Lesson 16: Interpreting inequalities

Goals

- Critique (orally) a solution method for a problem involving an inequality.
- Identify the inequality that represents a situation, and justify (in writing) the choice.
- Present (orally, in writing, and using other representations) the solution method for a problem involving an inequality, and interpret the solution.

Learning Targets

- I can match an inequality to a situation it represents, solve it, and then explain what the solution means in the situation.
- If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation.

Lesson Narrative

In this lesson and the next, we move on to applying inequalities to solve problems. The warm-up is a review of the work in the previous lesson about solving inequalities when no context is given. Then students interpret and solve inequalities that represent real-life situations, making sense of quantities and their relationships in the problem.

Addressing

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Building Towards

- Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Instructional Routines

- Group Presentations
- Collect and Display
- Compare and Connect

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

Let's write inequalities.

16.1 Solve Some Inequalities!

Warm Up: 5 minutes

This warm-up is an opportunity for students to recall understandings and techniques from the previous lesson.

Launch

Optionally, provide access to blank number lines.

Anticipated Misconceptions

If students express the solution in words or by drawing on a number line, applaud their use of these representations. Encourage them to attempt to express the solution using the efficient notation, as well. Direct their attention to any displays or notes that remind them of the meaning of the symbols involved.

Student Task Statement

For each inequality, find the value or values of x that make it true.

1. $8x + 21 \leq 56$

2. $56 < 7(7 - x)$

Student Response

1. $x \leq 4.375$
2. $x < -1$

Activity Synthesis

Ask one student to share their process for reasoning about a solution to each problem. Address and resolve any discrepancies that arise.

16.2 Club Activities Matching

10 minutes

In this activity, students analyse four situations and select the inequality that best represents the situation. (In the activity that follows, students will work in small groups to create a visual display showing the solution for one of these situations.)

Instructional Routines

- Collect and Display

Launch

Tell students that their job in this activity is to read four situations carefully and decide which inequality best represents the situation. In the next activity, they will be responsible for writing a solution for one of these situations. Give 5–10 minutes of quiet work time.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use the same colour to highlight key words or phrases in the situation with its corresponding inequality sign in the matching inequality.
Supports accessibility for: Visual-spatial processing Reading, Representing: Collect and Display. As students work, circulate and collect examples of words and phrases students use in their written response to "Explain your reasoning" for each question. Look for different ways students describe what the variable represents, how they know which number is the constant term, how they know which number should be multiplied by the variable, and the direction of the inequality symbol that makes sense for each context. Organise the phrases for each of these considerations and display for all to see. This will help students to focus on all of the important elements of the inequality they are assigned in the next activity, with language they can use in small group discussions.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Choose the inequality that best matches each given situation. Explain your reasoning.

1. The Garden Club is planting fruit trees in their school's garden. There is one large tree that needs 5 pounds of fertiliser. The rest are newly planted trees that need $\frac{1}{2}$ pound fertiliser each.

   a. $25x + 5 \leq \frac{1}{2}$

   b. $\frac{1}{2}x + 5 \leq 25$

   c. $\frac{1}{2}x + 25 \leq 5$

   d. $5x + \frac{1}{2} \leq 25$

2. The Chemistry Club is experimenting with different mixtures of water with a certain chemical (sodium polyacrylate) to make fake snow.
   To make each mixture, the students start with some amount of water, and then add $\frac{1}{7}$ of that amount of the chemical, and then 9 more grams of the chemical. The chemical is expensive, so there can't be more than a certain number of grams of the chemical in any one mixture.

a. $\frac{1}{7}x + 9 \leq 26.25$

b. $9x + \frac{1}{7} \leq 26.25$

c. $26.25x + 9 \leq \frac{1}{7}$

d. $\frac{1}{7}x + 26.25 \leq 9$

3. The Hiking Club is on a hike down a cliff. They begin at an height of 12 feet and descend at the rate of 3 feet per minute.

a. $37x - 3 \geq 12$

b. $3x - 37 \geq 12$

c. $12 - 3x \geq -37$

d. $12x - 37 \geq -3$

4. The Science Club is researching boiling points. They learn that at high altitudes, water boils at lower temperatures. At sea level, water boils at 212°F. With each increase of 500 feet in height, the boiling point of water is lowered by about 1°F.

a. $212 - \frac{1}{500}e < 195$

b. $\frac{1}{500}e - 195 < 212$

c. $195 - 212e < \frac{1}{500}$

d. $212 - 195e < \frac{1}{500}$

Student Response

1. b, because $\frac{1}{2}$ a pound for each of an unknown number of trees, plus 5 pounds of fertiliser, is less than or equal to 25, which is likely the maximum amount of fertiliser available.

