Optimization Problems

Introduction: The Problem

- For this problem, we want to use a model to answer the following:

A farmer has 24 meters of fencing and wants to fence off a rectangular field that borders the river. No fence along the river is need. What are the dimensions of the fence with the largest area?

First, we will construct a model which will show a changing rectangle representing the fenced lot. Then, we will look at the function on a graph.

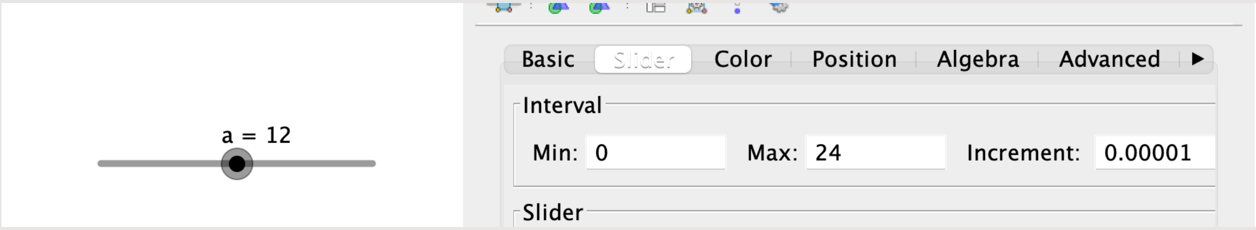
The Model: Step 1

- First, we make a horizontal line (in blue) which will represent the river. This horizontal line can be the x-axis. So, we input y=0, and change the color.

- Second, we start constructing the fence. The variable “a” will represent the length of the side parallel to the river, and “b” will represent the sides perpendicular to the river. Since there are only 24 meters of fencing, then our dimensions are restricted by the following equation:

$$a + 2b = 24 \quad \text{or} \quad b = 12 - a/2$$

- So, as b depends on a, we can make a slider for a to change the dimensions of the fence. Note that a is at most 24, so the slider goes from 0 to 24, and we set and increment of 0.00001 for smoother transition.

