Lesson 1: One of these things is not like the others

Goals

- Choose and create representations to compare ratios in the context of recipes or scaled copies.
- Coordinate (orally) different representations of a situation involving equivalent ratios, e.g., discrete diagrams, tables, or double number line diagrams.
- Determine which recipes or geometric figures involve equivalent ratios, and justify (orally, in writing, and through other representations) that they are equivalent.

Learning Targets

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Lesson Narrative

The activities in the lesson are intended to support initial, informal conversations about the key ideas in proportional relationships before the next lesson introduces the terms for those ideas. At the same time, there are opportunities to review work from earlier in KS2/3 in representing ratios with tables and diagrams.

The tasks are intentionally not well-posed, that is, they do not have exact solutions. They are designed to give students an opportunity to think about how we can bring a mathematical lens to better understand common perceptual experiences, such as things that taste or look the same or different. Other possibilities include experiments with mixtures of paint, looking at videos of vehicles moving at different constant speeds, looking at taps or other water sources that flow at different rates, and so on. The focus is on examination of a feature that can be represented as a *unit rate* (flavour, colour intensity, speed, etc.) and beginning to analyse differences in that feature in terms of the two quantities involved (drink mix and water, two paint colours, time and distance, and so on). In the next lesson, this will be identified as the key idea motivating the concept of a proportional relationship. Students may recognise, from their work earlier in KS3, associated quantities as **equivalent ratios** and reason in terms of scale factors and unit rates.

The second activity provides a bridge from students' work with scale drawings in an earlier unit. In the first activity, students are given the relevant measurements; in the second, they are asked to think about how to quantify what they see, in particular, what measurements might help describe the picture.

The amount of time students spend on these activities can be adjusted based on the results of the diagnostic assessment. This lesson can be used to support just-in-time review of any ratio concepts from earlier in KS2/3 that students struggled with.

Alignments

Building On

- Understand ratio concepts and use ratio reasoning to solve problems.

Addressing

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- Analyse proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- Collect and Display
- Compare and Connect
- Discussion Supports
- Think Pair Share

Required Materials

Coloured pencils
Drink mix
A powder that is mixed with water to create a fruit-flavoured or chocolate-flavoured drink. Using a sugar-free drink mix is recommended, but *not* a mix that calls for adding a separate sweetener when mixing up the drink.

Graph paper
Measuring cup
Measuring spoons
Mixing containers
Small disposable cups
Water

Required Preparation

Make three mixtures:

- 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix

- 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix
- 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

Students will need three small cups each; they just need a few sips of the mixture in each cup.

Student Learning Goals

Let's remember what equivalent ratios are.

1.1 Remembering Double Number Lines

Warm Up: 5 minutes

This activity prompts students to reason about equivalent ratios on a double number line and think of reasonable scenarios for these ratios as a review of their earlier work in KS2/3. As students discuss their answers with their partner, select students to share their answers during the whole-class discussion.

Instructional Routines

- Think Pair Share

Launch

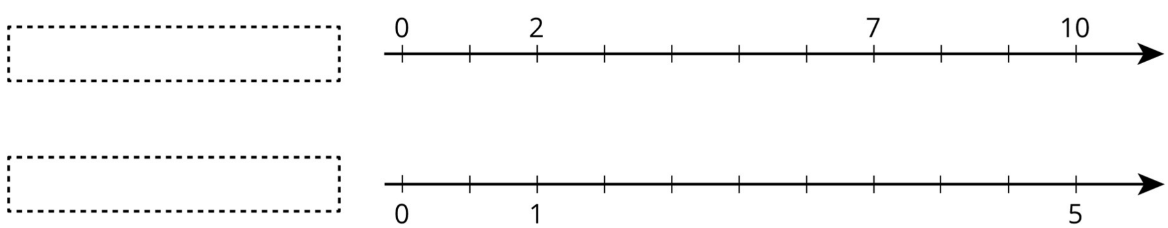
Arrange students in groups of 2. Display the double number line for all to see. Give students 2 minutes of quiet think time and ask them to give a signal when they have found the missing values. Ask students to compare their double number line with a partner and share the values they placed on the number line and their reasoning for each.

Anticipated Misconceptions

Students may struggle thinking of a scenario with a 1: 2 ratio. For those students, ask them if they can draw a picture that would represent that ratio and label each line accordingly.

