Mathematics Education Innovation

Lesson 13: Cube roots

Goals

- Determine the whole numbers that a cube root lies between, and explain (orally) the reasoning.
- Generalise a process for approximating the value of a cube root, and justify (orally and in writing) that if $x^3 = a$, then $x = \sqrt[3]{a}$.

Learning Targets

- When I have a cube root, I can reason about which two whole numbers it is between.

Lesson Narrative

In this lesson, students continue to work with cube roots, moving away from the geometric interpretation in favour of the algebraic definition. They approximate cube roots and locate them on the number line. They see their first negative cube root, and locate it on the number line.

Addressing

- Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue to get better approximations.

Instructional Routines

- Compare and Connect
- Discussion Supports
- Think Pair Share
- True or False

Required Materials

Coloured pencils

Student Learning Goals

Let’s compare cube roots.

13.1 True or False: Cubed

Warm Up: 5 minutes

The purpose of this warm-up is for students to analyse symbolic statements about cube roots and decide if they are true or not based on the meaning of the cube root symbol.

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have an answer and a strategy. After each problem, give students 1 minute of quiet think time and follow with a whole-class discussion.

Student Task Statement

Decide if each statement is true or false.

$\left(\sqrt[3]{5}\right)^3 = 5$

$\left(\sqrt[3]{27}\right)^3 = 3$

$7 = \left(\sqrt[3]{7}\right)^3$

$\left(\sqrt[3]{10}\right)^3 = 1\,000$

$\left(\sqrt[3]{64}\right) = 2^3$

Student Response

True, false, true, false, false

Activity Synthesis

Poll students on their response for each problem. Record and display their responses for all to see. If all students agree, ask 1 or 2 students to share their reasoning. If there is disagreement, ask students to share their reasoning until an agreement is reached.

13.2 Cube Root Values

10 minutes

The purpose of this activity is for students to think about cube roots in relation to the two whole number values they are closest to. Students are encouraged to use the fact that $\sqrt[3]{a}$ is a solution to the equation $x^3 = a$. Students can draw a number line if that helps them reason about the magnitude of the given cube roots, but this is not required. However students reason, they need to explain their thinking.

Monitor students multiplying non-integers by hand to try and approximate. While this isn't what the problem is asking for, their work could be used to think about which integer the square root is closest to and should be brought up during the whole-class discussion.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Do not give students access to calculators. Students in groups of 2. Two minutes of quiet work time, followed by partner then whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Provide students with access to a number line that includes rational numbers to support information processing.
Supports accessibility for: Visual-spatial processing; Organisation

Student Task Statement

What two whole numbers does each cube root lie between? Be prepared to explain your reasoning.

1. $\sqrt[3]{5}$
2. $\sqrt[3]{23}$
3. $\sqrt[3]{81}$
4. $\sqrt[3]{999}$

Student Response

1. 1 and 2. $1^3 = 1$ and $2^3 = 8$, so $\sqrt[3]{5}$ is between 1 and 2.
2. 2 and 3. $2^3 = 8$ and $3^3 = 27$, so $\sqrt[3]{23}$ is between 2 and 3.
3. 4 and 5. $4^3 = 64$ and $5^3 = 125$, so $\sqrt[3]{81}$ is between 4 and 5.
4. 9 and 10. $10^3 = 1\,000$ so $\sqrt[3]{999}$ is a little bit less than 10.

Activity Synthesis

Discuss:

- "What strategy did you use to figure out the two whole numbers?"
  (I made a list of perfect cubes and then found which two the number was between.)

- "How can we write one of these answers using inequality symbols?" (For example, $2 < \sqrt[3]{23} < 3$.)

Speaking: Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language to restate and/or revoice the strategy (or strategies) presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language that describes strategies to figure out which two whole numbers each cube root lies between.
Design Principle(s): Support sense-making; Maximise meta-awareness

13.3 Solutions on a Number Line

10 minutes

The purpose of this activity is for students to use rational approximations of irrational numbers to place both rational and irrational numbers on a number line, and to reinforce the definition of a cube root as a solution to the equation of the form $x^3 = a$. This is also the first time that students have thought about negative cube roots.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

No access to calculators. Students in groups of 2. Two minutes of quiet work time, followed by partner then whole-class discussion.

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the following term with a number line and sample calculations from previous activities that students will need to access for this activity: cube root.
Supports accessibility for: Memory; Language

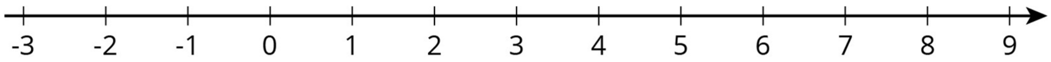
Student Task Statement

The numbers x, y, and z are positive, and:

$x^3 = 5$

$y^3 = 27$

$z^3 = 700$



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.

