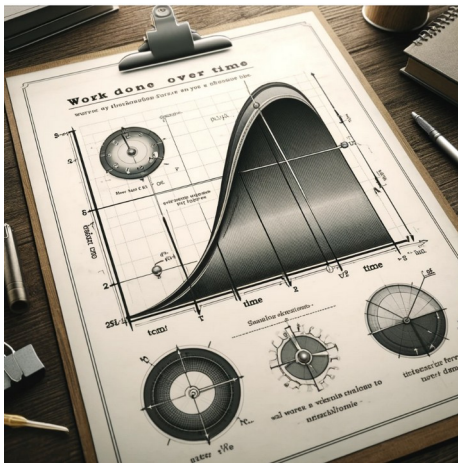
Integration by parts AAHL5.20



Imagine you're trying to figure out the work done over time by a variable force. In physics, work is defined as the integral of force times distance, but what if the force changes as the object moves? Consider force (F) as one function and displacement (s) as another.

Now, you want to integrate F with respect to s over a specific interval, but F is a function of s, making it tricky to integrate directly. Here's how integration by parts helps:

1. Choose u to be the force F(s), which you understand well, and dv to be ds, the differential element of displacement.
2. Compute du, which would be F'(s) ds, representing the rate of change of force with displacement.
3. Compute v from dv, which simply integrates ds to s, representing the total displacement from the start.

With these choices, integration by parts formula becomes:
$\int F(s)\,ds = F(s) * s - \int s * F'(s)\,ds$

This formula effectively transforms the original problem into one where you compute:
- The total force at a point multiplied by the displacement up to that point, minus
- The integral of the displacement multiplied by the rate of change of the force.

This decomposition helps to tackle integrals that are otherwise difficult to solve directly, particularly when one function is easy to differentiate and the other is easy to integrate.