# Lesson 11: Variability and MAD

## Goals

- Calculate the mean absolute deviation (MAD) for a data set and interpret what it tells us about the situation.
- Compare and contrast (in writing) distributions that have the same mean, but different amounts of variability.
- Comprehend that "mean absolute deviation (MAD)" is a measure of variability, i.e., a single number summarising how spread out the data set is.

## Learning Targets

- I can find the MAD for a set of data.
- I know what the mean absolute deviation (MAD) measures and what information it provides.

## Lesson Narrative

In a previous lesson, students computed and interpreted distances of data points from the mean. In this lesson, they take that experience to make sense of the formal idea of **mean absolute deviation (MAD)**. Students learn that the MAD is the mean distance of data points from the mean. They use their knowledge of how to calculate and interpret the mean to calculate and interpret the MAD.

Students also learn that we think of the MAD as a *measure of variability* or a *measure of spread* of a distribution. They compare distributions with the same mean but different MADs, and recognise that the centres are the same but the distribution with the larger MAD has greater variability or spread.

### Addressing

- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its centre, spread, and overall shape.
- Recognise that a measure of centre for a numerical/quantitative data set summarises all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Giving quantitative measures of centre (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

### Instructional Routines

- Stronger and Clearer Each Time

- Collect and Display
- Discussion Supports
- Poll the Class

### Required Materials

**Decks of playing cards**

### Required Preparation

If students are playing the optional Game of 22, prepare one standard deck of 52 playing cards for every 2–3 students.

### Student Learning Goals

Let's study distances between data points and the mean and see what they tell us.

## 11.1 Shooting Hoops (Part 1)

**Warm Up: 5 minutes**

The purpose of this warm-up is for students to first reason about the mean of a data set without calculating and then practise calculating mean. The context will be used in an upcoming activity in this lesson so this warm-up familiarises students with the context for talking about deviation from the mean.

In their predictions, students may think that Elena will have the highest mean, because she has a few very high scores (7, 8, and 9 points). They may also think that Lin and Jada will have very close means because they each have 5 higher scores than one another and their other scores are the same. Even though each player has the same mean, all of these ideas are reasonable things for students to consider when looking at the data. Record and display their predictions without further questions until they have calculated and compared the mean of their individual data sets.

### Instructional Routines

- Poll the Class

### Launch

Arrange students in groups of 3. Display the data sets for all to see. Ask students to predict which player has the largest mean and which has the smallest mean. Give students 1 minute of quiet think time and then poll students on the player who they think has the largest and smallest mean. Ask a few students to share their reasoning.

Tell each group member to calculate the mean of the data set for one player in the task, share their work in the small group, and complete the remaining questions.

## Student Task Statement

Elena, Jada, and Lin enjoy playing basketball during breaktime. Lately, they have been practising free throws. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 5 | 1 | 6 | 9 | 7 | 2 | 8 | 3 | 3 | 5 | 7 |

Jada

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 5 | 4 | 6 | 6 | 4 | 7 | 3 | 4 | 8 | 7 |

Lin

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 6 | 6 | 4 | 5 | 5 | 3 | 5 | 4 | 6 | 6 | 7 |

1. Calculate the mean number of baskets each player made, and compare the means. What do you notice?

2. What do the means tell us in this context?

## Student Response

1. Elena's mean score is $\frac{4+5+1+6+9+7+2+8+3+3+5+7}{12} = 5$. Jada's mean score is $\frac{2+4+5+4+6+6+4+7+3+4+8+7}{12} = 5$. Lin's mean score is $\frac{3+6+6+4+5+5+3+5+4+6+6+7}{12} = 5$. I noticed that all three players have the same mean score.

2. Answers vary. Sample explanation: The means show that all three students make, on average, half of the 10 attempts to get the basketball in the hoop.

## Activity Synthesis

Ask students to share the mean for each player's data set. Record and display their responses for all to see. After each student shares, ask the class if they agree or disagree and what the mean tells us in this context. If the idea that the means show that all three students make, on average, half of the 10 attempts to get the basketball in the hoop does not arise, make that idea explicit.

