Lesson 10: Multiply!

Goals

- Identify multiplication expressions that are equal, and justify (orally) that they are equal.
- Multiply rational numbers, including multiplication expressions with three factors, and explain (orally and in writing) the reasoning.

Learning Targets

- I can solve problems that involve multiplying rational numbers.

Lesson Narrative

The purpose of this optional lesson is to provide students with practice multiplying rational numbers. Students see products of three numbers, and figure out what to do with a product of three negative numbers by grouping two of them together and multiplying those first. They see products involving unit fractions, reminding them that dividing by a whole number is the same as multiplying by its reciprocal. This will be useful in the next lesson when they start to divide rational numbers.

Building On

- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

Addressing

- Apply properties of operations as strategies to multiply and divide rational numbers.

Building Towards

- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Instructional Routines

- Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Required Materials

Pre-printed slips, cut from copies of the blackline master

Card Sort: Matching Expressions -1×12	Card Sort: Matching Expressions $-64 \times \frac{1}{8}$	Card Sort: Matching Expressions 1×15
Card Sort: Matching Expressions $-1 \times (-3) \times (-5)$	Card Sort: Matching Expressions $-1 \times (-2) \times 6$	Card Sort: Matching Expressions $-1 \times (-12)$
Card Sort: Matching Expressions $1 \times (-3) \times (-5)$	Card Sort: Matching Expressions $-\frac{1}{4} \times -32$	Card Sort: Matching Expressions -2×6
Card Sort: Matching Expressions $-\frac{1}{2} \times (-16)$	Card Sort: Matching Expressions -3×5	Card Sort: Matching Expressions $2 \times (-4)$
Card Sort: Matching Expressions $-\frac{1}{2} \times 16$	Card Sort: Matching Expressions $-1 \times (-3) \times (-4)$	Card Sort: Matching Expressions 2×4
Card Sort: Matching Expressions $-1 \times (-3) \times 4$	Card Sort: Matching Expressions $(-3) \times (-5)$	Card Sort: Matching Expressions 1×-15

Required Preparation

For the Card Sort: Matching Expressions activity, prepare 1 copy of the blackline master for each group of 2 students. If possible, copy each complete set on a different colour of paper, so that a stray card can quickly be put back.

Student Learning Goals

Let's get more practice multiplying directed numbers.

10.1 Which One Doesn't Belong: Expressions

Warm Up: 5 minutes

This warm-up prompts students to compare four expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to

hear how they use terminology and talk about the expressions in comparison to one another. To allow all students to access the activity, each expression has one obvious reason it does not belong.

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Ask students to indicate when they have noticed one expression that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression does not belong and together find at least one reason each expression doesn't belong.

Student Task Statement

Which expression doesn't belong?

$7.9x$

$7.9 \times (-10)$

$7.9 + x$

-79

Student Response

Answers vary. Sample responses: $7.9x$ is the only product of a decimal and a variable.

$7.9 \times (-10)$ is the only one with parentheses; is the only one that is a product of two known numbers.

$7.9 + x$ is the only sum.

-79 is the only plain number.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given make sense.

10.2 Card Sort: Matching Expressions

Optional: 10 minutes

This activity reminds students of the links between positive fractions and multiplication and prepares them to think about division as multiplication by the reciprocal; this will be important for dividing negative numbers. Students will use earlier work in KS3 and their work in previous lessons in this unit to extend what they know about division of positive rationals to all rational numbers.

Ask students as they are working if there is an easy way to tell if two expressions are not equivalent, making note of students who reason about how many negative numbers are multiplied, and what the outcome will be. For example, they may have first gone through and marked whether each product would be positive or negative before doing any arithmetic.

Instructional Routines

- Discussion Supports

Launch

Ask students to recall the rules they have previously used about multiplication of directed numbers.

This is the first encounter with an expression where three integers are multiplied, so students might need to see an example of evaluating an expression like this one step at a time. Display an expression like this for all to see, and ask students how they might go about evaluating it: $(-2) \times (-3) \times (-4)$ The key insight is that you can consider only one product, and replace a pair of numbers with the product. In this example, you can replace $(-2) \times (-3)$ with 6. Then, you are just looking at $(6) \times (-4)$, which we already know how to evaluate.

