610 - Colouring Vertices 222
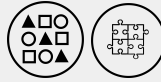
MATHS / COMBINATORICS

TEACHER

Logifaces

DESCRIPTION

Students colour the vertices of the block 222 (or 111 or 333) with 2 colours (red and blue) and consider the number of possible colourings. (Two colourings are different, if they cannot be moved into each other.)

LEVEL 1   1 red vertex or 1 blue vertex.

LEVEL 2   2 red vertices or 2 blue vertices.

LEVEL 3   3 red vertices or 3 blue vertices.

LEVEL 4   All possibilities.

SOLUTIONS / EXAMPLES

SOLUTION   There are 16 colourings of the vertices of the block 222.

There are 1, 1, 4, 4, 4, 1, 1 vertex colourings with 0, 1, 2, 3, 4, 5, 6 red vertices respectively, see the Figure. By symmetry, the number of colorings with k red vertices is the same as with $6 - k$ red vertices.

Level 2 and 3: It is worth discussing that the last two colourings with 2 red vertices are really different, and the same holds for the 2nd and 3rd with 3 red vertices. One possible reasoning is that the two red vertices can be connected with a diagonal of the rectangular face and the top right − bottom left diagonal and the top left − bottom right diagonal cannot be moved to each other, they are the mirror images of each other. It is a similar phenomenon to the fact that the blocks 123 and 132 are not the same, see exercise 601 - Matchmaking.
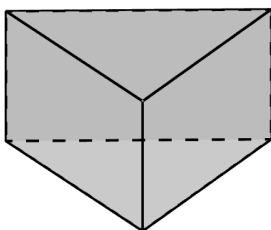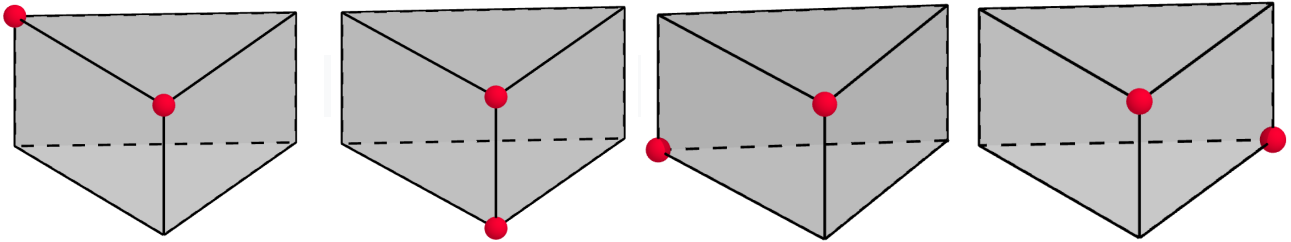
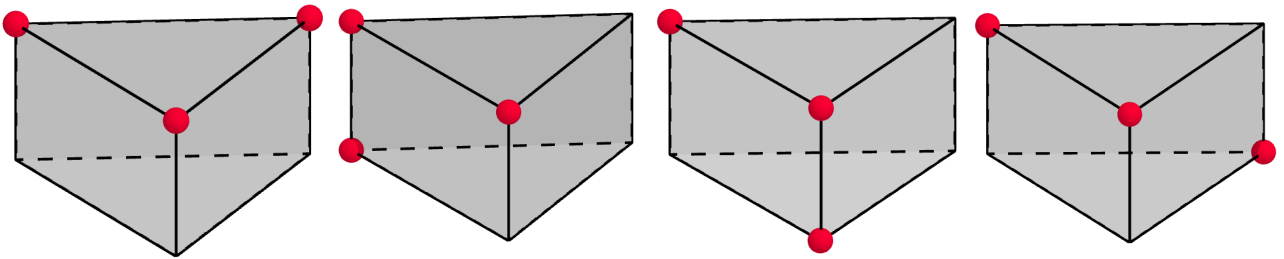0 red vertex   1 red vertex
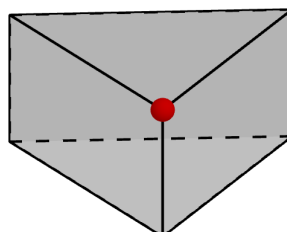
2 red vertices

3 red vertices

GUIDELINES FOR TEACHERS

Discuss the symmetries that imply that there is only one colouring with one red vertex.

Discuss the symmetries in the case of 2 red vertices.

Discuss that by symmetry, there is the same number of colourings with n red vertices as with n blue vertices.

PRIOR KNOWLEDGE

Vertex of polyhedron, Symmetry

RECOMMENDATIONS / COMMENTS

This exercise develops combinatorial and logical thinking, while also builds on spatial vision.

Levels 1-3 can be given to different students or groups, each level is more difficult than the previous one.

The previous level helps answer the next one, but each question can be asked independently of the others.

Level 4 is the combination of Levels 1-3.