Lesson 12: Solving problems about percentage increase or decrease

Goals

- Solve word problems leading to equations of the form $px + q = r$ or $p(x + q) = r$

Learning Targets

- I can solve story problems about percentage increase or decrease by drawing and reasoning about a bar model or by writing and solving an equation.

Lesson Narrative

This lesson is an opportunity for students to revisit percentages of and percentage change to solve word problems. Minimal scaffolding is provided, so students will need to make sense of the problems and perhaps attempt different solution pathways. Now, they can choose to use their deeper understanding of bar models and writing and solving equations.

Addressing

- Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."
- Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Group Presentations
- Three Reads
- Discussion Supports
- Think Pair Share

Required Materials

Sticky notes

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Decide if students will conduct group presentations or a gallery walk for the last activity. If so, prepare tools for creating a visual display and around 3 sticky notes per student. If not, these materials are not necessary.

Student Learning Goals

Let's use bar models, equations, and reasoning to solve problems with negatives and percentages.

12.1 20% Off

Warm Up: 10 minutes

The purpose of this warm-up is for students to review how to solve for percentage change and represent these situations with expressions. Analysing the structure of equivalent expressions for the same situation helps students see how the quantities in it are related. Since there are many equivalent expressions to represent how to find percentage change in a situation, encourage students to look for relationships between the expressions.

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by 2 minutes to compare their responses with their partner. During the partner discussion, tell students to discuss the expressions they have in common, ones they don't and then try to come to an agreement on the correct expressions that represent the price of the item after the discount. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may choose expressions that represent the discount itself instead of the price of the item after the discount. Ask those students to refer back to the situation to identify which piece of the problem the expression they chose finds. If students are still unclear, it may be helpful to give students a price for x such as £10 and ask students if 20% of £10 makes sense as the new price of the item after the discount and then what piece of the problem they found.

Student Task Statement

An item costs x pounds and then a 20% discount is applied. Select **all** the expressions that could represent the price of the item after the discount.

1. $\frac{20}{100}x$
2. $x - \frac{20}{100}x$
3. $(1 - 0.20)x$
4. $\frac{100-20}{100}x$
5. $0.80x$
6. $(100 - 20)x$

Student Response

Expressions 2, 3, 4, and 5 represent the price of the item after the discount.

Activity Synthesis

Ask students to indicate whether each expression represents the price of the item after the discount. If all students agree on an expression, ask 1 or 2 students to explain their reasoning and move to the next expression. Record and display their responses for all to see. If there is a disagreement on an expression, ask students to explain their reasoning for both choices and come to an agreement.

After the class has agreed on the four expressions that represent the price of the item after the discount, record them as a list and display them all to see. Ask students to discuss any connections they see between the expressions to show they are equivalent.

12.2 Walking More Each Day

10 minutes

The first three questions help students recall how bar models can be used to represent a percentage increase situation and draw connections to an equation representing percentage increase. Monitor for different approaches in the first three questions (see solutions for different approaches).

The last question is a review of previous work in this unit. This fourth question can be used for additional practice, but it can be safely skipped if time is short.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Keep students in the same groups. Tell students to work on the first three questions and pause for discussion. Give 5 minutes of quiet work time and time to share their responses

with a partner, followed by a whole-class discussion. If time permits, the last question can be used as more practice on work from earlier in the unit.

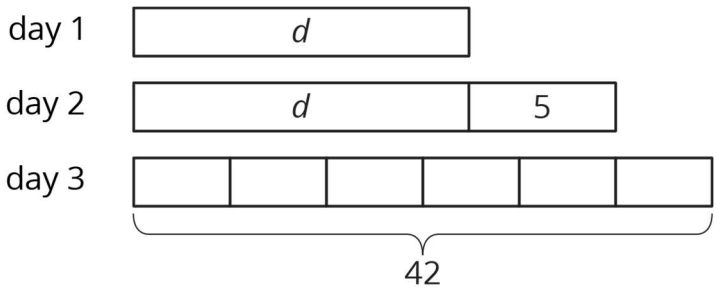
Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Look for students who use equivalent fractions and percentages in explaining their understanding of the diagram.
Supports accessibility for: Memory; Organisation Representing, Speaking: Discussion Supports. Use this routine to help students understand how bar models can be used to represent a percentage increase situation. Invite students to label their bar models to show what each section represents for Day 1, 2, and 3. Arrange students in groups of 2. Ask groups to compare how they used the bar models to solve each question. Listen for common language students use to describe different approaches.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