If there is time, consider revisiting the predictions and asking how the mean of Elena's data set can be the same as the others when she has more high scores?

# 11.2 Shooting Hoops (Part 2)

**15 minutes**

In this activity, students continue to develop their understanding of what could be considered typical for a group as well as variability in a data set. Students compare distributions with the same mean but different spreads and interpret them in the context of a situation. The context given here (basketball score) prompts them to connect the mean to the notion of how "well" a player plays in general, and deviations from the mean to how "consistently" that player plays.

They encounter the idea of calculating the *mean absolute deviation from the mean* as a way to describe variability in data.

### Instructional Routines

- Collect and Display

### Launch

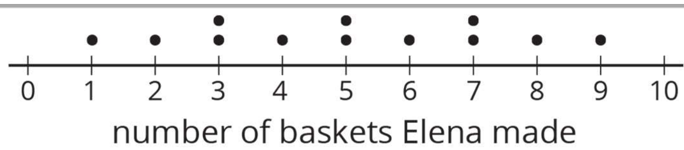
Arrange students in groups of 3–4. Give groups 6–7 minutes to answer the questions and follow with a whole-class discussion.

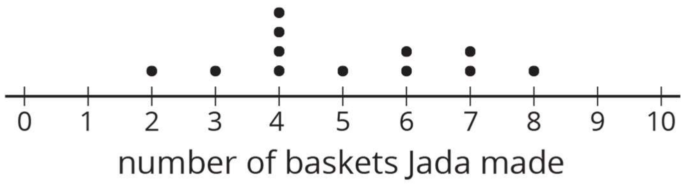
*Action and Expression: Internalise Executive Functions.* Begin with a small-group or whole-class demonstration and think aloud to remind students how calculate mean using data from a dot plot. Keep the worked-out calculations on display for students to reference as they work.
*Supports accessibility for: Memory; Conceptual processing Listening, Speaking: Collect and Display.* Use this routine to collect student ideas and language about playing "well" and "consistently" as they discuss the players' data in the three dot plots. Record students' language on a display for all to see. This will help students connect their reasoning about the visual representations of data to statistical terms such as mean, deviation from the mean, and variability in data.
*Design Principle(s): Support sense-making*

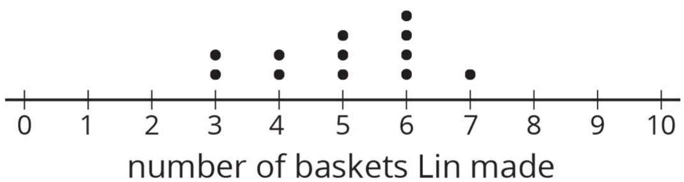
### Student Task Statement

Here are the dot plots showing the number of baskets Elena, Jada, and Lin each made over 12 school days.

1. On each dot plot, mark the location of the mean with a triangle (*Δ*). Then, contrast the dot plot distributions. Write 2–3 sentences to describe the shape and spread of each distribution.




2. Discuss the following questions with your group. Explain your reasoning.

    a. Would you say that all three students play equally well?

    b. Would you say that all three students play equally consistently?

    c. If you could choose one player to be on your basketball team based on their records, who would you choose?

### Student Response

1. Each dot plot should show a triangle at 5. Descriptions vary. Sample response: The data distributions for all players are centred at 5. Elena's data set is symmetric and very spread out; she has scores between 1 and 9 points. Jada's data set is not symmetric. Her data is less spread out than Elena's; they span between 2 and 8. Lin's data set has the narrowest spread, spanning from 3 to 7.

2. Answers vary.

### Activity Synthesis

The purpose of the discussion is to highlight that the centre of the distribution is not always the only consideration when discussing data. The variability or spread can also influence how we understand the data.

There are many ways to answer the second set of questions. Invite students or groups who have different interpretations of "playing well" and "playing consistently" to share their thinking. Allow as many interpretations to be shared as time permits. Discuss:

- "How might we use the given data to quantify 'playing well' and 'playing consistently?'"

- "Is there a way to describe variability and consistency in playing precisely and in an objective way (rather than using broad, verbal descriptions)?"