2. Plot -$\sqrt[3]{2}$ on the number line.

Student Response

The point x should be between 1 and 2, the point y should be at 3, the point z should be between 8 and 9, and the point -$\sqrt[3]{2}$ should be between -1 and -2.

Are You Ready for More?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

Student Response

Three of Diego's statements are correct, but $\sqrt{-64}$ does not equal -8 because $(-8)^2$ is 64, not -64.

Activity Synthesis

Display the number line from the activity for all to see. Select groups to share how they chose to place values onto the number line. Place the values on the displayed number line as groups share, and after each placement poll the class to ask if students used the same reasoning or different reasoning. If any students used different reasoning, invite them to share with the class.

Conclude the discussion by asking students to share how they placed -$\sqrt[3]{2}$ on the number line.

Representing, Conversing, Listening: Compare and Connect. As students prepare their work before the discussion, look for approaches that favour the algebraic definition, and approaches that use the geometric interpretation. Call students' attention to the different ways students describe how they operated on numbers cubed to determine the approximate cube roots, and to the different ways these operations are made visible in each representation (e.g., using the fact that $\sqrt[3]{a}$ is a solution to the equation $x^3 = a$, and finding rational approximations of the irrational numbers to plot the cube roots on the

number line). Emphasise language used to make sense of strategies reinforcing the definition of a cube root as a solution to the equation of the form $x^3 = a$.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

The purpose of this discussion is to reinforce the definition of a cube root.

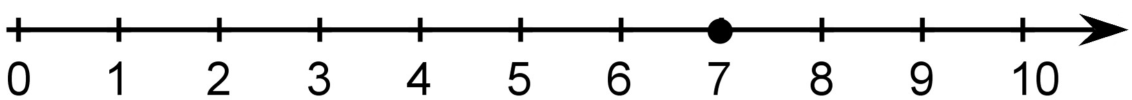
- "What is the solution to the equation $a^3 = 47$?" ($\sqrt[3]{47}$)
- "What is the solution to the equation $a^3 = 64$?" ($\sqrt[3]{64} = 4$)
- "What is the solution to the equation $a^3 = -64$?" ($\sqrt[3]{-64} = -4$)
- "How can we plot cube roots on the number line?" (Find the two whole numbers they lie between, and determine the approximate location between them.)

13.4 Different Types of Roots

Cool Down: 5 minutes

Student Task Statement

Lin is asked to place a point on a number line to represent the value of $\sqrt[3]{49}$ and she writes:



Where should $\sqrt[3]{49}$ actually be on the number line? How do you think Lin got the answer she did?

Student Response

Answers vary. Sample response: $\sqrt[3]{49}$ should be between 3 and 4 on the number line. I think Lin placed the point at $\sqrt{49}$ because she forgot it was a cube root instead of a square root.

Student Lesson Summary

Remember that square roots of whole numbers are defined as side lengths of squares. For example, $\sqrt{17}$ is the side length of a square whose area is 17. We define cube roots similarly, but using cubes instead of squares. The number $\sqrt[3]{17}$, pronounced "the cube root of 17," is the edge length of a cube which has a volume of 17.

We can approximate the values of cube roots by observing the whole numbers around it and remembering the relationship between cube roots and cubes. For example, $\sqrt[3]{20}$ is between 2 and 3 since $2^3 = 8$ and $3^3 = 27$, and 20 is between 8 and 27. Similarly, since 100 is between 4^3 and 5^3, we know $\sqrt[3]{100}$ is between 4 and 5. Many calculators have a cube root function which can be used to approximate the value of a cube root more precisely. Using our numbers from before, a calculator will show that $\sqrt[3]{20} \approx 2.7144$ and that $\sqrt[3]{100} \approx 4.6416$.

Also like square roots, most cube roots of whole numbers are irrational. The only time the cube root of a number is a whole number is when the original number is a perfect cube.

