Lesson 13: Dividing decimals by decimals

Goals

- Compare and contrast (orally and using other representations) division problems with whole-number and decimal divisors.
- Divide whole numbers or decimals by decimals, and explain the reasoning (orally and using other representations), including choosing to divide a different expression that gets the same quotient.
- Generate another division expression that has the same value as a given expression, and justify (orally) that they are equal.

Learning Targets

- I can explain how multiplying dividend and divisor by the same power of 10 can help me find a quotient of two decimals.
- I can find the quotient of two decimals.

Lesson Narrative

In the previous lesson, students learned how to divide a decimal by a whole number. They also saw that multiplying both the dividend and the divisor by the same power of 10 does not change the quotient. In this lesson, students integrate these two understandings to find the quotient of two decimals. They see that to divide a number by a decimal, they can simply multiply both the dividend and divisor by a power of 10 so that both numbers are whole numbers. Doing so makes it simpler to use long division, or another method, to find the quotient. Students then practise using this principle to divide decimals in both abstract and contextual situations.

Addressing

- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- Fluently divide multi-digit numbers using the standard algorithm.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- Fluently divide multi-digit numbers using the standard algorithm.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Clarify, Critique, Correct
- Compare and Connect
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's divide decimals by decimals.

13.1 Same Values

Warm Up: 5 minutes

In this warm-up, students continue the decimal division work from the previous lesson and do so in the context of money. The work reinforces the idea that the value of a quotient does not change if the numerator and denominator are both multiplied by the same power of 10.

Launch

Give students a moment to read the first question and to estimate whether the quotient will be less than 1 or more than 1. Ask them to give a signal when they have an estimate and can explain it. Ask one student from the "more than 1" group to explain their reasoning and another from the "less than 1" group to do the same. Clarify that the quotient will be less than 1 and give students a few minutes to complete the questions. If time is limited, ask students to work only on the second question. Follow with a whole-class discussion.

Student Task Statement

1. Use long division to find the value of $5.04 \div 7$.
2. Select **all** of the quotients that have the same value as $5.04 \div 7$. Be prepared to explain how you know.
   a. $5.04 \div 70$
   b. $50.4 \div 70$
   c. $504\,000 \div 700$
   d. $504\,000 \div 700\,000$

Student Response

1. 0.72. Student work should show long division.
2. B and D. Sample reasoning:

- Long division can be used to divide 50.4 by 70 and still get 0.72.
- $5.04 \div 7 = 50.4 \div 70$ because 5.04 and 7 can be multiplied by 10 to get 50.4 and 70. Multiplying both numbers by the same non-zero factor does not change the quotient.
- 504 000 is 5.04 groups of 100 000, and 700 000 is 7 groups of 100 000. Since the unit of the groups is the same (100 000), examining how many times 7 goes into 5.04, which could simply be represented with $5.04 \div 7$, gives the answer.

Activity Synthesis

Focus the whole-class discussion on the second question. Select several students to explain why choices b and d are correct and why a and c are not. Students should see that $5.04 \div 70$ and $504\,000 \div 700$ are not equivalent expressions to $5.04 \div 7$ because the dividend and divisors in each pair are *not* results of multiplying the 5.04 and 7 by the same factor or the same power of 10.

13.2 Placing Decimal Points in Quotients

15 minutes

The goal of this task is to show that we can calculate quotients of two decimals by multiplying both numbers by an appropriate power of 10 and, as a result, work only with whole numbers. Students can calculate the quotient of whole numbers using long division or another method of their choice. Students also have an opportunity to evaluate and critique another's reasoning.

Students use the structure of base-ten numbers to change the place value of the digits (through multiplication by an appropriate power of 10), and they use their understanding of equivalent expressions to know that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Both pieces of knowledge allow students to replace a quotient of decimal numbers with a quotient of whole numbers.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet time to consider how to find the first quotient. Encourage them to think of more than one way to do so, if possible. Then, give partners 2–3 minutes to discuss their methods and another 2–3 minutes to find the second quotient together. Follow with a brief whole-class discussion, reviewing the first two questions. If not brought up by a student, discuss the equivalent expressions $300 \div 12$ and $1\,800 \div 4$. Consider bringing up the first expression and asking students to find an analogous expression for the second problem.

Ask students to finish the last problem and follow with a whole-class discussion.