Explain that we can describe variability more formally and precisely—using a number to sum it up; we will look at how to do so in the next activity.

## 11.3 Shooting Hoops (Part 3)

**15 minutes**

In the previous activity, students evaluated the performance of three students based on the mean and variability. Here they learn the term *mean absolute deviation (MAD)* as a way to quantify variability and calculate it by finding distances between the mean and each data value. Students compare data sets with the same mean but different MADs and interpret the variations in context.

While this process of calculating MAD involves taking the absolute value of the difference between each data point and the mean, this formal language is downplayed here. Instead, the idea of "finding the distance," which is always positive, is used. This is done for a couple of reasons. One reason is to focus students' attention on the statistical work rather than on terminology or symbolic work. Another reason is that finding these differences may involve operations with directed numbers, which some students may not have mastered in year 7.

### Instructional Routines

- Discussion Supports

### Launch

Remind students in the last lesson they found distance between each data point and the mean, and found that the sum of those distances on the left and the sum on the right were equal, which allows us to think of the mean as the balancing point or the centre of the data. Explain that the distance between each point and the mean can tell us something else about a distribution.

Arrange students in groups of 2. Give students 4–5 minutes to complete the first set of questions with their partner, and then 4–5 minutes of quiet time to complete the remaining questions. Follow with a whole-class discussion.

### Anticipated Misconceptions

Students may recall the previous lesson about thinking of the mean as a balance point and think that the MAD should always be zero since the left and right distances should be equal. Remind them that distances are always positive, so finding the mean of these distances to the mean can only be zero if all the data points are exactly at the mean.

### Student Task Statement

The tables show Elena, Jada, and Lin's basketball data from an earlier activity. Recall that the mean of Elena's data, as well as that of Jada and Lin's data, was 5.

1. Record the distance between each of Elena's scores and the mean.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Elena** | 4 | 5 | 1 | 6 | 9 | 7 | 2 | 8 | 3 | 3 | 5 | 7 |
| **distance from 5** | 1 | | | 1 | | | | | | | | |

Now find *the mean of the distances* in the table. Show your reasoning and round your answer to the nearest tenth.

This value is the **mean absolute deviation (MAD)** of Elena's data.

Elena's MAD: ________

2. Find the mean absolute deviation of Jada's data. Round it to the nearest tenth.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Jada** | 2 | 4 | 5 | 4 | 6 | 6 | 4 | 7 | 3 | 4 | 8 | 7 |
| **distance from 5** | | | | | | | | | | | | |

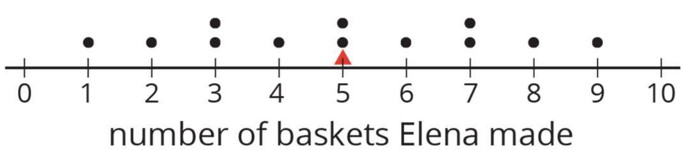
Jada's MAD: ________

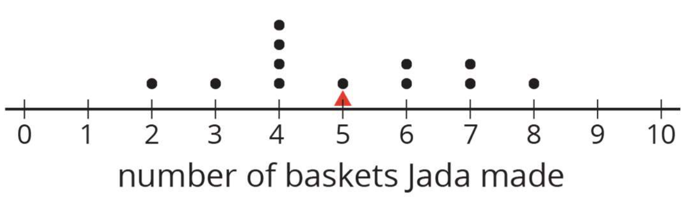
3. Find the mean absolute deviation of Lin's data. Round it to the nearest tenth.

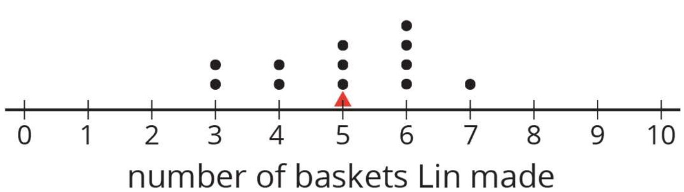
| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Lin** | 3 | 6 | 6 | 4 | 5 | 5 | 3 | 5 | 4 | 6 | 6 | 7 |
| **distance from 5** | | | | | | | | | | | | |

Lin's MAD: ________

4. Compare the MADs and dot plots of the three students' data. Do you see a relationship between each student's MAD and the distribution on her dot plot? Explain your reasoning.