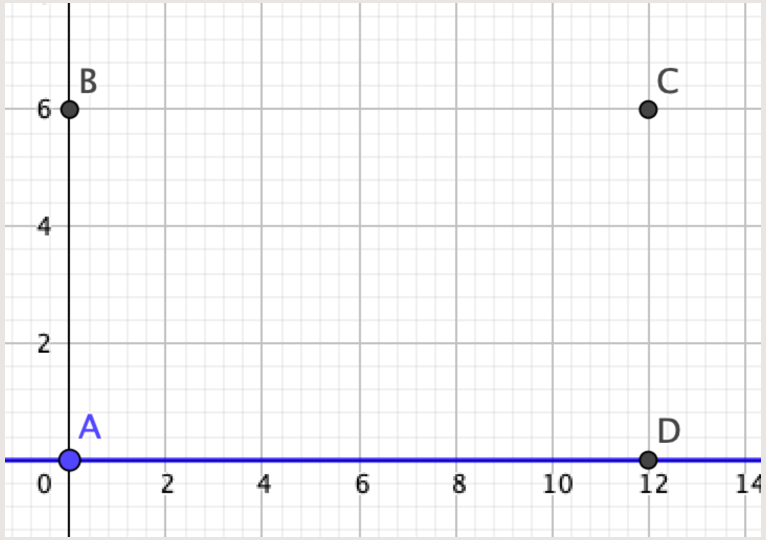


The Model: Step 2

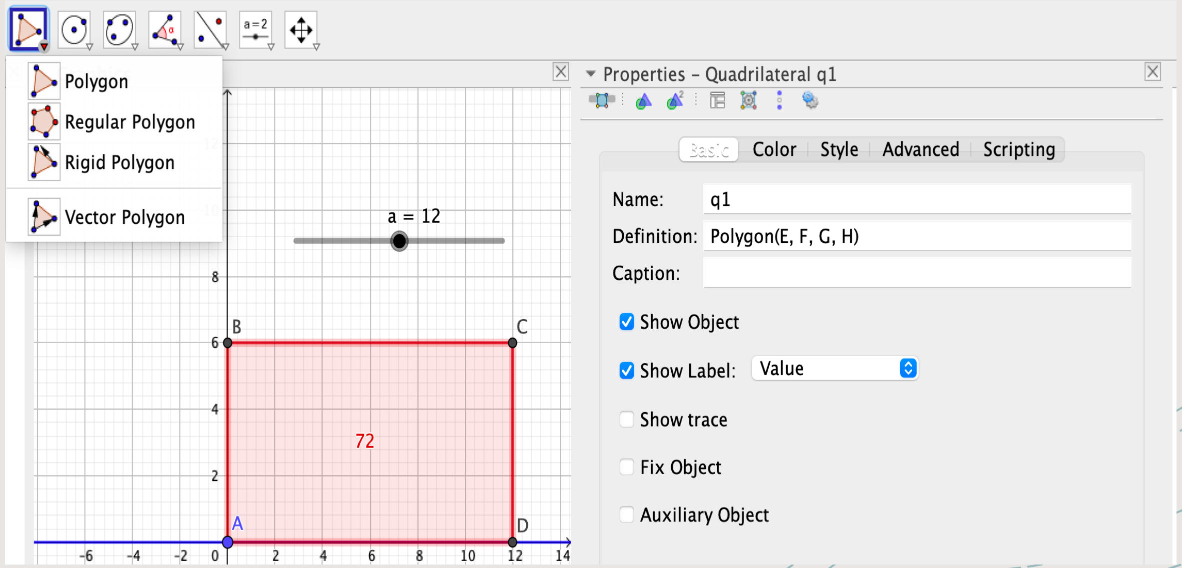
Now we construct the fence:

- First, we make the four corners of hour fence lot using the origin as a point of reference. So, we have the following four points:

$$(0,0) \quad (0,b) \quad (a,b) \quad (a,0)$$

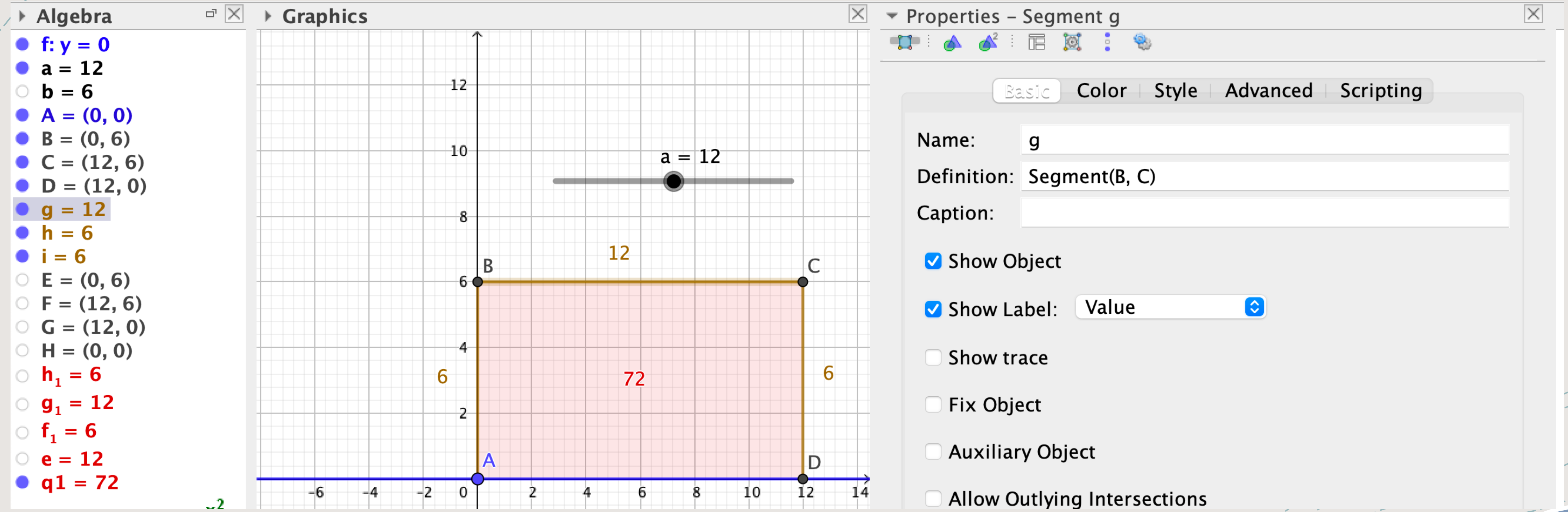


- Second, we use the polygon feature to construct a rectangle which will represent the fence. And we set the label to value, which will display the area of the rectangle.



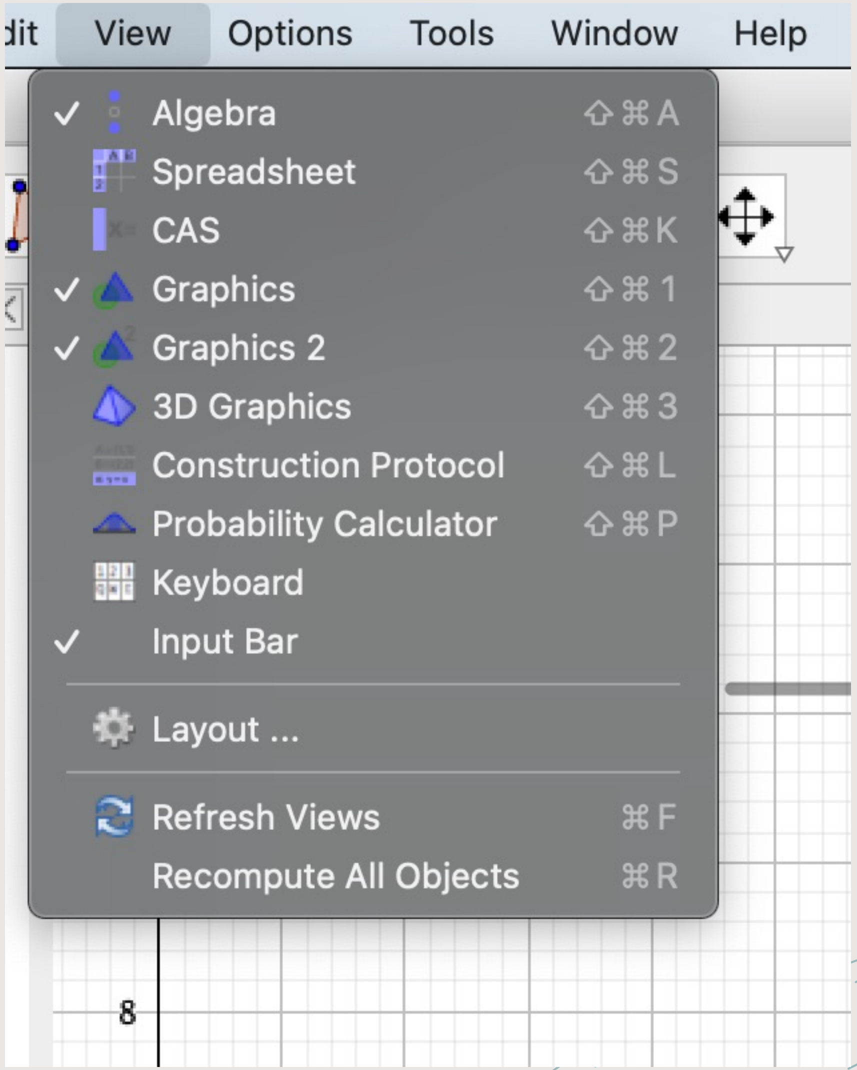
The Model: Step 3

– To finalize the construction, we use line segments (color brown) over the edges of the polygon which are meant to be the fence. This way, we can display the dimensions, by setting the label of the edges to “Value”, and we hide the edges of the polygon.