Lesson 13 Practice Problems

1. **Problem 1 Statement**

   Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

   a. $t^3 = 216$

   b. $a^2 = 15$

   c. $m^3 = 8$

   d. $c^3 = 343$

   e. $f^3 = 181$

   Solution

   a. $t = 6$

   b. $a = \sqrt{15}$

   c. $m = 2$

   d. $c = 7$

   e. $f = \sqrt[3]{181}$

2. **Problem 2 Statement**

   For each cube root, find the two whole numbers that it lies between.

   a. $\sqrt[3]{11}$

   b. $\sqrt[3]{80}$

   c. $\sqrt[3]{120}$

   d. $\sqrt[3]{250}$

   Solution

a. 2 and 3

b. 4 and 5

c. 4 and 5

d. 6 and 7

3. **Problem 3 Statement**

Order the following values from least to greatest:

$\sqrt[3]{530}$, $\sqrt{48}$, π, $\sqrt{121}$, $\sqrt[3]{27}$, $\frac{19}{2}$

Solution

$\sqrt[3]{27}, \pi, \sqrt{48}, \sqrt[3]{530}, \frac{19}{2}, \sqrt{121}$

4. **Problem 4 Statement**

Select **all** the equations that have a solution of $\frac{2}{7}$:

a. $x^2 = \frac{2}{7}$

b. $x^2 = \frac{4}{14}$

c. $x^2 = \frac{4}{49}$

d. $x^3 = \frac{6}{21}$

e. $x^3 = \frac{8}{343}$

f. $x^3 = \frac{6}{7}$

Solution ["C", "E"]

5. **Problem 5 Statement**

The equation $x^2 = 25$ has *two* solutions. This is because both $5 \times 5 = 25$, and also -5 × -5 $= 25$. So, 5 is a solution, and also -5 is a solution. But! The equation $x^3 = 125$ only has one solution, which is 5. This is because $5 \times 5 \times 5 = 125$, and there are no other numbers you can cube to make 125. (Think about why -5 is not a solution!)

Find all the solutions to each equation.

a. $x^3 = 8$

b. $\sqrt[3]{x} = 3$

c. $x^2 = 49$

d. $x^3 = \frac{64}{125}$

Solution

a. 2

b. 27

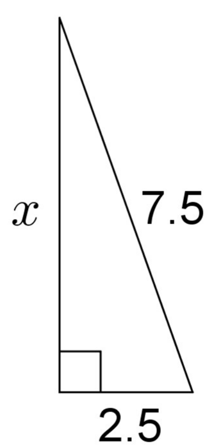
c. 7 and -7

d. $\frac{4}{5}$

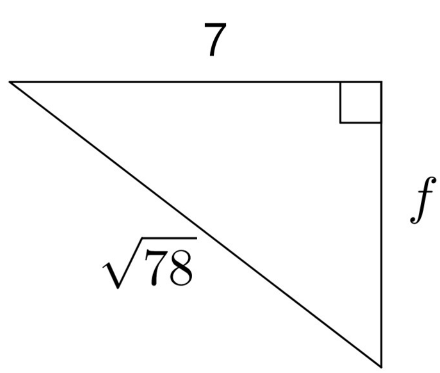
6. **Problem 6 Statement**

Find the value of each variable, to the nearest tenth.

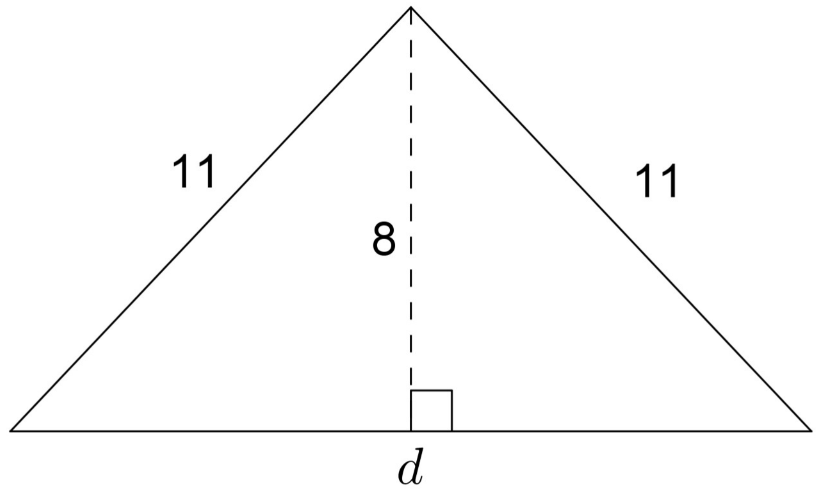
a.



b.



c.



Solution

a. $x \approx 7.1$

b. $f \approx 5.4$

c. $d \approx 15.1$

7. **Problem 7 Statement**

A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round your answer to the nearest metre.

Solution

The walk through the park is about 68 m shorter than the walk around the park. Along the streets: $80 + 270 = 350$. Along the diagonal: solve $80^2 + 270^2 = x^2$ and get approximately 282. $350 - 282 = 68$.