Student Task Statement

1. Complete the double number line diagram with the missing numbers.



2. What could each of the number lines represent? Invent a situation and label the diagram.

3. Make sure your labels include appropriate units of measure.

Student Response

1. The ratios are all equivalent to 1: 2
2. Answers vary.
3. Answers vary.

Activity Synthesis

Invite selected students to explain how they reasoned about possible labels for each of the number lines and the units of each. After each student shares, invite others to agree, disagree, or question the reasonableness of the number line descriptions. If there is time, ask students to name other equivalent ratios that would appear if the double number line continued to the right.

1.2 Mystery Mixtures

15 minutes

The purpose of this activity is for students to articulate that the taste of the mixture depends both on the amount of water and the amount of drink mix used to make the mixture.

Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyse contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can’t, make strategic pairings of students for this task.

Instructional Routines

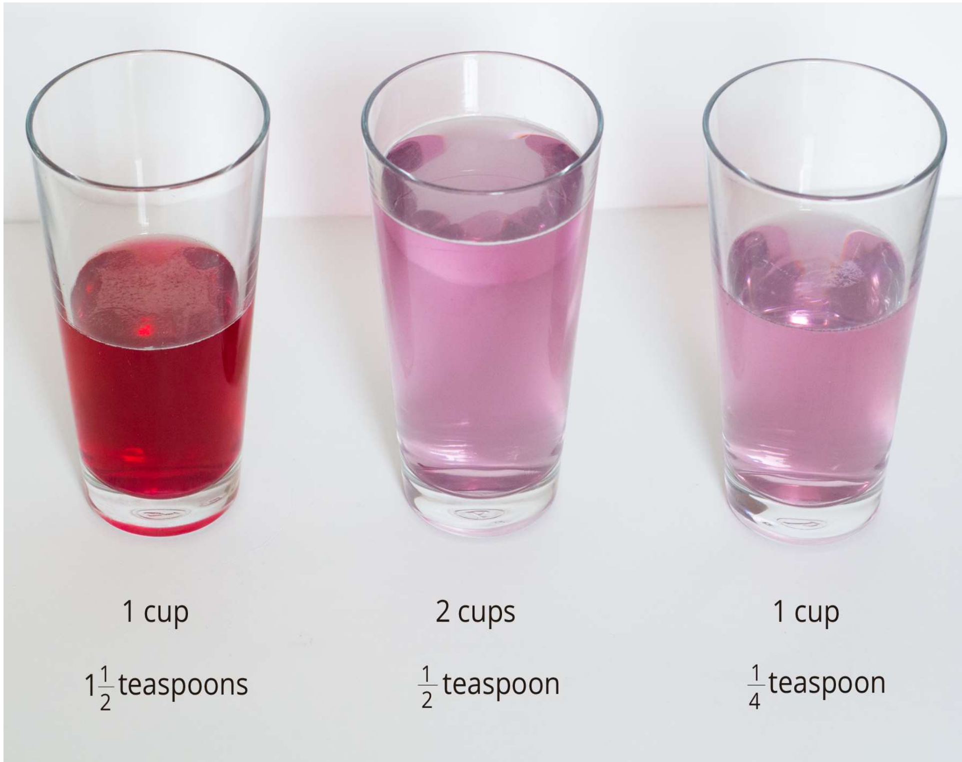
- Collect and Display
- Compare and Connect
- Discussion Supports

Launch

Show students images of the drinks.


$\frac{1}{2}$ teaspoon
2 cups


$\frac{1}{4}$ teaspoon
1 cup



If possible, give each student three cups containing the drink mixtures.

Tell students to work through the first question and pause for a discussion. Ask questions like,

- "What does it mean to say that it has more drink mix in it?"
- "Imagine you take different amounts of the two that taste the same. There will be more drink mix in the larger amount, but it will not taste different. Why is that?"

The goal is to see that in the same quantity of each mixture (say a teaspoon), the more flavoured drink mixture has more drink mix for the same amount of water. (Alternatively, we can say the more flavoured drink mixture has less water for the same amount of drink mix.) Use Discussion Supports by making gestures or acting out facial expressions for "strength" of the mixture.

After the students have made some progress understanding this idea, the class should continue to the second question. If students finish quickly, press them to find the amount of drink mix per cup of water in each recipe, thus emphasising the unit rate.