Arrange students in groups of 2. Distribute sets of cards.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organisation Conversing: Discussion Supports. Arrange students in groups of 2. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: " ___ and ___ are equal because . . .", and "I noticed ___ , so I matched . . ." Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about multiplication of directed numbers.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Your teacher will give you cards with multiplication expressions on them. Match the expressions that are equal to each other. There will be 3 cards in each group.

Student Response

-12	-1×12	$-1 \times (-3) \times (-4)$	-2×6
-15	$-1 \times (-3) \times (-5)$	-3×5	1×-15
+15	$1 \times (-3) \times (-5)$	$(-3) \times (-5)$	1×15
+8	$-\frac{1}{2} \times (-16)$	$-\frac{1}{4} \times -32$	2×4
-8	$-\frac{1}{2} \times 16$	$-64 \times \frac{1}{8}$	$2 \times (-4)$
+12	$-1 \times (-3) \times 4$	$-1 \times (-2) \times 6$	$-1 \times (-12)$

Activity Synthesis

Ask the previously identified students to share their rationale for identifying those that do not match.

Consider highlighting the link between multiplying by a fraction and dividing by a whole number. If desired, ask students to predict the values of some division expressions with directed numbers. For example, students could use the expression $-64 \times \frac{1}{8}$ to predict the value of $-64 \div 8$. However, it is not necessary for students to learn rules for dividing directed numbers at this point. That will be the focus of future lessons.

10.3 Row Game: Multiplying Rational Numbers

Optional: 10 minutes

This optional activity gives students an opportunity to practise multiplying directed numbers. The solutions to the problems in each row are the same, so students can check their work with a partner.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Make sure students know how to play a row game. Give students 5–6 minutes of partner work time followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first 2-3 rows of the table, check-in with either select groups of students or the whole class. Invite students to share how they have applied generalisations about multiplying directed numbers from the previous activity so far.
Supports accessibility for: Conceptual processing; Organisation; Memory

Student Task Statement

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$790 \div 10$	$(7.9) \times 10$
$-\frac{6}{7} \times 7$	$(0.1) \times -60$
$(2.1) \times -2$	$(-8.4) \times \frac{1}{2}$
$(2.5) \times (-3.25)$	$-\frac{5}{2} \times \frac{13}{4}$
$-10 \times (3.2) \times (-7.3)$	$5 \times (-1.6) \times (-29.2)$

Student Response

Row 1: 79

Row 2: -6

Row 3: -4.2

Row 4: -8.125

Row 5: 233.6

Are You Ready for More?

A sequence of rational numbers is made by starting with 1, and from then on, each term is one more than the reciprocal of the previous term. Evaluate the first few expressions in the sequence. Can you find any patterns? Find the 10th term in this sequence.

$$1 \qquad 1+\frac{1}{1} \qquad 1+\frac{1}{1+1} \qquad 1+\frac{1}{1+\frac{1}{1+1}} \qquad 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}} \qquad \ldots$$

Student Response

The 10th term is $\frac{89}{55}$. Instead of calculating this from an extremely long fraction representation, we can look for patterns in applying the "one more than the reciprocal" rule. The first few terms are 1, 2, $\frac{3}{2}, \frac{5}{3}, \frac{8}{5}$. To find the next one, applying that rule gives $1 + \frac{8}{5} = 1 + \frac{5}{8} = \frac{8}{8} + \frac{5}{8} = \frac{13}{8}$. This leads to an interesting sequence in the numerators and

denominators: Given a term $\frac{a}{b}$ in the sequence, the next term will be $\frac{a+b}{a}$. That is, the old numerator becomes the new denominator, and to get the new numerator, you add together the old numerator and old denominator. This observation makes it very easy to continue the sequence to the 10th term.

$1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}$.

Activity Synthesis

Ask students, "Were there any rows that you and your partner did not get the same answer?" Invite students to share how they came to an agreement on the final answer for the problems in those rows.

Consider asking some of the following questions:

- "Did you and your partner use the same strategy for each row?"
- "What was the same and different about both of your strategies?"
- "Did you learn a new strategy from your partner?"
- "Did you try a new strategy while working on these questions?"