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students about the quotients that have the same value as $5.04 \div 7$ in the previous activity. Explain to students that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Ask students how they can use this idea to find a quotient that has the same value as $3 \div 0.12$.
Supports accessibility for: Social-emotional skills; Conceptual processing Speaking, Listening: Compare and Connect. After students have 3 minutes of quiet time to consider how to find the first quotient, ask them to create a visual display that shows their strategy and a brief explanation. Give students time to meet with 2–3 partners, to share discuss connections they notice between their different approaches. Follow up with a whole-class discussion to identify and highlight correspondences between different approaches or representations you observe in the room. Listen for and amplify key phrases such as "multiply both numbers by 10", "use whole numbers" or "create an equivalent expression." This will help students make sense of mathematical strategies by relating and connecting other approaches to their own.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

Student Task Statement

1. Think of one or more ways to find $3 \div 0.12$. Show your reasoning.

2. Find $1.8 \div 0.004$. Show your reasoning. If you get stuck, think about what equivalent division expression you could write.

3. Diego said, "To divide decimals, we can start by moving each digit in both the dividend and divisor by the same number of places and in the same direction. Then we find the quotient of the resulting numbers."

   Do you agree with Diego? Use the division expression $7.5 \div 1.25$ to support your answer.

Student Response

1. $3 \div 0.12 = 25$. Reasonings vary. Sample reasonings:

   – $10 \times (0.12) = 1.2$ and $20 \times (0.12) = 2.4$. These mean there are at least 20 groups of 0.12 in 3. The value 3 is 0.6 away from 2.4. There are 5 groups of 0.12 in 0.6 (i.e., $5 \times (0.12) = 0.6$. So in total, there are $(20 + 5)$ or 25 groups of 0.12 in 3.

   – $3 \div 12$ is $\frac{1}{4}$ or 0.25. Because the divisor 0.12 is a hundredth of 12, the quotient $3 \div 0.12$ must be 100 times 0.25, which is 25.

   – 0.12 can be written as $\frac{12}{100}$, so the division can be written as $3 \div \frac{12}{100}$, which equals $3 \times \frac{100}{12}$ or $\frac{300}{12}$. The quotient is 25 because $300 \div 12 = 25$.

- We can multiply both 3 and 0.12 by 100 to get 300 and 12, and then simply find $300 \div 12$. Multiplying both the dividend and divisor by the same number does not change the quotient. $300 \div 12 = 25$.

2. $1.8 \div 0.004 = 450$. Reasoning vary. Sample reasonings:

   - 1.8 is $\frac{18}{10}$ and 0.004 is $\frac{4}{1\,000}$. The quotient $\frac{18}{10} \div \frac{4}{1\,000}$ can be found by multiplying $\frac{18}{10} \times \frac{1\,000}{4}$, which equals $\frac{18\,000}{40}$ or 450.
   - $1.8 \div 0.004$ is equivalent to $1\,800 \div 4$, which is 450.

3. Agree. Sample explanation: We can multiply both 7.5 and 1.25 in $7.5 \div 1.25$ by 100 to have a whole-number dividend and divisor. Multiplying both by 100 moves each digit 2 places to the left, so 7.5 becomes 750 and 1.25 becomes 125. Then, we can divide 750 by 125.

Are You Ready for More?

Can we create an equivalent division expression by multiplying both the dividend and divisor by a number that is *not* a multiple of 10 (for example: 4, 20, or $\frac{1}{2}$)? Would doing so produce the same quotient? Explain or show your reasoning.

Student Response

Yes. Explanations vary. Sample response: A division expression such as $1.8 \div 0.004$ can be thought of as the fraction $\frac{1.8}{0.004}$. Equivalent fractions can be found by multiplying the numerator and denominator by the same number. This means that equivalent division expressions can be found by multiplying the dividend and divisor by the same number.

Activity Synthesis

The goal of this discussion is to help students recognise when division expressions are equivalent.

Ask students to write a division expression that looks like it might be equivalent to either $3 \div 0.12$ or $1.8 \div 0.004$ but has different decimal point locations. Select a few students to share their expression with the class.

13.3 Two Ways to Calculate Quotients of Decimals

Optional: 15 minutes

This lesson demonstrates how the division of two equivalent expressions (e.g., $48.78 \div 9$ and $4\,878 \div 900$) result in the same quotient. By looking at worked-out calculations, students reinforce their understanding about what each part of the calculations represent. The advantage of representing a quotient with a different equivalent expression is so students can simplify problems involving division of decimals by rewriting expressions using whole numbers.

