Lesson 14: Percentage error

Goals

- Calculate the percentage error, correct amount, or erroneous amount, given the other two of these three quantities, and explain (orally and using other representations) the solution method.
- Compare and contrast (orally) strategies used for solving problems about percentage error with strategies used for solving problems about percentage increase or decrease.

Learning Targets

- I can solve problems that involve percentage error.

Lesson Narrative

Situations involving percentage error can be more difficult than situations involving percentage increase or percentage decrease because the student has to decide which amount represents the whole. In this lesson students get practice using the language of percentage error in various different situations, and identifying the correct amount, which is the whole, and the incorrect amount. They work with a multi-step problem involving percentage error. They also see a common usage of percentage error to express a range of possible values by thinking about a scale that claims to be accurate to within 0.5%. Understanding and finding percentage error is important for solving real-world problems.

Building On

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Building Towards

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Discussion Supports

- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

For the Measuring in the Heat activity, student will need access to calculators.

Student Learning Goals

Let's use percentages to describe other situations that involve error.

14.1 Number Talk: Estimating a Percentage of a Number

Warm Up: 5 minutes

The purpose of this number talk is for students to reason about a percentage of a number based on percentages they already know or could easily find. The percentages and numbers were purposefully chosen so that it would be cumbersome to calculate the exact answer and encourage making an estimate. During the whole-class discussion, highlight the percentages students found helpful and ask them to explain how they used these percentages. For example, if a student is estimating 9% of 38 and says, "I know 10% of 38 is 3.8..." ask the student to explain how they found 10% of 38.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display each problem one at a time. Give students 30 seconds of quiet think time followed by a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

If students try to figure out exact answers, encourage them to think about numbers that are *close to* the numbers in the problem, in order to estimate the percentage for each question.

Student Task Statement

Estimate.

25% of 15.8

9% of 38

1.2% of 127

0.53% of 6

0.06% of 202

Student Response

Answers vary. Sample responses:

- About 4, because 15.8 is close to 16, and 25% of 16 is 4.
- About 3.8, because 9% is close to 10% and 10% of 38 is 3.8.
- About 1.5, because 1.2% is close to 1% but a little bigger, and 1% of 127 is 1.27.
- About 0.03, because 0.53% is close to 0.5% which is half of 1%. Since 1% of 6 is 0.06, and half of that is 0.03.
- About 0.1, because 0.06% is close to 0.05% which is half of 0.1%. Since 0.1% of 202 is 0.202, and half of that is about 0.1.

Activity Synthesis

Ask students to share their responses for each question. Record and display student responses for all to see. After each response, ask students:

- "What benchmark percentages do you find it helpful to think about, when estimating?"
- "Is your estimate more or less than the actual answer? How do you know?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

14.2 Plants, Bicycles, and Crowds

10 minutes

The purpose of this activity is to give students practice using the language of percentage error. The problems here are similar in structure to percentage increase or decrease problems, but the language is different. Students may need some help interpreting the language used for percentage error and drawing parallels to the language used for percentage increase and decrease. Students should use similar strategies they used to calculate percentage increase or decrease. This activity includes one of each type of problem:

- finding the erroneous amount given the correct amount and the percentage error
- finding the correct amount given the erroneous amount and the percentage error
- finding the percentage error given the erroneous amount and the correct amount

Students may need help understanding that a different approach is needed for each question. As students work, monitor for those who use strategies similar to those used for percentage increase and decrease, and ask them to share during the whole-class discussion.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Before students begin working, read the first question aloud. It is helpful to tie the language used in the task to the phrase "percentage increase." Ask several students to explain in their own words what information is given in the problem, and what it is asking them to find. The plant is supposed to get $\frac{3}{4}$ cup of water, but it is getting 25% more than that. We can think of this as $\frac{3}{4}$ cup increased by 25%.

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time. Partner then by whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Represent the same information through different modalities. If students are unsure where to begin, suggest that they draw a double number line to help organise the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Students might struggle with figuring out how to calculate how much water the plant has been getting. Ask students, "How much more water has the plant been getting? How do you calculate that total?"