Conversing, Writing: Collect and Display. Before students begin writing a response to the first question, invite them to discuss their thinking with a partner. Listen for vocabulary and phrases students use to describe how the amount of water and the amount of drink mix affects the taste of the mixture. Collect and display words and phrases such as "more drink mix," "more water," "tastes stronger/weaker," etc., and then encourage students to

use this language in their written responses, and during discussion.
Design Principle(s): Support sense-making

Student Task Statement

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture tastes different? Describe how it is different.

2. Here are the recipes that were used to make the three mixtures:

   - 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix

   - 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix

   - 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

   Which of these recipes is for the stronger tasting mixture? Explain how you know.

Student Response

1. The first mixture is different—it is stronger because it has more drink mix in it.

2. Answers vary. Possible responses: The first one has more drink mix, so it is the strongest one. There is more drink mix for every cup in the first one.

Are You Ready for More?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar

- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar

- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

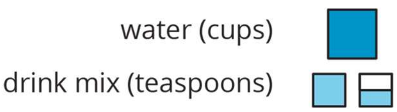
Student Response

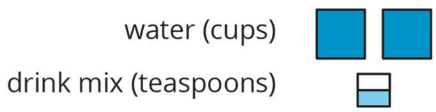
Mixture A and Mixture C will taste exactly the same. Mixture B will taste equally salty, but will be a little bit sweeter.

Activity Synthesis

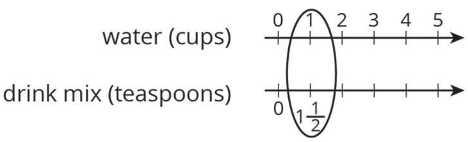
The key takeaway from this activity is that the flavour depends on both how much drink mix *and* how much water there is in the mixture. For a given amount of water, the more drink mix you add, the stronger the mixture tastes. Likewise, for a given amount of drink mix, the more water you add, the weaker the mixture tastes. To compare the amount of flavour of two mixtures, when both the amounts of drink mix and the amounts of water are different in the two mixtures, we can write ratios equivalent to each situation so that we are comparing the amount of drink mix for the same amount of water or the amount of water for the same amount of drink mix. Computing a *unit rate* for each situation is a particular instance of this strategy. Make these ideas explicit if the students do not express them.

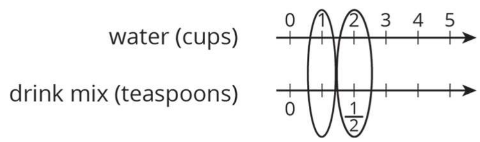
If students do not create them, draw discrete diagrams like this:





Or double number line diagrams like this:





For each mixture, identify correspondences between the discrete and number line diagrams, and between the diagrams and tables:

water (cups)	drink mix (teaspoons)
1	$1\frac{1}{2}$
2	3
water (cups)	drink mix (teaspoons)
2	$\frac{1}{2}$
1	$\frac{1}{4}$

Ask questions like, "On the double number line diagram we see the 1 to $1\frac{1}{2}$ relationship at the first tick mark. Where do we see that relationship in the double bar model? In the table?"

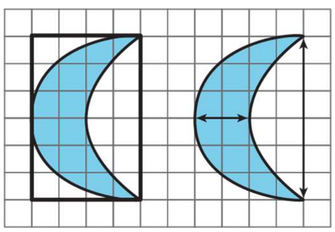
Use Compare and Connect for students to compare methods of how they knew which recipe was strongest. Who used multiplication? Who used division? Who used a unit rate of water per drink mix teaspoon? Who used a unit rate of drink mix per water cup?

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use the same colour to illustrate correspondences between the number line diagrams and ratio tables for each mixture.
Supports accessibility for: Visual-spatial processing

1.3 Crescent Moons

15 minutes (there is a digital version of this activity)

In a previous unit, students studied scale drawings of real-world objects. Later in KS3, they will study dilations and similarity. The purpose of this activity is to use students' recent study of scale drawings as a transition to the study of proportional relationships. Initially, students may describe the difference between Moons A, B, C, and D in qualitative terms, e.g., "D is more squished than the others." They may also use the term "scaled copies," which appeared in the work of a previous unit, but struggle to identify measurements to use in these figures that consist only of curved sides. It is important to ask students to articulate what they mean by "squished" in quantitative terms, for example, by talking about the height relative to the width and helping students to define "height" and "width" of a moon in some appropriate way. Once students have that, they can note that the height of the enclosing rectangle is always one and a half times its width for Moons A, B, and C, but not D, or they might note that the distance tip to tip is three times the width of the widest part of the moon for Moons A, B, and C, but not so for D.