If students bring up that the diagram represents 120% or $\frac{6}{5}$, or if they refer to each equal part as 20% or $\frac{1}{5}$, ask what whole the fraction or percentage refers to. They should understand that the whole is the amount from Day 2, $d + 5$.

Student Task Statement

1. Mai started a new exercise program. On the second day, she walked 5 minutes more than on the first day. On the third day, she increased her walking time from day 2 by 20% and walked for 42 minutes. Mai drew a diagram to show her progress.



   Explain how the diagram represents the situation.

2. Noah said the equation $1.20(d + 5) = 42$ also represents the situation. Do you agree with Noah? Explain your reasoning.

3. Find the number of minutes Mai walked on the first day. Did you use the diagram, the equation, or another strategy? Explain or show your reasoning.

4. Mai has been walking indoors because of cold temperatures. On Day 4 at noon, Mai hears a report that the temperature is only 9 degrees Celsius. She remembers the morning news reporting that the temperature had doubled since midnight and was expected to rise 15 degrees by noon. Mai is pretty sure she can draw a diagram to represent this situation but isn't sure if the equation is $9 = 15 + 2t$ or $2(t + 15) = 9$.

What would you tell Mai about the diagram and the equation and how they might be useful to find the temperature, t, at midnight?

Student Response

1. Answers vary. Sample response: The last day is day 2 plus $\frac{1}{5}$ (20%) of day 2. Day 2 is 5 more than Day 1.

2. Answers vary. Sample responses: Yes, she walked 42 minutes on Day 3, which is the same as (equal to) 20% more than (1.20 times) 5 more than Day 1 ($d + 5$). No, I wrote the equation $\frac{6}{5}(d + 5) = 42$ because the diagram shows that 42 is $\frac{1}{5}$ more than $d + 5$.

3. 30 minutes. Answers vary. Sample responses:

   - From the diagram, $42 \div 6 = 7, 7 \times 5 = 35, 35 - 5 = 30$
   - From the equation, $1.2(d + 5) = 42, d + 5 = 42 \div 1.2, d + 5 = 35, d = 30$
   - From another version of the equation: $\frac{6}{5}(d + 5) = 42, d + 5 = \frac{5}{6} \times 42, d + 5 = 35, d = 30$

4. Answers vary. Sample response: Since the temperature doubled and then increased by 15, the diagram would show two equal parts and another part of 15, all with a total of 9. The equation would be $2t + 15 = 9$ (or equivalent). The temperature at midnight can be found with the equation or the diagram by subtracting 15 from 9 to get -6 and dividing by 2 to get -3. The temperature, t, at midnight was -3 degrees Celsius.

Activity Synthesis

Select groups with different approaches to share their responses to the first three questions.

12.3 A Sale on Shoes

15 minutes

This activity offers four word problems. Depending on time constraints, you may have all students complete all four problems or assign a different problem to each group. The problems increase in difficulty. It is suggested that students create a visual display of one of the problems and do a gallery walk or presentation, but if time is short you may choose to just have students work in their workbooks or devices.

Instructional Routines

- Group Presentations
- Three Reads

Launch

Keep students in the same groups. Either instruct students to complete all four problems or assign one problem to each group. If opting to have students do presentations or a gallery walk, distribute tools for making a visual display.

Give students 5–6 minutes quiet work time and partner discussion followed by a whole-class discussion or gallery walk.