The Function: Step 4

- Before we display the function, we have to go to view and select Graphics 2. This will open a second window where we can plot the objective function, showing the absolute maximum of the function.



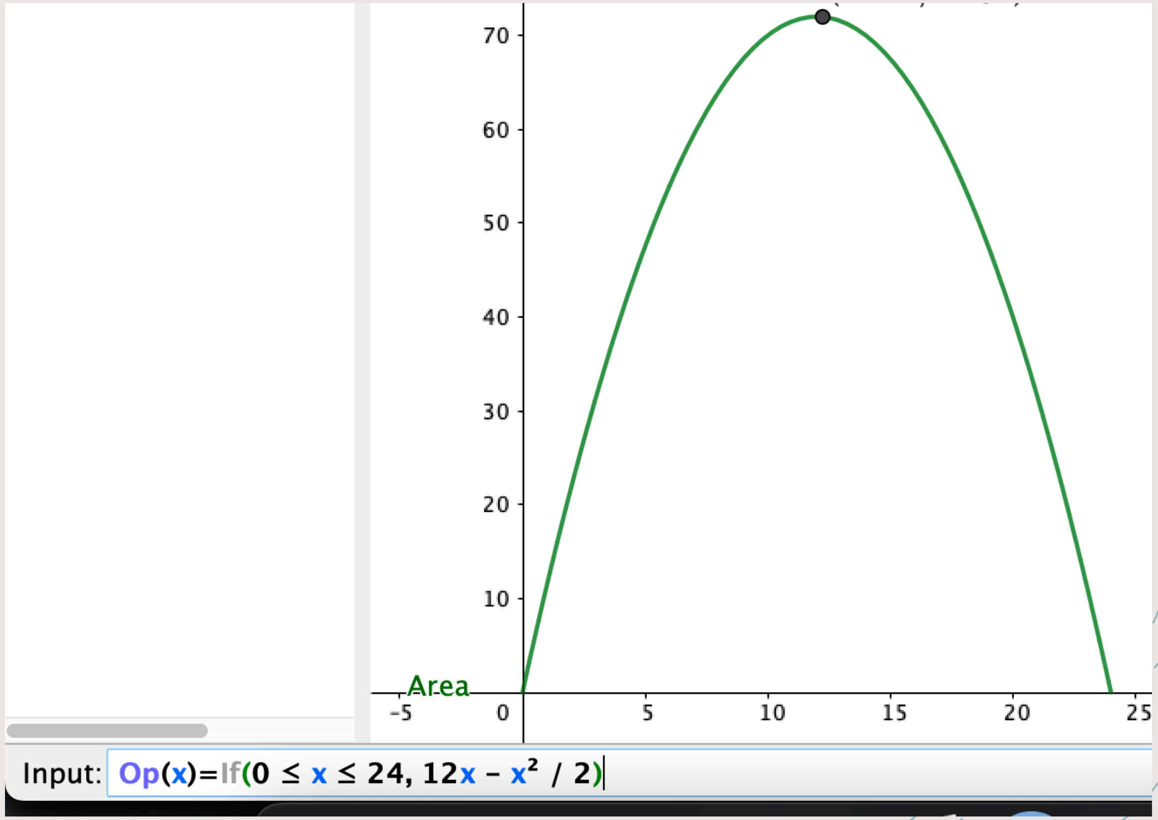
The Function: Step 5

- Now we plot the function on the second window. Since the objective is to maximize the area, then how function is

$$A = ab = a(12 - a/2)$$

So, to plot the function. We input:

Op(a) = If(0<x<24, a*(12-a/2))



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- To keep track of the change in area as the dimension changes, we set a point on the graph by inputting the point whose x-coord is ”a”, and y-coord is Op(a): (a, Op(a)) and set the label to be value to display the point.