As students explore these transformations, ask questions with the goal of having students articulate that for two images to look like scaled copies of each other, the ratios of the side lengths need to be the same.

In the third question, students are asked to represent the situation with tables and double number line diagrams, providing students with an opportunity to recall these representations from their work earlier in KS2/3.

Instructional Routines

- Collect and Display
- Discussion Supports
- Think Pair Share

Launch

Give students 3 minutes of quiet think time and tell them to pause after the first two questions. After the quiet think time, ask students to discuss their answers with a partner to describe how Moon D is different. Use 'Collect and Display' as students share. Record the explanations that students are using to describe the moons. Ensure that students see some ways to measure lengths associated with the moons, then complete the last question.

If using the digital activity, give students 3 minutes of quiet think time and tell them to pause after the first two questions. After the quiet think time, students can discuss answers with a partner. Based on student conversations, you may want to have a whole-group discussion to ensure they see a way to measure the lengths associated with the moons before they attempt to answer the last question.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have a method for comparing the measurements of each moon. For example, create a rectangle around each moon and compare the width-height ratios.
Supports accessibility for: Memory; Organisation

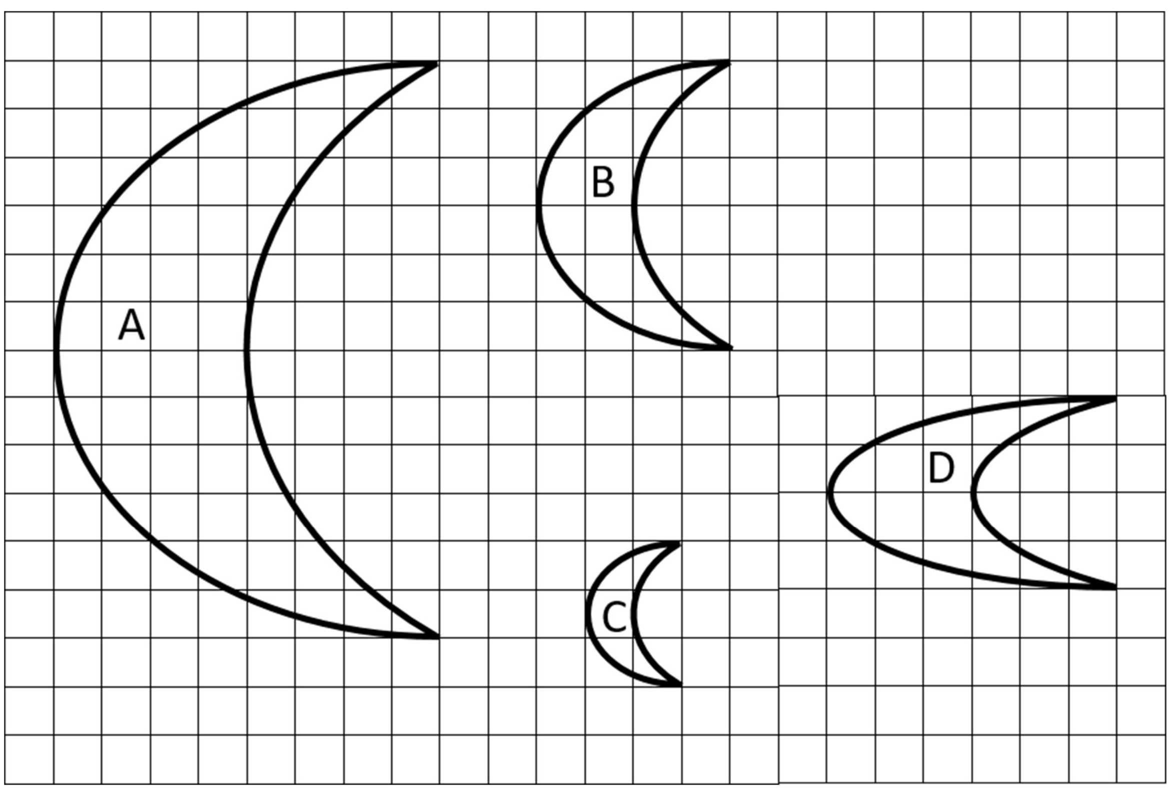
Anticipated Misconceptions

For question 2, students might attempt to find the area of each moon by counting individual square units. Suggest that they create a rectangle around each moon instead and compare the width-height ratios.