They use the structure of base-ten numbers to move the digits (through multiplication by an appropriate power of 10). They also use their understanding of equivalent expressions to know that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Both pieces of knowledge allow students to replace a quotient of decimal numbers with a quotient of whole numbers.

Instructional Routines

- Discussion Supports

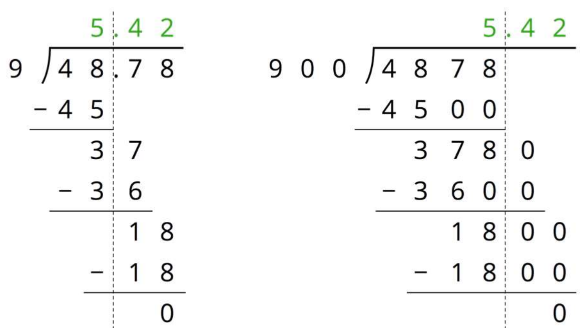
Launch

Arrange students in groups of 2. Give partners 5 minutes to discuss the first problem and then quiet work time for the second problem. Follow with a whole-class discussion.

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as “Both _____ and _____ are alike because…” and “_____ and _____ are different because…”
Supports accessibility for: Language; Organisation

Student Task Statement

1. Here are two calculations of 48.78 ÷ 9. Work with your partner to answer the following questions.



Calculation A   Calculation B

   a. How are the two calculations the same? How are they different?

   b. Look at Calculation A. Explain how you can tell that the 36 means “36 tenths” and the 18 means “18 hundredths.”

   c. Look at Calculation B. What do the 3 600 and 1 800 mean?

   d. We can think of $48.78 \div 9 = 5.42$ as saying, “There are 9 groups of 5.42 in 48.78.” We can think of $4\,878 \div 900 = 5.42$ as saying, “There are 900 groups of 5.42 in 4 878.” How might we show that both statements are true?

2.

a. Explain why $51.2 \div 6.4$ has the same value as $5.12 \div 0.64$.

b. Write a division expression that has the same value as $51.2 \div 6.4$ but is easier to use to find the value. Then, find the value using long division.

Student Response

1.

a. Answers vary. Sample response: Both calculations have the same quotient. They both have the same non-zero digits in the dividend and divisor. In Calculation A, the decimal point in the dividend stays. In Calculation B, both the divisor and dividend have been multiplied by 100 so that they are whole numbers.

b. The 3 is in the ones place and the 6 is in the tenths place, so 36 means 36 tenths. The 1 in 18 is in the tenths place, so it has the same value as 10 hundredths. The 8 is in the hundredth place. Together, they make 18 hundredths.

c. The 3 600 represents 3 600 tenths, and the 1 800 represents 1 800 hundredths.

d. Answers vary. Sample response:

- We can multiply the number of groups and the total amount by 100. $9 \times (5.42) = 48.78$ and $900 \times (5.42) = 4\,878$.
- 900 is 100 times 9. So if 9 groups of 5.42 equal 48.78, then 900 groups of 5.42 equal 100 times 48.78, which is 4 878.

2.

a. $51.2 = 10 \times (5.12)$ and $6.4 = 10 \times (0.64)$. Multiplying the dividend and the divisor by the same number (10) does not affect the value of the quotient.

b. Answers vary. Sample responses: $512 \div 64$ and $5\,120 \div 640$. The long division should show a quotient of 8.

Activity Synthesis

The goal of this discussion is for students to contrast the two division methods used in the task. Discuss:

- Why is the value of $48.78 \div 9$ the same as the value of $4\,878 \div 900$? (The numbers in the second expression are both multiplied by 100, and this does not change the value of the quotient.)

- What are some advantages of calculating 48.78 ÷ 9 with the dividend intact (the method on the left)? (It is fast, and we don't need to deal with a bunch of 0's. Also, if the numbers are from a contextual problem, we could better make meaning of them in their original form.)
- What are some advantages of calculating 4 878 ÷ 900 with long division? (These are whole numbers, and we are familiar with how to divide whole numbers. Also, we could express this as a fraction and write an equivalent fraction of $\frac{543}{100}$, which then tells us that its value is 5.43.)