### Student Response

1.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Elena | 4 | 5 | 1 | 6 | 9 | 7 | 2 | 8 | 3 | 3 | 5 | 7 |
| distance from 5 | 1 | 0 | 4 | 1 | 4 | 2 | 3 | 3 | 2 | 2 | 0 | 2 |

Elena's MAD: 2. $\frac{1+0+4+1+4+2+3+3+2+2+0+2}{12} = \frac{24}{12} = 2$

2.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jada | 2 | 4 | 5 | 4 | 6 | 6 | 4 | 7 | 3 | 4 | 8 | 7 |
| distance from 5 | 3 | 1 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 3 | 2 |

Jada's MAD: 1.5. $\frac{3+1+0+1+1+1+1+2+2+1+3+2}{12} = 1.5$

3.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lin | 3 | 6 | 6 | 4 | 5 | 5 | 3 | 5 | 4 | 6 | 6 | 7 |
| distance from 5 | 2 | 1 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 1 | 2 |

Lin's MAD: 1. $\frac{2+1+1+1+0+0+2+0+1+1+1+2}{12} = 1$

4. Answers vary. Sample response: Yes, I see a relationship between the MAD and the distribution of data. The largest MAD value corresponds to the dot plot with the widest spread. The smallest MAD value corresponds to the dot plot with the narrowest spread.

### Are You Ready for More?

Invent another data set that also has a mean of 5 but has a MAD greater than 2. Remember, the values in the data set must be whole numbers from 0 to 10.

### Student Response

Answers vary. Sample response: 0, 0, 0, 0, 0, 0, 10, 10, 10, 10, 10, 10

### Activity Synthesis

During discussion, highlight that finding how far away, on average, the data points are from the mean is a way to describe the variability of a distribution. Discuss:

- "What can we say about a data set whose data points have very small distances from the mean?"
- "What about a data set with points that show large distances from the mean?"
- "Does a data set with smaller distances (and therefore smaller mean distances) show less or more variability?"

- "What do MAD values of 2, 1.5, and 1 mean in this context?"

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: mean absolute deviation (MAD). Invite students to suggest language or diagrams to include on the display that will support their understanding of this term.
*Supports accessibility for: Memory; Language Listening, Speaking, Representing: Discussion Supports.* To provide a support as students explain their reasoning, invite students to use the sentence frame: "The largest (or smallest) mean absolute deviation value matches with the dot plot with the _____ spread because . . .". This will help students produce explanations connecting visual and numeric representations of data distribution.
*Design Principle(s): Support sense-making*

## 11.4 Game of 22

**Optional: 15 minutes**

This optional activity uses a game to help students develop the idea of variability. Use this activity if students could benefit from more concrete experiences around the idea of distance from the mean. Students will draw from a standard deck of playing cards and find the sum. The player with the least mean distance from 22 wins the round.

### Instructional Routines

- Stronger and Clearer Each Time

### Launch

Students in groups of 2–3. A deck of playing cards per group. Play 4–6 rounds.

*Representation: Internalise Comprehension.* Begin with a demonstration of the steps for the activity to support understanding. Invite a group of students to play an example round while the class observes.
*Supports accessibility for: Conceptual processing; Visual-spatial processing*

### Student Task Statement

Your teacher will give your group a deck of cards. Shuffle the cards, and put the deck face down on the playing surface.

- To play: Draw 3 cards and add up the values. An ace is a 1. A jack, queen, and king are each worth 10. Cards 2–10 are each worth their face value. If your sum is anything other than 22 (either above or below 22), say: "My sum deviated from 22 by ____ ," or "My sum was off from 22 by ____ ."