For question 3, if students are not sure how to set up these representations, providing a template may be helpful.

Student Task Statement

Here are four different crescent moon shapes.



1. What do Moons A, B, and C all have in common that Moon D doesn't?

2. Use numbers to describe how Moons A, B, and C are different from Moon D.

3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Student Response

1. Answers vary. Possible responses: Moon D is smashed down more than Moons A, B, and C. Moons A, B, and C are all taller than they are wide while Moon D is wider than it is tall.

2. Answers vary. Possible response: We could enclose the Moon with a rectangle and compare the ratio of width to height for each moon.

3. Answers vary. Possible responses: (Note that students might put the rows or columns in another order.)

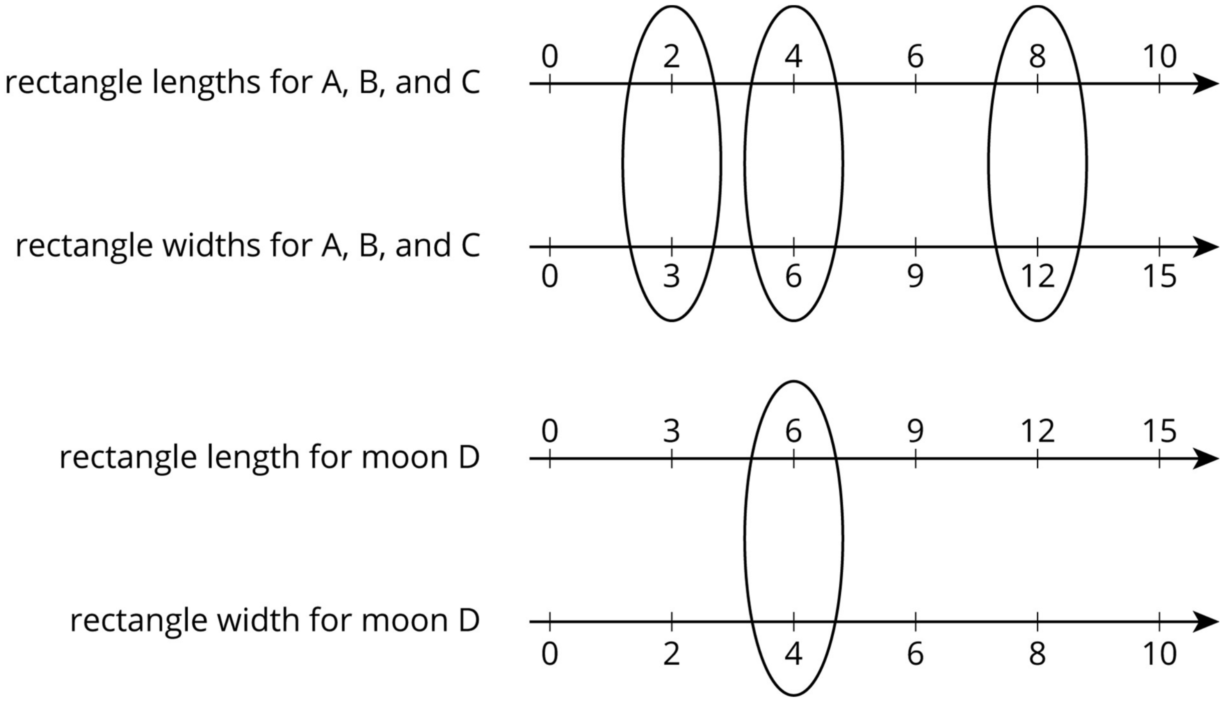
For students enclosing the moons with a rectangle:

moon	rectangle length (units)	rectangle width (units)	length ÷ width
C	2	3	$\frac{2}{3}$
B	4	6	$\frac{4}{6}=\frac{2}{3}$
A	8	12	$\frac{8}{12}=\frac{2}{3}$
D	6	4	$\frac{6}{4}$