End the discussion by telling students that they will next look at quotients where both the divisor and the dividend are decimals. The method used here of multiplying both numbers by a power of 10 will apply in that situation as well.

Speaking: Discussion Supports. As students discuss contrasts between the two calculations of 48.78 ÷ 9, press for details and mathematical language in their explanations. Encourage students to elaborate on the idea that multiplying the dividend and divisor by the same factor does not change the value of the quotient. Provide these sentence frames to support the discussion: "The values are the same because ….", "I can use powers of 10 by ….", "Some advantages of Calculation A/Calculation B are ….". This will help students make sense of the structure of base-ten numbers to move the digits through multiplication by an appropriate power of 10.
Design Principles(s): Support sense-making

13.4 Practising Division with Decimals

15 minutes

In this activity, students practise calculating quotients of decimals by using any method they prefer. Then, they extend their practice to calculate the division of decimals in a real-world context. While students could use ratio techniques (e.g., a ratio table) to answer the last question, encourage them to use the division of decimal numbers.

As students work on the first three problems, monitor for groups in which students have different strategies used on the same question.

Instructional Routines

- Clarify, Critique, Correct

Launch

Arrange students in groups of 3–5. Give groups 5–7 minutes to work through and discuss the first three questions. Ask them to consult with you if there is a disagreement about a correct answer in their group. (If this happens, let them know which student's work is correct and have that student explain their thinking so all group members are in agreement.)

After all group members have answered the first three questions and have the same answer, have them complete the last question. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students might have trouble calculating because their numbers are not aligned so the place-value associations are lost. Suggest that they use graph paper for their calculations. They can place one digit in each box for proper decimal point and place-value alignment.

Student Task Statement

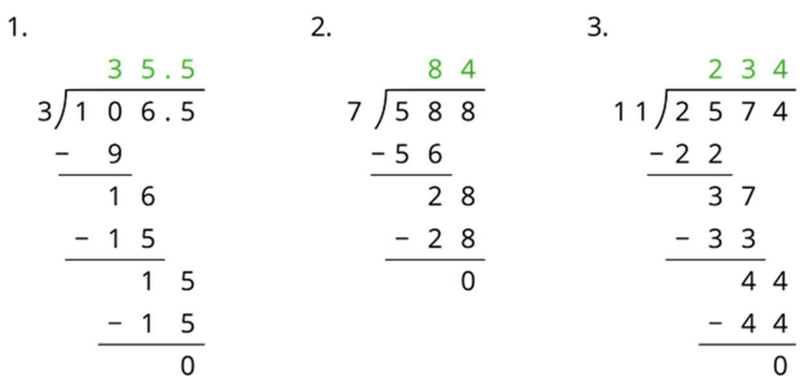
Find each quotient. Discuss your quotients with your group and agree on the correct answers. Consult your teacher if the group can't agree.

1. $106.5 \div 3$
2. $58.8 \div 0.7$
3. $257.4 \div 1.1$
4. Mai is making friendship bracelets. Each bracelet is made from 24.3 cm of string. If she has 170.1 cm of string, how many bracelets can she make? Explain or show your reasoning.

Student Response

(Long division calculations for questions 1–3 are shown after question 3.)

1. 35.5.
2. 84. The dividend and divisor in $58.8 \div 0.7$ can each be multiplied by 10 to get 588 and 7. The quotient $58.8 \div 0.7$ has the same value as the quotient $588 \div 7$, which equals 84.
3. 234. The dividend and divisor can be multiplied by 10 to get 2 574 and 11. The quotient $257.4 \div 1.1$ has the same value as the quotient $2\,574 \div 11$, which equals 234.



4. She can make 7 bracelets. $170.1 \div 24.3$ is equivalent to $1\,701 \div 243$, which is 7.

Activity Synthesis

The purpose of this discussion is to highlight the different strategies used to answer the division questions. Select a previously identified group that used different strategies on one of the first three questions. Ask each student in the group to explain their strategy and why they chose it. For the fourth question, ask students:

- "Could the answer be found by calculating the quotient of this expression: 24.3 ÷ 170.1?" (No, because the question is asking how many pieces of string of length 24.3 are in the long string of length 170.1. This is equivalent to asking how many groups of 24.3 are in 170.1 or 170.1 ÷ 24.3.)
- "What would the quotient 24.3 ÷ 170.1 represent in the context of the problem?" (This quotient would represent what fraction the length of bracelet string is of the full length of string.)