Representation: Internalise Comprehension. Represent the same information through different modalities by using diagrams, equations, or other drawings to depict the situation. If students are unsure where to begin, suggest that they draw a diagram to help organise the information provided. During the Synthesis, annotate drawings to illustrate connections between representations.
Supports accessibility for: Conceptual processing; Visual-spatial processing Reading, Representing: Three Reads. Use this routine to support reading comprehension of the first problem without solving it for students. For the first read, read the situation to students, without revealing the final question. Ask students "What is this question about?" (A store is having a sale. Diego is buying shoes with a coupon). In the second read, ask students to name the important quantities (e.g., discount of 20%, coupon for £3 off of the regular price, Diego pays £18.40), and then create a diagram to represent the relationships among these quantities. After the third read, ask students to brainstorm possible strategies to answer the question, “What was the original price before the sale and without the coupon?” This will help students connect the language in the word problem and reasoning needed to solve percentage problems.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

1. A store is having a sale where all shoes are discounted by 20%. Diego has a coupon for £3 off of the regular price for one pair of shoes. The store first applies the coupon and then takes 20% off of the reduced price. If Diego pays £18.40 for a pair of shoes, what was their original price before the sale and without the coupon?

2. Before the sale, the store had 100 pairs of flip flops in stock. After selling some, they notice that $\frac{3}{5}$ of the flip flops they have left are blue. If the store has 39 pairs of blue flip flops, how many pairs of flip flops (any colour) have they sold?

3. When the store had sold $\frac{2}{9}$ of the boots that were on display, they brought out another 34 pairs from the stock room. If that gave them 174 pairs of boots out, how many pairs were on display originally?

4. On the morning of the sale, the store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during the sale. If the store had 288

pairs after the donation and the sale, how many pairs of shoes did they have at the start?

Student Response

1. £26. Explanations vary. Sample response: I wrote the equation $0.8(x - 3) = 18.40$ to show that Diego paid 80% (0.8) of the original price x less the £3 coupon $(x - 3)$, which came to a discounted price of £18.40.

2. 35 pairs of flip flops. $\frac{3}{5}(100 - x) = 39,\ x = 35$.

3. 180 pairs of boots. $\frac{7}{9}x + 34 = 174, x = 180$.

4. 850 pairs of shoes. $0.36(x - 50) = 288, x = 850$.

Are You Ready for More?

A coffee shop offers a special: 33% extra free or 33% off the regular price. Which offer is a better deal? Explain your reasoning.

Student Response

Answers vary. Sample response: 33% off the price is a better deal. Suppose you buy 1 cup of coffee at price p. 33% off means you pay $0.67p$ for one cup. 33% extra free means you pay p for 1.33 cups of coffee, or $\frac{p}{1.33}$ for 1 cup, which is about $0.75p$. The unit price for 1 cup of coffee is less with 33% off the price.

Activity Synthesis

If students created a visual display and you opt to conduct a gallery walk, ask students to post their solutions. Distribute sticky notes and ask students to read others' solutions, using the sticky notes to leave questions or comments. Give students a moment to review any questions or comments left on their display.

Invite any students who chose to draw a diagram to share; have the class agree or disagree with their diagrams and suggest any revisions. Next, invite students who did not try to draw a diagram to share strategies. Ask students about any difficulties they had creating the expressions and equations. Highlight equivalent expressions that represent the same quantity and different strategies for solving equations.

Lesson Synthesis

Ask students to reflect on the work done in this unit so far. What strategies have they learned? What kinds of problems can they solve that they weren't able to previously? Ask them to write down or share with a partner one new thing they have learned and one thing they still have questions or confusion about.

12.4 Timing the Relay Race

Cool Down: 5 minutes

Student Task Statement

The track team is trying to reduce their time for a relay race. First they reduce their time by 2.1 minutes. Then they are able to reduce that time by $\frac{1}{10}$. If their final time is 3.96 minutes, what was their beginning time? Show or explain your reasoning.