For students measuring widest part and tip-to-tip:

moon	**widest part (units)**	**tip-to-tip (units)**	**widest part ÷ tip-to-tip**
C	1	3	$\frac{1}{3}$
B	2	6	$\frac{2}{6}=\frac{1}{3}$
A	4	12	$\frac{4}{12}=\frac{1}{3}$
D	3	4	$\frac{3}{4}$

Here is an example of double number line diagrams for enclosing rectangles:



Activity Synthesis

As students suggest ways to characterise the difference between Moons A, B, and C and Moon D, ask questions that help them clarify and make their statements more precise. For example,

- "What does it mean to be 'smashed down?' What measurements might you make to show that this is true?"
- "Is there anything else about what A, B, and C have in common that you can identify?"
- "What things might we measure about these moons to be able to talk about what makes them different in a more precise way?"
- "How can you represent the ratios of the measurements you are comparing using a table and a double number line diagram?"

If some students are still struggling with the tables or diagrams, ask students who were successful to share their representations with the class.

Speaking: Discussion Supports. To aid students in producing statements about comparisons of proportional relationships. Provide sentence frames for students to use when they are comparing and contrasting such as: "All _____ have _____ except _____," "What makes _____ different from the others is _____." Ask students to further add to the statement to clarify the comparisons if they used descriptor words such as "wider, narrower, etc." Improved statements should include mathematical language to describe the measurements (such as length, width, ratio, etc.). This will help students practice and develop language for

comparisons.
Design Principle(s): Support sense-making

Lesson Synthesis

Revisit each activity (the drink mixture and the moons), and note that in each, there are two quantities. The ratios of those quantities are equivalent for all but one of the things, the one that is different in an important way. This unit is the study of situations where **equivalent ratios** characterise something important about a situation. As part of the discussion, use and emphasise ratio and rate language in contexts and review representations like double number line diagrams and tables of equivalent ratios.

- "In what important way were the drink mixtures the same and different?"
- "How could we tell using ratios that these were the same and different?"
- "In what important way were the moons the same and different?"
- "How could we tell using numbers that these were the same and different?"

1.4 Orangey-Pineapple Juice

Cool Down: 5 minutes

Student Task Statement

Here are three different recipes for Orangey-Pineapple Juice. Two of these mixtures taste the same and one tastes different.

- Recipe 1: Mix 4 cups of orange juice with 6 cups of pineapple juice.
- Recipe 2: Mix 6 cups of orange juice with 9 cups of pineapple juice.
- Recipe 3: Mix 9 cups of orange juice with 12 cups of pineapple juice.

Which two recipes will taste the same, and which one will taste different? Explain or show your reasoning.

Student Response

Recipe 3, which requires $1\frac{1}{3}$ cups of pineapple juice for every 1 cup of orange juice, is different from Recipes 1 and 2, which both require $1\frac{1}{2}$ cups of pineapple juice for every 1 cup of orange juice.

recipe 1

orange juice (cups)	pineapple juice (cups)
4	6
2	3
1	$1\frac{1}{2}$

recipe 2

orange juice (cups)	pineapple juice (cups)
6	9
2	3
1	$1\frac{1}{2}$

recipe 3

orange juice (cups)	pineapple juice (cups)
9	12
3	4
1	$1\frac{1}{3}$

Double number line diagrams can be used to compare the recipes, for instance, by noting that for Recipes 1 and 2, you use 2 cups of orange juice for every 3 cups of pineapple juice, whereas with Recipe 3, you use $2\frac{1}{4}$ cups of orange juice for 3 cups of pineapple juice.

Student Lesson Summary

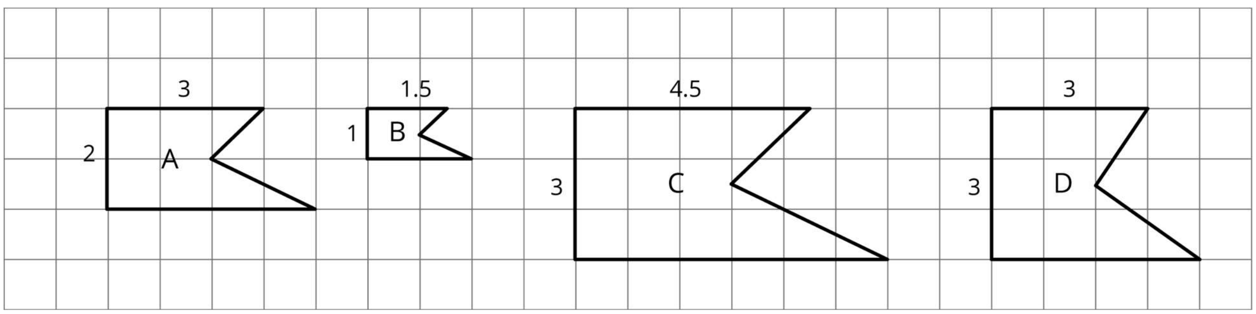
When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were 6: 4, then the mixture would taste different.