Student Task Statement

1. Instructions to care for a plant say to water it with $\frac{3}{4}$ cup of water every day. The plant has been getting 25% too much water. How much water has the plant been getting?
2. The pressure on a bicycle tyre is 63 psi. This is 5% higher than what the manual says is the correct pressure. What is the correct pressure?
3. The crowd at a sporting event is estimated to be 3 000 people. The exact attendance is 2 486 people. What is the **percentage error**?

Student Response

1. $\frac{15}{16}$ of a cup. 25% of $\frac{3}{4}$ is one quarter of $\frac{3}{4}$, which is $\frac{3}{16}$. So the plant has been getting $\frac{15}{16}$ cups of water each day, since $\frac{3}{4} + \frac{3}{16} = \frac{15}{16}$. That is almost 1 cup.

2. 60 psi. We know that 63 psi is 5% more than the correct pressure. If the correct pressure is p, this means that $63 = p + 0.05p$ or $1.05p$. Since $63 \div 1.05 = 60$, the correct pressure is 60 psi.

3. Approximately 21%. The estimate is more than the exact amount by 514 people, so the percentage error is $\frac{514}{2\,486} \approx 0.21$

Are You Ready for More?

A micrometre is an instrument that can measure lengths to the nearest micron (a micron is a millionth of a metre). Would this instrument be useful for measuring any of the following things? If so, what would the largest percentage error be?

1. The thickness of an eyelash, which is typically about 0.1 millimetres.

2. The diameter of a red blood cell, which is typically about 8 microns.

3. The diameter of a hydrogen atom, which is about 100 picometres (a picometre is a trillionth of a metre).

Student Response

1. Yes. The biggest the error could be is half of a micron, or 0.0000005 metres. If we divide this by the thickness, which is 0.0001 metres, the percentage error would be 0.5%.

2. Yes. The biggest the error could be is 0.5 microns. If we divide this by the length, which is 8 microns, the percentage error would be 6.25%.

3. No. The diameter of a hydrogen atom is much smaller than a micron.

Activity Synthesis

Select students who used these strategies for each problem to share:

- Calculate a quarter of $\frac{3}{4}$ and add it to $\frac{3}{4}$.
- Use 1.05 to divide into 63.
- Calculate percentage error using $\frac{14}{2\,486}$.

After each student has shared, ask the class if they agree or disagree or if they had a different way to calculate the solution. If students use strategies similar to ones they did calculating percentage increase or decrease, ask students if they see a connection. If no

student brings it up, ask students how the solution strategies here are similar to the ones used with percentage increase and decrease.

Speaking: Discussion Supports. As students describe their strategies for calculating the erroneous amount, correct amount, and percentage error, revoice student ideas to demonstrate mathematical language use. Press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. This will help students to produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Support sense-making; Optimise output (for explanation)

14.3 Measuring in the Heat

10 minutes

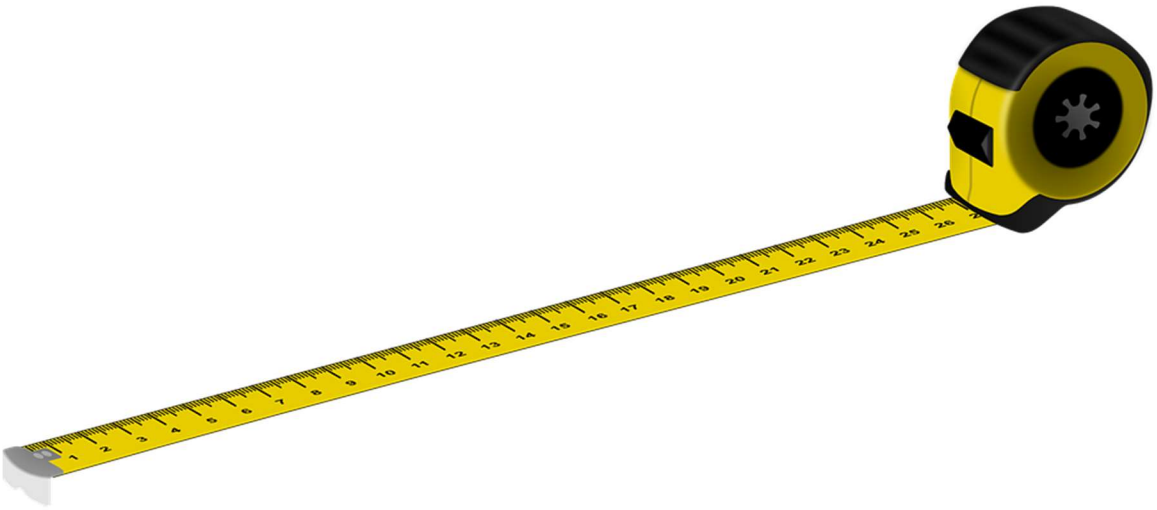
In this activity students use what they have learned about percentage error in a multi-step problem.

