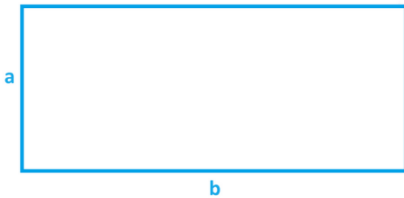


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## Umfang und Fläche gleich

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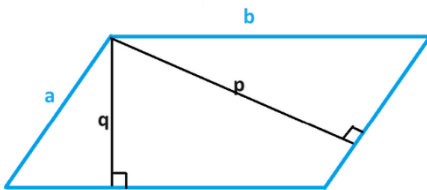
Rechteck:  $\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$



$$A = ab = U = 2(a+b) \Rightarrow \frac{1}{ab} = \frac{1}{2(a+b)}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{1}{2} \Rightarrow \frac{1}{b} + \frac{1}{a} = \frac{1}{2}$$

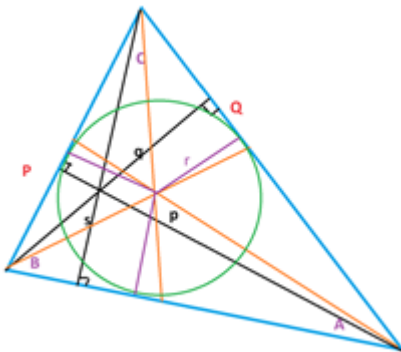
Parallelogramm:  $\frac{q}{2a} = \frac{p}{2b} = \frac{1}{a} + \frac{1}{b}$



$$A = ap = bq = U = 2(a+b) \Rightarrow \frac{q}{a} = \frac{p}{b}, \frac{ap}{2} = \frac{bq}{2} = a + b \quad | : ab$$

$$\Rightarrow \frac{p}{2b} = \frac{q}{2a} = \frac{a+b}{ab} = \frac{1}{b} + \frac{1}{a}$$

Dreieck:  $\frac{1}{p} + \frac{1}{q} + \frac{1}{s} = \frac{1}{r}$

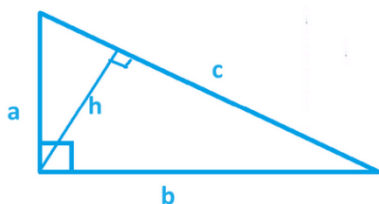


a,b,c ... Seiten des Dreiecks  
p,q,s ... Höhen auf die Seiten  
r ... Inkreisradius

$$Fl = \frac{1}{2}(a + b + c)r = \frac{1}{2}ap = \frac{1}{2}bq = \frac{1}{2}cs \quad | : rap = rbq = rcs$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{s} = \frac{1}{r}$$

Rechtwinkeliges Dreieck:  $\frac{1}{a} + \frac{1}{b} + \frac{1}{h} = \frac{1}{2}$



$$A = ab/2 = ch/2 = U = a+b+c \quad | : ab$$

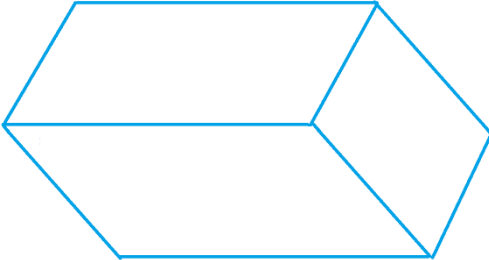
$$\Rightarrow \frac{1}{2} = \frac{1}{b} + \frac{1}{a} + \frac{c}{ab} \quad \text{mit } c = \frac{ab}{h}$$

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## Oberfläche und Volumen gleich

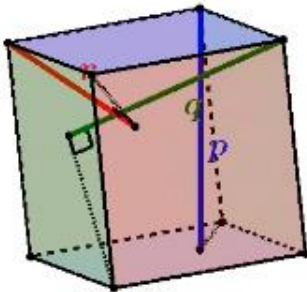
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Quader:  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$



$$V = abc = O = 2(ab+bc+ac) \quad | :abc \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$$

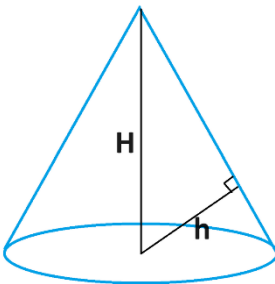
Parallelepiped:  $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{1}{2}$



$$V = pA_1 = qA_2 = rA_3 = O = 2(A_1+A_2+A_3) = 2\left(\frac{V}{p} + \frac{V}{q} + \frac{V}{r}\right)$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{1}{2}$$

Kreiskegel:  $\frac{1}{H} + \frac{1}{h} = \frac{1}{3}$



$$A = \pi r(\sqrt{r^2 + H^2} + \pi r^2)$$

$$V = \frac{\pi r^2 H}{3}$$

$$V = A$$

$$\Rightarrow \frac{rH}{3} = \sqrt{r^2 + H^2} + r$$

$$\Rightarrow \frac{1}{3} = \sqrt{\frac{1}{H^2} + \frac{1}{r^2}} + \frac{1}{H}$$

$$s^2 = H^2 + r^2$$

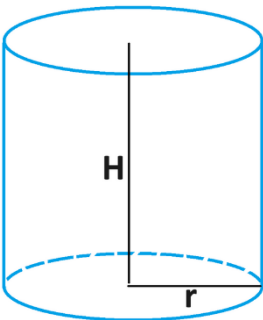
$$\frac{h}{r} = \frac{H}{s} \Rightarrow hs = rH$$

$$\frac{1}{h^2} = \frac{1}{r^2} + \frac{1}{H^2} \Rightarrow \frac{1}{r^2} = \frac{1}{h^2} - \frac{1}{H^2}$$

$$\Rightarrow \frac{1}{3} = \sqrt{\frac{1}{H^2} + \frac{1}{h^2} - \frac{1}{H^2}} + \frac{1}{H}$$

$$= \frac{1}{h} + \frac{1}{H}$$

Kreiszylinder:  $\frac{1}{H} + \frac{1}{r} = \frac{1}{2}$



$$V = r^2\pi H = O = 2r^2\pi + 2r\pi H \quad | : r\pi \Rightarrow$$

$$rH = 2r + 2H \Rightarrow \frac{1}{2} = \frac{1}{H} + \frac{1}{r}$$