- To keep score: Record each sum and each distance from 22 in the table. After five rounds, calculate the mean of the distances. The player with the lowest mean distance from 22 wins the game.

| **player A** | round 1 | round 2 | round 3 | round 4 | round 5 |
|---|---|---|---|---|---|
| sum of cards | | | | | |
| distance from 22 | | | | | |

Mean distance from 22: ___________

| **player B** | round 1 | round 2 | round 3 | round 4 | round 5 |
|---|---|---|---|---|---|
| sum of cards | | | | | |
| distance from 22 | | | | | |

Mean distance from 22: ___________

| **player C** | round 1 | round 2 | round 3 | round 4 | round 5 |
|---|---|---|---|---|---|
| sum of cards | | | | | |
| distance from 22 | | | | | |

Mean distance from 22: ___________

Whose mean distance from 22 is the smallest? Who won the game?

### Student Response

Playing results vary.

### Activity Synthesis

Ask students to think about how *mean distance from a number* can be used to summarise variability and invite a couple of students to share their thinking.

In the game, we can think of the player with the least mean distance from 22 as having cards that are, on the whole, closest to 22 or the "least different" from 22. By the same token, a player with the greatest mean distance from 22 can be seen as having cards that are, on the whole, farthest away from 22 or the "most different" from 22. Connect this to the idea that a data set with a large MAD means it has many values that vary from what we could consider a typical member of the group.

*Writing, Conversing: Stronger and Clearer Each Time.* Use this routine with 2–3 successive pair shares to give students a structured opportunity to revise and refine their response to "How can the mean distance from a number to the mean be used to summarise variability?" Provide students with prompts for feedback (e.g., "Could you give a specific example from a round of Game of 22?" or, "Could you use the term 'mean absolute deviation' to explain your example further?"). Students can borrow ideas and language from each other to strengthen their final product. This will help students strengthen their ideas and clarify their language.
*Design Principle(s): Optimise output (for explanation); Cultivate conversation*

## Lesson Synthesis

In an earlier lesson, we learned that the mean can tell us about what is typical for (or what is characteristic of) a data set because it measures the centre of a distribution. In this lesson, we learn that we can also use a measure of spread to tell us about how much the values in a data set vary.

- “How did we measure spread?”
- “We learned about a measure called the mean absolute deviation (or MAD). What is the meaning of this term?”
- “What does the MAD tell us?” (The mean distance between data points and the mean. It tells us how spread out the data values are.)
- “How do we find the MAD?”
- “How is MAD related to the variability of a data set?”

# 11.5 Text Messages, Again

**Cool Down: 5 minutes**

### Student Task Statement

These three data sets show the number of text messages sent by Jada, Diego, and Lin over 6 days as well as the mean number of text messages sent by each student per day.

Jada

mean: 5

4 4 4 6 6 6

Diego

mean: 6

4 5 5 6 8 8

Lin

mean: 4

1 1 2 2 9 9

1. Predict which data set has the largest MAD and which has the smallest MAD.

2. Compute the MAD for each data set to check your prediction.

### Student Response

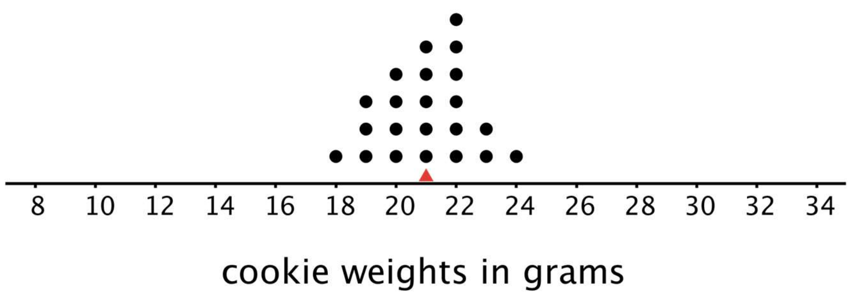
1. Lin's data set will have the largest MAD, because the data has the most variability. Jada's data set will have the smallest MAD, because the data has the least variability.

2. Jada's MAD is $\frac{1+1+1+1+1+1}{6} = 1$; Diego's MAD is $\frac{2+1+1+0+2+2}{6} = 1.33$; Lin's MAD is $\frac{3+3+2+2+5+5}{6} = 3.33$.

