Función Gamma.

Definimos para cualquier $\alpha>0$ la función Gamma como:

$$\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}\cdot e^{-\alpha}\cdot dx$$

que es un valor finito para $\forall\,\alpha>0$.

Además, se cumple:

- $\Gamma(1)=\int_0^\infty e^{-x}\cdot dx=\left[-e^{-x}\right]_0^\infty=1$
- $\Gamma\left(\frac{1}{2}\right)=\left\{\begin{matrix} Haciendo\, el\, cambio \\ x=\frac{1}{2}\cdot y^2 \end{matrix}\right\}=\int_0^\infty x^{-\frac{1}{2}}e^{-x}\cdot dx=\sqrt{2}\int_0^\infty e^{-\frac{1}{2}\cdot y^2}\cdot dy=$

  $=\sqrt{2}\cdot\frac{\sqrt{2\cdot\pi}}{2}\cdot\int_{-\infty}^\infty\frac{1}{\sqrt{2\cdot\pi}}\cdot e^{-\frac{1}{2}\cdot y^2}\cdot dy=\sqrt{\pi};$ Dado que: $\int_{-\infty}^\infty\frac{1}{\sqrt{2\cdot\pi}}\cdot e^{-\frac{1}{2}\cdot y^2}\cdot dy=1$
- $Si\ \ \alpha>1;\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}\cdot e^{-x}\cdot dx=$

  $=$ integrando por partes $=\left\{\begin{matrix} u=x^{\alpha-1}; & du=(\alpha-1)\cdot x^{\alpha-2}\cdot dx \\ dv=e^{-x}\cdot dx; & v=-e^{-x} \end{matrix}\right\}=$

  $=\left[-x^{\alpha-1}\cdot e^{-x}\right]_0^\infty+(\alpha-1)\cdot\int_0^\infty x^{\alpha-2}\cdot e^{-x}\cdot dx\int_0^\infty x^{-\frac{1}{2}}e^{-x}\cdot dx=\sqrt{2}\int_0^\infty e^{-\frac{1}{2}\cdot y^2}\cdot dy=$

  $=(\alpha-1)\cdot\Gamma(\alpha-1)$
- Si $\alpha=n\in\mathbb{N},n>2;\Gamma(n)=(n-1)\cdot\Gamma(n-1)=(n-1)\cdot(n-2)\cdot\Gamma(n-2)=$
  $=(n-1)\cdot(n-2)\cdot\ldots\cdot\Gamma(1)=(n-1)!$.
- si $\alpha=n+\frac{1}{2};n\in\mathbb{N}$ y $n>1\,\Gamma\left(n+\frac{1}{2}\right)=\left(n-\frac{1}{2}\right)\cdot\Gamma\left(n-\frac{1}{2}\right)$.
- si $\alpha=n+\frac{1}{2};n\in\mathbb{N}$ y $n>2;\Gamma\left(n+\frac{1}{2}\right)=\left(n-\frac{1}{2}\right)\cdot\left(n-\frac{3}{2}\right)\cdot\ldots\left(\frac{1}{2}\right)\cdot\Gamma\left(\frac{1}{2}\right)=$
  $=\left(n-\frac{1}{2}\right)\cdot\left(n-\frac{3}{2}\right)\cdot\ldots\left(\frac{1}{2}\right)\cdot\sqrt{\pi}$.

$\forall\,t\in(0,\infty)$ se cumple:

$$\Gamma(\alpha)=\int_0^\infty x^{\alpha-1}\cdot e^{-\alpha}\cdot dx=\int_0^t x^{\alpha-1}\cdot e^{-\alpha}\cdot dx+\int_t^\infty x^{\alpha-1}\cdot e^{-\alpha}\cdot dx$$

Denominando:

$\gamma(\alpha,t)=\int_0^t x^{\alpha-1}\cdot e^{-\alpha}\cdot dx$ Gamma incompleta inferior.

$\Gamma(\alpha,t)=\int_t^\infty x^{\alpha-1}\cdot e^{-\alpha}\cdot dx$ Gamma incompleta superior.