Lesson Synthesis

Display a number line with the numbers -1, 0, and 1 labelled. Ask students to give examples of multiplications problems with a product that is:

- greater than 1 (Sample responses: 5×3 or -5×-3)
- less than -1 (Sample responses: 5×-3 or $-5 \times -3 \times -1$)
- between 0 and 1 (Sample responses: $\frac{1}{5} \times \frac{1}{3}$ or $-\frac{1}{5} \times -\frac{1}{3}$)
- between -1 and 0 (Sample responses: $\frac{1}{5} \times -\frac{1}{3}$ or $-\frac{1}{5} \times -\frac{1}{3} \times -1$)

10.4 Making Mistakes

Cool Down: 5 minutes

Student Task Statement

Noah was doing some homework and answered the following questions. Do you agree with his answers? If you disagree, explain your reasoning.

1. $2.7 \times -2.5 = -6.75$
2. $-\frac{3}{4} \times -\frac{5}{7} = -\frac{15}{28}$

3. $5.5 \times -\frac{3}{5} = 3.3$

Student Response

1. agree
2. disagree; a negative times a negative is positive
3. disagree; a positive times a negative is negative

Student Lesson Summary

- A positive times a positive is always positive.

  For example, $\frac{3}{5} \times \frac{7}{8} = \frac{21}{40}$.

- A negative times a negative is also positive.

  For example, $-\frac{3}{5} \times -\frac{7}{8} = \frac{21}{40}$.

- A negative times a positive or a positive times a negative is always negative.

  For example, $\frac{3}{5} \times -\frac{7}{8} = -\frac{3}{5} \times \frac{7}{8} = -\frac{21}{40}$.

- A negative times a negative times a negative is also negative.

  For example, $-3 \times -4 \times -5 = -60$.

Lesson 10 Practice Problems

1. **Problem 1 Statement**

   Evaluate each expression:

   a. $-12 \times \frac{1}{3}$

   b. $-12 \times -\frac{1}{3}$

   c. $12 \times \left(-\frac{5}{4}\right)$

   d. $-12 \times \left(-\frac{5}{4}\right)$

   Solution

   a. -4

   b. 4

   c. -15

d. 15

2. **Problem 2 Statement**

Evaluate each expression:

a. $-1 \times 2 \times 3$

b. $-1 \times (-2) \times 3$

c. $-1 \times (-2) \times (-3)$

Solution

a. -6

b. 6

c. -6

3. **Problem 3 Statement**

Order each set of numbers from least to greatest.

a. 4, 8, -2, -6, 0

b. $-5, -5.2, 5.5, -5\frac{1}{2}, \frac{-5}{2}$

Solution

a. -6, -2, 0, 4, 8

b. $-5\frac{1}{2}, -5.2, -5, \frac{-5}{2}, 5.5$

4. **Problem 4 Statement**

$30 + -30 = 0$.

a. Write another sum of two numbers that equals 0.

b. Write a sum of three numbers that equals 0.

c. Write a sum of four numbers that equals 0, none of which are opposites.

Solution

Answers vary. Sample response:

a. $-589 + 589$

b. $-589 + 500 + 89$

c. $-589 + 500 + 90 + (-1)$

5. **Problem 5 Statement**

A submarine is searching for underwater features. It is accompanied by a small aircraft and an underwater robotic vehicle.

At one time the aircraft is 200 m above the surface, the submarine is 55 m below the surface, and the underwater robotic vehicle is 227 m below the surface.

a. What is the difference in height between the submarine and the aircraft?

b. What is the distance between the underwater robotic vehicle and the submarine?

Solution

We have to assume they are all directly above or below each other to answer the question.

a. 255 m

b. 172 m

6. **Problem 6 Statement**

a. Clare is cycling at a speed of 12 miles per hour. If she starts at a position chosen as zero, what will her position be after 45 minutes?

b. Han is cycling at a speed of -8 miles per hour; if he starts at the same zero point, what will his position be after 45 minutes?

c. What will the distance between them be after 45 minutes?

Solution

a. 9 miles ($12 \times 0.75 = 9$)

b. -6 miles ($-8 \times 0.75 = -6$)

c. 15 miles

7. **Problem 7 Statement**

Fill in the missing numbers in these equations

a. $(-7) \times ? = -14$

b. $? \times 3 = -15$

c. $? \times 4 = 32$

d. $-49 \times 3 = ?$

Solution

a. $(-7) \times 2 = -14$

b. $(-5) \times 3 = -15$

c. $8 \times 4 = 32$

d. $(-49) \times 3 = -147$