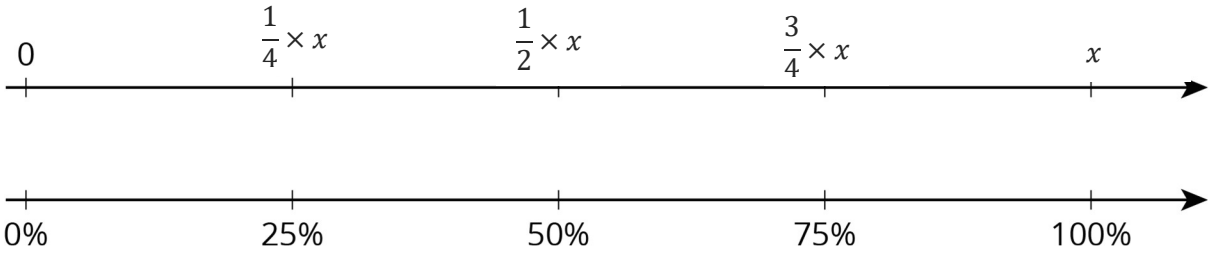
3. How long is 75% of 60 minutes?

Student Response

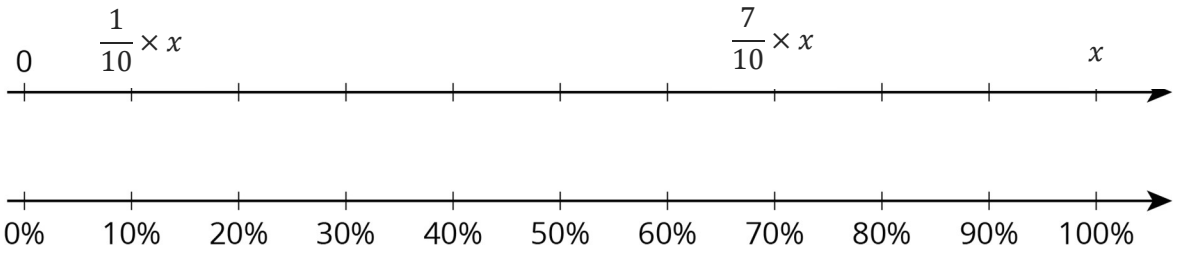
1. 30 minutes because it is $\frac{1}{2}$ of an hour
2. 6 minutes because it is $\frac{1}{10}$ of an hour
3. 45 minutes because it is $\frac{3}{4}$ of an hour

Student Lesson Summary

Certain percentages are easy to think about in terms of fractions.



- 25% of a number is always $\frac{1}{4}$ of that number.
  For example, 25% of 40 litres is $\frac{1}{4} \times 40$ or 10 litres.
- 50% of a number is always $\frac{1}{2}$ of that number.
  For example, 50% of 82 kilometres $\frac{1}{2} \times 82$ or 41 kilometres.
- 75% of a number is always $\frac{3}{4}$ of that number.
  For example, 75% of 1 pound is $\frac{3}{4}$ pound.
- 10% of a number is always $\frac{1}{10}$ of that number.
  For example, 10% of 95 metres is 9.5 metres.
- We can also find multiples of 10% using tenths.
  For example, 70% of a number is always $\frac{7}{10}$ of that number, so 70% of 30 days is $\frac{7}{10} \times 30$ or 21 days.



Lesson 13 Practice Problems

Problem 1 Statement

a. How can you find 50% of a number quickly in your head?

b. Andre lives 1.6 km from school. What is 50% of 1.6 km?

c. Diego lives $\frac{1}{2}$ mile from school. What is 50% of $\frac{1}{2}$ mile?

Solution

a. Answers vary. Sample response: Divide the number by 2 (or multiply it by $\frac{1}{2}$).

b. 0.8 km (or equivalent)

c. $\frac{1}{4}$ mile (or equivalent)

Problem 2 Statement

There is a 10% off sale on laptop computers. If someone saves £35 on a laptop, what was its original cost? If you get stuck, consider using the table.

savings (pounds)	percentage
35	10
?	100

Solution

£350

Problem 3 Statement

Explain how to calculate these mentally.

a. 15 is what percentage of 30?

b. 3 is what percentage of 12?

c. 6 is what percentage of 10?

Solution

Answers vary. Sample response:

a. 50%. 15 is $\frac{1}{2}$ of 30, so that is 50%.

b. 25%. 3 is $\frac{1}{4}$ of 12, so that is 25%.

c. 60%. $\frac{6}{10}$ is the same as $\frac{3}{5}$, and each $\frac{1}{5}$ is 20%.

Problem 4 Statement

Noah says that to find 20% of a number he divides the number by 5. For example, 20% of 60 is 12, because $60 \div 5 = 12$. Does Noah's method always work? Explain why or why not.

Solution

Yes. Answers vary. Sample response: 20% of a number is $\frac{20}{100}$ times the number and $\frac{20}{100} = \frac{1}{5}$. Multiplying by $\frac{1}{5}$ gives the same result as dividing by 5.

Problem 5 Statement

Diego has 75% of £10. Noah has 25% of £30. Diego thinks he has more money than Noah, but Noah thinks they have an equal amount of money. Who is right? Explain your reasoning.

Solution

They each have £7.50 ($10 \times 0.75 = 7.50$ and $30 \times 0.25 = 7.50$).

Problem 6 Statement

Lin and Andre start walking toward each other at the same time from opposite ends of 22-mile walking trail. Lin walks at a speed of 2.5 miles per hour. Andre walks at a speed of 3 miles per hour.

Here is a table showing the distances travelled and how far apart Lin and Andre were over time. Use the table to find how much time passes before they meet.

elapsed time (hour)	Lin's distance (miles)	Andre's distance (miles)	distance apart (miles)
0	0	0	22
1	2.5	3	16.5

			0

Solution

4 hours. Possible strategy:

elapsed time (hour)	Lin's distance (miles)	Andre's distance (miles)	distance apart (miles)
0	0	0	22
1	2.5	3	16.5
2	5	6	11
3	7.5	9	5.5
4	10	12	0