Writing, Speaking, Conversing: Clarify, Critique, and Correct. Before students share their answers, present an incorrect solution that uses long division to find the quotient of 106.5 ÷ 3. Consider using this statement to open the discussion: "Sam believes the quotient of 106.5 ÷ 3 is 355 because he multiplied 106.5 by 10 and then got his answer." Ask pairs to identify the ambiguity or error, and critique the reasoning presented in the statement. Guide discussion by asking, "What do you think Sam was thinking when performing this division problem?", "What strategy did he use to find the quotient?" and/or "What is unclear?" Invite pairs to offer a response that includes a correct version of Sam's long division (e.g., the long division work for 1 065 ÷ 30). This will help students explain how to use multiplication by powers of 10 in both the divisor and dividend to create equivalent expressions.
Design Principle(s): Maximise meta-awareness; Cultivate conversation

Lesson Synthesis

In this lesson, we saw that we can divide decimals by decimals by first making the decimals whole numbers. As long as we multiply both numbers by the same number, the value of the quotient will not change.

- What equivalent expression can we write to help us find 18.4 ÷ 0.2? (We can multiply both numbers by 10 to get 184 ÷ 2, which is equivalent to the original expression.)
- How might we find the quotient of 184 ÷ 2? (We can use any methods learned so far: base-ten diagrams, partial quotients, or long division.)
- Do we always multiply the dividend and divisor by 10? For example, what number should we multiply by to enable us to find 1.25 ÷ 0.005? (We can multiply by any power of 10. In this example, we should multiply both numbers by 1 000 to turn the 0.005 into 5, so that we can find 1 250 ÷ 5.)

- Why is it helpful to multiply by a power of 10 instead of another number that is not a power of 10? (Because we are working with base-ten numbers, multiplying by a power of 10 allows us to easily convert so that we end up with a whole number.)

13.5 The Quotient of Two Decimals

Cool Down: 5 minutes

Student Task Statement

1. Write two division expressions that have the same value as 36.8 ÷ 2.3.

2. Find the value of 36.8 ÷ 2.3. Show your reasoning.

Student Response

1. Answers vary. Sample responses: 3.68 ÷ 0.23 and 368 ÷ 23.

2. 16. Sample reasoning:

```
        1 6
     ______
23 ) 3 6 8
   - 2 3
   -------
     1 3 8
   - 1 3 8
   -------
         8
       - 8
       ---
         0
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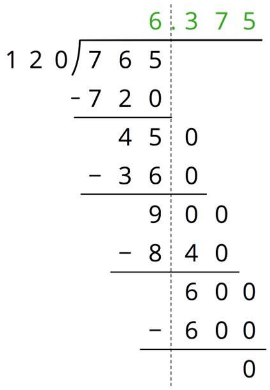
Student Lesson Summary

One way to find a quotient of two decimals is to multiply each decimal by a power of 10 so that both products are whole numbers.

If we multiply both decimals by the same power of 10, this does not change the value of the quotient. For example, the quotient 7.65 ÷ 1.2 can be found by multiplying the two decimals by 10 (or by 100) and instead finding 76.5 ÷ 12 or 765 ÷ 120.

To calculate 765 ÷ 120, which is equivalent to 76.5 ÷ 12, we could use base-ten diagrams, partial quotients, or long division. Here is the calculation with long division:



Lesson 13 Practice Problems

1. **Problem 1 Statement**

   A student said, “To find the value of 109.2 ÷ 6, I can divide 1 092 by 60.”

   a. Do you agree with her? Explain your reasoning.

   b. Calculate the quotient of 109.2 ÷ 6 using any method of your choice.

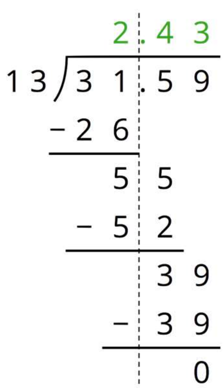
   Solution

   a. Yes. Reasoning varies. Sample reasoning: As long as both dividend and divisor are multiplied by the same power of 10 (or just the same non-zero number), the quotient has the same value.

   b. 18.2. Methods vary. Sample response: 109.2 (dividend) and 6 (divisor) can be multiplied by 10 to get 1 092 ÷ 60. The value of this quotient is 18.2.

2. **Problem 2 Statement**

   Here is how Han found 31.59 ÷ 13:



   a. At the second step, Han subtracts 52 from 55. How do you know that these numbers represent tenths?

b. At the third step, Han subtracts 39 from 39. How do you know that these numbers represent hundredths?

c. Check that Han's answer is correct by calculating the product of 2.43 and 13.

Solution

a. Explanations vary. Sample explanation: The second 5 of the 55 is written in the tenths column (directly under the tenths place of 31.59), so it represents 5 tenths. The first 5 of 55 is written in the ones column (directly under the ones place of 31.59), so it represents 5 ones, which is 50 tenths. So, the 55 represents 55 tenths. The 2 of 52 is written in the tenths column and the 5 is in the ones column, so the 52 represents 52 tenths.

b. Explanations vary. Sample explanation: The 9 of 39 is written in the hundredths column (directly under the hundredths place of 31.59), so it represents 9 hundredths. The 3 of 39 is written in the tenths column (directly under the tenths place of 31.59), so it represents 3 tenths, which is 30 hundredths. So, the 39 represents 39 hundredths.

c. $(2.43) \times 13 = 31.59$, so Han is correct. Calculations vary. Sample calculation: $(2.43) \times 13 = (2.43) \times (10 + 3) = 24.3 + 6 + 1.29 = 31.59$.

3. **Problem 3 Statement**

a. Write two division expressions that have the same value as $61.12 \div 3.2$.

b. Find the value of $61.12 \div 3.2$. Show your reasoning.

Solution

a. Answers vary. Sample responses: $611.2 \div 32$ and $6.112 \div 0.32$.

b. 19.1 Reasoning varies. Sample reasoning:

```
      1 9.1
32 ) 6 1 1.2
   - 3 2 0
     2 9 1
   - 2 8 8
         3 2
   -     3 2
           0
```

4. **Problem 4 Statement**

A bag of pennies weighs 5.1 kilograms. Each penny weighs 2.5 grams. About how many pennies are in the bag?

a. 20

b. 200

c. 2 000

d. 20 000

Solution C

5. **Problem 5 Statement**

Find each difference. If you get stuck, consider drawing a diagram.

a. $2.5 - 1.6$

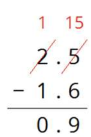
b. $0.72 - 0.4$

c. $11.3 - 1.75$

d. $73 - 1.3$

Solution

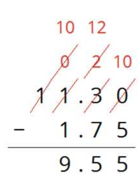
a. $2.5 - 1.6 = 0.9$



b. $0.72 - 0.4 = 0.32$

$$\begin{array}{r} 0.72 \\ -\ 0.40 \\ \hline 0.32 \end{array}$$

c. $11.3 - 1.75 = 9.55$



d. $73 - 1.3 = 71.7$

$$\begin{array}{r} {\scriptstyle 2\ \ 10} \\ 7\,\not{3}.\not{0} \\ -\ \ 1.3 \\ \hline 71.7 \end{array}$$

6. **Problem 6 Statement**

Plant B is $6\frac{2}{3}$ inches tall. Plant C is $4\frac{4}{15}$ inches tall. Complete the sentences and show your reasoning.

a. Plant C is _______ times as tall as Plant B.

b. Plant C is _______ inches ___________ (taller or shorter) than Plant B.

Solution

a. Plant C is $\frac{16}{25}$ times as tall as Plant B. Plant B is $6\frac{2}{3}$ inches and Plant C is $4\frac{4}{15}$ inches tall. Reasoning varies. Sample reasoning: $4\frac{4}{15} \div 6\frac{2}{3} = \frac{64}{15} \div \frac{20}{3} = \frac{64}{15} \times \frac{3}{20} = \frac{16}{25}$

b. Plant C is $2\frac{2}{5}$ inches shorter than Plant B. ($6\frac{2}{3} - 4\frac{4}{15} = 6\frac{10}{15} - 4\frac{4}{15} = 2\frac{6}{15} = 2\frac{2}{5}$)

7. **Problem 7 Statement**

At a school, 460 of the students walk to school.

a. The number of students who take public transport is 20% of the number of students who walk. How many students take public transport?

b. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?

c. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

Solution

a. 92 students ($460 \times 0.2 = 92$)

b. 23 students ($460 \times 0.05 = 23$)

c. 506 students ($460 \times 1.10 = 506$)