2. a, because $\frac{1}{7}$ of the amount of water plus 9 grams is the amount of the chemical used. This total must be less than 26.25 grams, which is likely the maximum amount of the chemical that can be used in a mixture.

3. c, because they start at 12 feet and then lose 3 feet per minute. If x is the number of minutes they hike, then $3x$ is the change in height. Their height must be above -37 feet; perhaps this is the bottom of the cliff.

4. a, because the boiling point is 212 at sea level, and decreases $\frac{1}{500}$ of a degree for every foot of height. The solution will tell us for which heights the temperature is below 195 degrees.

Activity Synthesis

At this time, consider *not* validating which inequalities are correct. When students get into groups for the next activity, they can compare their responses with the members of their groups and resolve any discrepancies at that time.

16.3 Club Activities Display

20 minutes

In this activity, students interpret parts of an inequality in context, term by term; for example, what quantity must $\frac{1}{2}x$ represent? Then they make sense of the entire inequality by thinking about what question would be answered by the solution to the inequality. Notice groups that create displays that communicate their mathematical thinking clearly, contain an error that would be instructive to discuss, or organise the information in a way that is useful for all to see. At this point, there is very little scaffolding for the solving of the inequality itself.

Instructional Routines

- Group Presentations
- Compare and Connect

Launch

Arrange students in groups of 2–3 and provide tools for making a visual display. Assign one situation to each group. Note that the level of difficulty increases for the situations, so this is an opportunity to differentiate by assigning more or less challenging situations to different groups.

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, create an exemplar display including all required components, highlighting different ways to communicate mathematical thinking clearly.
Supports accessibility for: Attention; Social-emotional skills

Student Task Statement

Your teacher will assign your group *one* of the situations from the last task. Create a visual display about your situation. In your display:

- Explain what the variable and each part of the inequality represent
- Write a question that can be answered by the solution to the inequality

- Show how you solved the inequality
- Explain what the solution means in terms of the situation

Student Response

- x represents the number of small trees that can be fertilised with the remaining fertiliser. 5 is the number of pounds of fertiliser for the large tree. $\frac{1}{2}x$ represents the number of pounds of fertiliser needed to grow x small trees. 25 is the total weight of fertiliser available in pounds.
- How many small trees can be planted with the available fertiliser?
- $\frac{1}{2}x + 5 \leq 25, \frac{1}{2}x \leq 20, x \leq 40$
- Up to 40 small trees can be planted with the fertiliser available.
- x represents the amount (in grams) of water used in a given mixture. $\frac{x}{7} + 9$ represents the amount of chemical (in grams) that is added to the water.
- How much water can you start with so that you don't use up too much of the chemical?
- $\frac{x}{7} + 9 \leq 26.25, \frac{x}{7} \leq 17.25, x \leq 120.75.$
- In order to make a mixture that doesn't use too much of the chemical, you have to start with 120.75 grams of water or less.
- -3 represents the height lost each minute. x is the number of minutes the students have been hiking. $-3x$ is the amount of height loss after x minutes. 12 is the initial height. $12 - 3x$ represents the students' height after hiking for x minutes. -37 represents the height at the bottom of the cliff: 37 feet below sea level.
- What are the times during which the students are hiking toward the bottom of the cliff?
- $12 - 3x \geq -37, -3x \geq -49, x \leq 16\frac{1}{3}$
- The students hike for a time period of $16\frac{1}{3}$ minutes, at which point they come to the bottom of the cliff.
- $\frac{1}{500}e$ represents the change in the boiling point of water after a 1-foot increase in height. $212 - \frac{1}{500}e$ is the boiling point of water at height e.
- At which heights is the boiling point of water below 195 degrees?

- $212 - \frac{1}{500}e < 195, -\frac{1}{500}e < -17, e > 8500$
- At heights greater than 8500 feet, the boiling point of water is less than 195 degrees.

Are You Ready for More?

{3,4,5,6} is a set of four consecutive integers whose sum is 18.