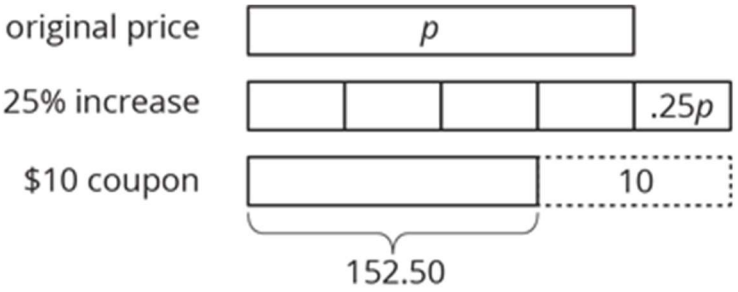
Student Response

6.5 minutes. Explanations vary. Sample responses:

- With equation: $0.9(x - 2.1) = 3.96$, $x - 2.1 = 4.4$, $x = 6.5$.
- Reasoning with or without a diagram: 9 out of 10 parts represent 3.96 minutes, so the $\frac{1}{10}$ reduction was $3.96 \div 9$ or 0.44 minutes. That makes the time before the 2.1 minute reduction $3.96 + 0.44$ or 4.4 minutes. The original time was $4.4 + 2.1$ or 6.5 minutes.

Student Lesson Summary

We can solve problems where there is a percentage increase or decrease by using what we know about equations. For example, a camping store in the US increases the price of a tent by 25%. A customer then uses a \$10 coupon for the tent and pays \$152.50. We can draw a diagram that shows first the 25% increase and then the \$10 coupon.



The price after the 25% increase is $p + 0.25p$ or $1.25p$. An equation that represents the situation could be $1.25p - 10 = 152.50$. To find the original price before the increase and discount, we can add 10 to each side and divide each side by 1.25, resulting in $p = 130$. The original price of the tent was \$130.

Lesson 12 Practice Problems

1. **Problem 1 Statement**

   A backpack normally costs £25 but it is on sale for £21. What percentage is the discount?

   Solution

   16%

2. **Problem 2 Statement**

   Find each product.

   a. $\frac{2}{5} \times (-10)$

   b. $-8 \times \left(\frac{-3}{2}\right)$

   c. $\frac{10}{6} \times 0.6$

   d. $\left(\frac{-100}{37}\right) \times (-0.37)$

   Solution

   a. -4

   b. 12

   c. 1

   d. 1

3. **Problem 3 Statement**

   Select **all** expressions that show x increased by 35%.

   a. $1.35x$

   b. $\frac{35}{100}x$

   c. $x + \frac{35}{100}x$

   d. $(1 + 0.35)x$

   e. $\frac{100+35}{100}x$

   f. $(100 + 35)x$

Solution ["A", "C", "D", "E"]

4. **Problem 4 Statement**

Complete each sentence with the word *discount*, *deposit*, or *withdrawal*.

a. Clare took £20 out of her bank account. She made a _____.

b. Kiran used a coupon when he bought a pair of shoes. He got a _____.

c. Priya put £20 into her bank account. She made a _____.

d. Lin paid less than usual for a pack of gum because it was on sale. She got a _____.

Solution

a. withdrawal

b. discount

c. deposit

d. discount

5. **Problem 5 Statement**

Here are two stories:

- The first year cohort at a university is 10% smaller than last year's cohort. But then during the first week of classes, 20 more students enrol. There are then 830 students in the first year cohort.
- A store reduces the price of a computer by £20. Then during a 10% off sale, a customer pays £830.

Here are two equations:

- $0.9x + 20 = 830$
- $0.9(x - 20) = 830$

a. Decide which equation represents each story.

b. Explain why one equation has brackets and the other doesn't.

c. Solve each equation, and explain what the solution means in the situation.

Solution

Answers vary. Sample responses:

a. The first year cohort: $0.9x + 20 = 830$, computer: $0.9(x - 20) = 830$

b. It depends on which came first, the additive increase or decrease (brackets needed) or the percentage decrease (no brackets needed, since the convention is to multiply before adding when there are no brackets).

c. The first year cohort: $x = 900$, which is the size of last year's first year cohort. Computer: $x = £942.22$ (rounding to the nearest penny), which is the original price of the computer.