Monitor for students who multiply 0.0000064 by 50 to answer the second part of the problem rather than using the calculation from the first part of the problem. These students should be asked to share during the whole-class discussion.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

Launch



Display the image of the metal measuring tape for all to see. Ask if any students have used a tool like this before, and for what purpose. Tell them that many measuring tapes like this

are made out of metal, and that some metals expand or contract slightly at warmer or colder temperatures.

In this problem, a metal measuring tape gets 0.00064% longer for every degree over 50° Fahrenheit. Ask students what would happen if a measuring tape was used to measure 10 feet, and then got 0.00064% longer. How much longer is that? Can they show the difference between two fingers? The difference would be very, very small, only 0.0000064 feet (about the width of a hair) . . . barely perceptible! For most uses, this difference wouldn't matter, but if someone needed a very, very precise measurement, they would want to know about it.

Arrange students in groups of 2. Give students 5 minutes of quiet work time. After 5 minutes, give students 3 minutes to discuss with a partner the ways they approached this problem.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have converted the percentage 0.00064 to its decimal form 0.0000064 prior to calculating how much longer the measuring tape is than its correct length of 30 feet.
Supports accessibility for: Memory; Organisation Representing, Speaking: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to refine their explanation of their strategy for calculating the added length of the measuring tape and the percentage error. Ask each student to meet with 2-3 other partners in a row for feedback. Provide listeners with prompts for feedback that will help teams strengthen their ideas and clarify their language. For example, “What did you do first?”, “How did the example help you?”, “How did you use 0.00064%?”, etc. Students can borrow ideas and language from each partner to strengthen their final response.
Design Principle(s): Optimise output (for explanation)

Anticipated Misconceptions

If students struggle with how to calculate the length increase, ask students

- "How many degrees over 50° is 100°?"
- "How do you calculate the length increase knowing that there is a 50° increase?"
- "What is the actual length of the tape measure?"

Student Task Statement

A metal measuring tape expands when the temperature goes above 50°F. For every degree Fahrenheit above 50, its length increases by 0.00064%.

1. The temperature is 100 degrees Fahrenheit. How much longer is a 30-foot measuring tape than its correct length?
2. What is the percentage error?

Student Response

1. 0.0096 feet. The temperature is 50 degrees over the ideal temperature, because $100 - 50 = 50$. So we need to compute $50 \times (0.0000064) = 0.00032$, and then we need that percentage of 30 feet. $30 \times (0.00032) = 0.0096$.

2. 0.032%, because $0.0096 \div 30 = 0.00032$.

Activity Synthesis

Select previously identified students to share how they obtained their answer to the second part of the activity. Ask students who multiplied 0.0000064 by 50 why that method works and if there is another method (using the answer from the first part of the activity). Encourage students to make the connection between the two methods.

Ask students how big the error is in inches, and to show approximately how big they think it is with their thumb and forefinger. 0.0096 feet is almost 3 millimetres. When measuring 30 feet, this may not seem like very much, but the importance of the error may depend on what is being measured. The length of a car may not be as important as a precise scientific instrument to measure the speed of light, for example.

Lesson Synthesis

Students should feel confident calculating percentage error given different contexts and information. Ask students:

- "What strategies did we use to solve percentage error problems?" (diagrams, tables, equations)
- "How are these strategies similar to the ones we used while solving percentage increase/decrease problems?" (the same)

14.4 Jumbo Eggs

Cool Down: 5 minutes

The cool-down assesses student ability to find percentage error from an expected and an actual measurement.