1. How many sets of three consecutive integers are there whose sum is between 51 and 60? Can you be sure you've found them all? Explain or show your reasoning.

2. How many sets of four consecutive integers are there whose sum is between 59 and 82? Can you be sure you've found them all? Explain or show your reasoning.

Student Response

Both of these problems can be solved by intelligent guess-and-check, or other more conceptual strategies, and by using the first answer one finds to generate the others. If students use these strategies, help them to crystalise their reasoning: how do they know they have all of the sets? Also encourage students to see if they can write inequalities in addition (not instead of!) whatever strategies they use.

1. 4 sets: {16,17,18},{17,18,19}, {18,19,20}, {19,20,21}
   $(x-1)+x+(x+1) \geq 51$ and $(x-1)+x+(x+1) \leq 60$, $17 \leq x \leq 20$.

2. 5 sets: {14,15,16,17}, {15,16,17,18}, {16,17,18,19}, {17,18,19,20},{18,19,20,21}.
   $x+(x+1)+(x+2)+(x+3) \geq 59$ and $x+(x+1)+(x+2)+(x+3) \leq 82$, $13.25 \leq x \leq 18.5$.

Activity Synthesis

Select groups to share their visual displays. Encourage students to ask questions about the mathematical thinking or design approach that went into creating the display. Here are questions for discussion, if not already mentioned by students:

- How did you figure out what the $\frac{x}{7}$ term represents?
- How did you decide on the direction of the inequality for the solutions?
- Did anyone with the same problem do one of the steps differently? Share what you did differently so we can learn from what happened.
- How do you know there are 25 pounds of fertiliser available?

Alternatively, have students do a "gallery walk" in which they leave written feedback on sticky notes for the other groups. Here is guidance for the kind of feedback students should aim to give each other:

- What is one thing that group did that would have made your project better if you had done it?
- What is one thing your group did that would have improved their project if they did it too?
- How did the group decide the direction of inequality for the solutions?
- Does their answer make sense in the situation?
- Is their mathematics clear and correct?
- If there was a mistake, what could they be more careful about in similar problems?

Representing, Conversing: Compare and Connect. During the launch, make sure at least two groups are assigned to each situation (assign fewer contexts if there are fewer than 8 groups). Assign groups who worked on the same situation to review each other's display. Ask groups to look closely at how the inequality was solved, then to identify and discuss what is the same and what is different, compared to their own display. If the other group's solution is the same, students should compare the strategies used. If the solution is different, students should look for any errors in reasoning, either in their own or the other group's method. Ask each group to leave a comment on a sticky note that describes the comparison they discussed. This will help students make sense of the reasoning of others by interpreting work that is similar to their own.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

In this lesson, we saw how inequalities can be applied to real-world situations. Some questions to bring this work together:

- Suppose your friend asks you to write some practice problems for solving inequalities. You want to write an inequality that has a solution of $x \leq -8\frac{2}{3}$. Describe how to write such an inequality.
- Think about an after-school activity in which you are involved. Write an inequality that represents a situation related to that activity. Be prepared to share the inequality and an explanation of its terms with the class.

If time allows, have students solve their inequalities.

16.4 Party Decorations

Cool Down: 5 minutes

Student Task Statement

Andre is making paper cranes to decorate for a party. He plans to make one large paper crane for a centrepiece and several smaller paper cranes to put around the table. It takes Andre 10 minutes to make the centrepiece and 3 minutes to make each small crane. He will only have 30 minutes to make the paper cranes once he gets home.

1. Andre wrote the inequality $3x + 10 \leq 30$ to plan his time. Describe what x, $3x$, 10, and 30 represent in this inequality.

2. Solve Andre's inequality and explain what the solution means.

Student Response

1. The variable x represents the number of small paper cranes Andre will make. 10 is the number of minutes it takes to make the centrepiece.
   $3x$ is the amount of time it takes to make x small cranes (it takes 3 minutes to make one crane). 30 is Andre's time limit in minutes.

2. $x \leq 6\frac{2}{3}$. Andre can make 6 or fewer small cranes.

Student Lesson Summary

We can represent and solve many real-world problems with inequalities. Writing the inequalities is very similar to writing equations to represent a situation. The expressions that make up the inequalities are the same as the ones we have seen in earlier lessons for equations. For inequalities, we also have to think about how expressions compare to each other, which one is bigger, and which one is smaller. Can they also be equal?