Notice that the ratios of pairs of corresponding side lengths are equivalent in figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other; this is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

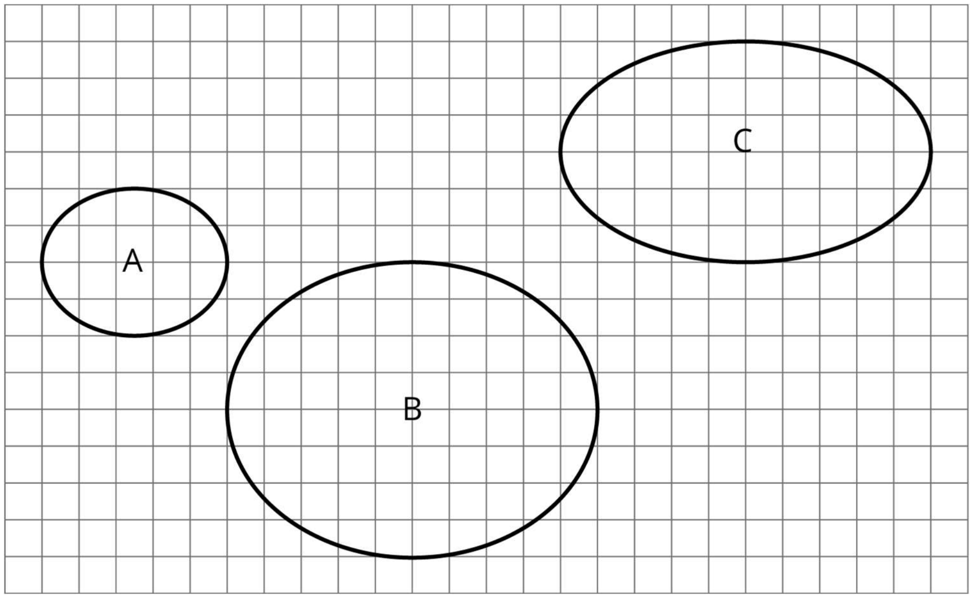
Glossary

- equivalent ratios

Lesson 1 Practice Problems

1. **Problem 1 Statement**

   Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.



Solution

C is different from A and B. For both A and B, the width:height ratio is 5:4. However, for C, the width is 10 units and the height is 6 units, so the width:height ratio is 5:3.

2. **Problem 2 Statement**

In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

Solution

Yes, since 3 times 1.5 is 4.5 and 2 times 1.5 is 3.

3. **Problem 3 Statement**

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects

a. A person

b. A football field (120 yards by $53\frac{1}{3}$ yards)

c. The state of Washington (about 240 miles by 360 miles)

d. The floor plan of a house

e. A rectangular farm (6 miles by 2 mile)

Scales

- 1 in : 1 ft
- 1 cm : 1 m
- 1: 1 000
- 1 ft: 1 mile
- 1: 100 000
- 1 mm: 1 km
- 1: 10 000 000

Solution

Answers vary. Sample responses:

a. 1 in :1 ft

b. 1: 1 000

c. 1: 10 000 000

d. 1cm: 1 m

e. 1: 100 000

4. **Problem 4 Statement**

Which scale is equivalent to 1 cm to 1 km?

a. 1 to 1 000

b. 10 000 to 1

c. 1 to 100 000

d. 100 000 to 1

e. 1 to 1 000 000

Solution D

5. **Problem 5 Statement**

a. Find 3 different ratios that are equivalent to 7: 3.

b. Explain why these ratios are equivalent.

Solution

a. Answers vary. Sample response: 14: 6, 21: 9, 28: 12

b. Answers vary. Sample response: 7 and 3 are each multiplied by 2, 3, and 4, respectively.