Student Task Statement

To be labelled as a jumbo egg, the egg is supposed to weigh 2.5 oz. Priya buys a carton of jumbo eggs and measures one of the eggs as 2.4 oz. What is the percentage error?

Student Response

4%. The correct weight is 2.5 oz and Priya's egg is off by 0.1 oz. so the percentage error is $0.1 \div 2.5 = 0.04$ or 4%.

Student Lesson Summary

Percentage error can be used to describe any situation where there is a correct value and an incorrect value, and we want to describe the relative difference between them. For example, if a milk carton is supposed to contain 16 fluid ounces and it only contains 15 fluid ounces:

- the measurement error is 1 oz, and
- the percentage error is 6.25% because $1 \div 16 = 0.0625$.

We can also use percentage error when talking about estimates. For example, a teacher estimates there are about 600 students at their school. If there are actually 625 students, then the percentage error for this estimate was 4%, because $625 - 600 = 25$ and $25 \div 625 = 0.04$.

Glossary

- percentage error

Lesson 14 Practice Problems

Problem 1 Statement

A student estimated that it would take 3 hours to write a book report, but it actually took her 5 hours. What is the percentage error for her estimate?

Solution

40%, because $5 - 3 = 2$ and $2 \div 5 = 0.4$

Problem 2 Statement

A radar gun measured the speed of a baseball at 103 miles per hour. If the baseball was actually going 102.8 miles per hour, what was the percentage error in this measurement?

Solution

0.19%, because $103 - 102.8 = 0.2$ and $0.2 \div 102.8 \approx 0.0019$

Problem 3 Statement

It took 48 minutes to drive downtown. An app estimated it would be less than that. If the error was 20%, what was the app's estimate?

Solution

38.4 minutes, because $48 - (0.2)48 = 38.4$.

Problem 4 Statement

A farmer estimated that there were 25 gallons of water left in a tank. If this is an underestimate by 16%, how much water was actually in the tank?

Solution

About 29.8 gallons, because $25 \div 0.84 \approx 29.8$

Problem 5 Statement

For each story, write an equation that describes the relationship between the two quantities.

a. Diego collected x kg of recycling. Lin collected $\frac{2}{5}$ more than that.

b. Lin cycled x km. Diego cycled $\frac{3}{10}$ less than that.

c. Diego read for x minutes. Lin read $\frac{4}{7}$ of that.

Solution

a. $y = \frac{7}{5}x$

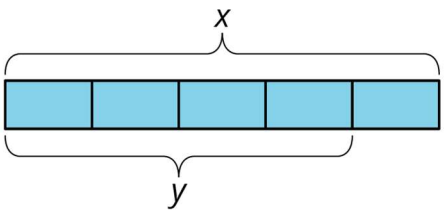
b. $y = \frac{7}{10}x$

c. $y = \frac{4}{7}x$

Problem 6 Statement

For each diagram, decide if y is an increase or a decrease of x. Then determine the percentage.

A



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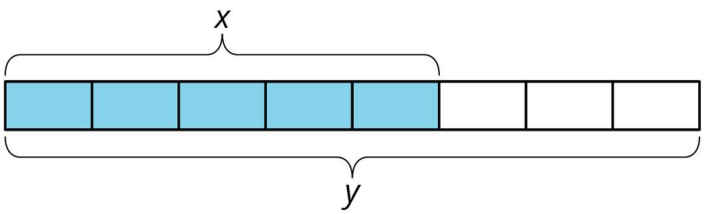

Solution

A Decrease: y is a 20% decrease of x

B Increase: y is a 60% increase of x

Problem 7 Statement

Lin is making a window covering for a window that has the shape of a half circle on top of a square of side length 3 feet. How much fabric does she need?

Solution

At least 12.5 square feet. The area of the square part of the window is 9 square feet, and the area of the half circle is about 3.5 square feet, because $\frac{1}{2} \times \pi \times (1.5)^2 \approx 3.5$. (Typically for sewing projects, you need more fabric than the area you are covering, so Lin would need a bit more fabric than that.)