For example, a school fundraiser has a minimum target of £500. Faculty have donated £100 and there are 12 student clubs that are participating with different activities. How much money should each club raise to meet the fundraising goal? If n is the amount of money that each club raises, then the solution to $100 + 12n = 500$ is the minimum amount each club has to raise to meet the goal. It is more realistic, though, to use the inequality $100 + 12n \geq 500$ since the more money we raise, the more successful the fundraiser will be. There are many solutions because there are many different amounts of money the clubs could raise that would get us above our minimum goal of £500.

Lesson 16 Practice Problems

1. **Problem 1 Statement**

   Priya looks at the inequality $12 - x > 5$ and says "I subtract a number from 12 and want a result that is bigger than 5. That means that the solutions should be values of x that are smaller than something."

   Do you agree with Priya? Explain your reasoning and include solutions to the inequality in your explanation.

   Solution

   Yes, Priya is correct. Explanations vary. Sample response: Try subtracting different numbers from 12. For example, $12 - 3$ is larger than $12 - 8$ because subtracting 3 is subtracting less. When $x = 7$, the inequality is not true anymore, but for anything smaller than 7, it is still true. The solution to the inequality is $x < 7$.

2. **Problem 2 Statement**

   When a store had sold $\frac{2}{5}$ of the shirts that were on display, they brought out another 30 from the stockroom. The store likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{3}{5}x + 30 \geq 150$ to describe the situation.

   a. Explain what $\frac{3}{5}$ means in the inequality.

   b. Solve the inequality.

   c. Explain what the solution means in the situation.

   Solution

   Answers vary. Sample responses:

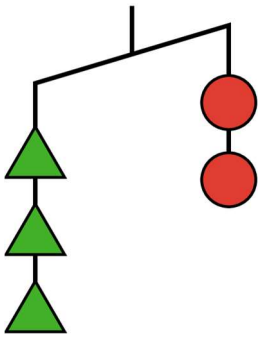
   a. Since $\frac{2}{5}$ of the original shirts were sold, $\frac{3}{5}$ of the original shirts remain on display.

   b. $\frac{3}{5}x + 30 \geq 150, \frac{3}{5}x \geq 120, x \geq 200$

   c. There were 200 or more shirts originally on display. At least 120 were left when they brought out 30 more.

3. **Problem 3 Statement**

   Here is an unbalanced balance.



a. If you knew each circle weighed 6 grams, what would that tell you about the weight of each triangle? Explain your reasoning.

b. If you knew each triangle weighed 3 grams, what would that tell you about the weight of each circle? Explain your reasoning.

Solution

a. The triangles would weigh more than 4 grams each. The 3 triangles weigh *more* than 2 circles. The 2 circles weigh 12 grams, so that means each triangle would weigh more than 4 grams.

b. The circles would weigh less than 4.5 grams each. The 3 triangles weigh 9 grams and this is *more* than 2 circles. So each circle weighs less than 4.5 grams.

4. **Problem 4 Statement**

Match each sentence with the inequality that could represent the situation.

A. Han got £2 from Clare, but still has less than £20.

B. Mai spent £2 and has less than £20.

C. If Tyler had twice the amount of money he has, he would have less than £20.

D. If Priya had half the money she has, she would have less than £20.

1. $x - 2 < 20$
2. $2x < 20$
3. $x + 2 < 20$
4. $\frac{1}{2}x < 20$

Solution

- A: 3
- B: 1

- C: 2
- D: 4

5. **Problem 5 Statement**

   You know x is a number less than 4. Select **all** the inequalities that *must* be true.

   a. $x < 2$

   b. $x + 6 < 10$

   c. $5x < 20$

   d. $x - 2 > 2$

   e. $x < 8$

   Solution ["B", "C", "E"]

6. **Problem 6 Statement**

   At a skateboard shop:

   a. The price tag on a shirt says £12.58. VAT is 7.5% of the price. How much will you pay for the shirt?

   b. The store buys a helmet for £19.00 and sells it for £31.50. What percentage was the markup?

   c. The shop pays workers £14.25 per hour plus 5.5% commission. If someone works 18 hours and sells £250 worth of merchandise, what is the total amount of their pay for this pay period? Explain or show your reasoning.

   Solution

   a. £13.52

   b. 65.8% or 66%

   c. £270.25, because $18 \times (14.25) + (0.055) \times 250 = 270.25$.