## Student Lesson Summary

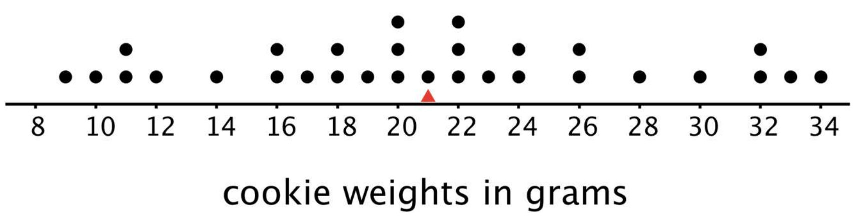
We use the mean of a data set as a measure of centre of its distribution, but two data sets with the same mean could have very different distributions.

This dot plot shows the weights, in grams, of 22 cookies.




The mean weight is 21 grams. All the weights are within 3 grams of the mean, and most of them are even closer. These cookies are all fairly close in weight.

This dot plot shows the weights, in grams, of a different set of 30 cookies.

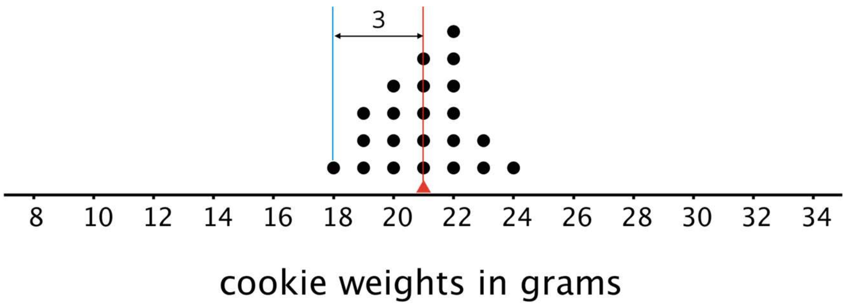



The mean weight for this set of cookies is also 21 grams, but some cookies are half that weight and others are one-and-a-half times that weight. There is a lot more variability in the weight.

There is a number we can use to describe how far away, or how spread out, data points generally are from the mean. This *measure of spread* is called the **mean absolute deviation (MAD)**.

Here the MAD tells us how far cookie weights typically are from 21 grams. To find the MAD, we find the distance between each data value and the mean, and then calculate the mean of those distances.

For instance, the point that represents 18 grams is 3 units away from the mean of 21 grams. We can find the distance between each point and the mean of 21 grams and organise the distances into a table, as shown.




| weight in grams | 18 | 19 | 19 | 19 | 20 | 20 | 20 | 20 | 21 | 21 | 21 | 21 | 21 | 22 | 22 | 22 | 22 | 22 | 22 | 23 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| distance from mean | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 |

The values in the first row of the table are the cookie weights for the first set of cookies. Their mean, 21 grams, is the *mean of the cookie weights*.

The values in the second row of the table are the *distances* between the values in the first row and 21. The mean of these distances is the *MAD of the cookie weights*.

What can we learn from the means of these distances once they are calculated?

- In the first set of cookies, the distances are all between 0 and 3. The MAD is 1.2 grams, which tells us that the cookie weights are typically within 1.2 grams of 21 grams. We could say that a typical cookie weighs between 19.8 and 22.2 grams.
- In the second set of cookies, the distances are all between 0 and 13. The MAD is 5.6 grams, which tells us that the cookie weights are typically within 5.6 grams of 21 grams. We could say a typical cookie weighs between 15.4 and 26.6 grams.

The MAD is also called a *measure of the variability* of the distribution. In these examples, it is easy to see that a higher MAD suggests a distribution that is more spread out, showing more variability.

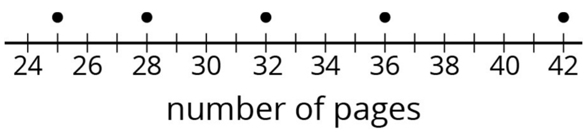
## Glossary

- mean absolute deviation (MAD)

## Lesson 11 Practice Problems

**Problem 1 Statement**

Han recorded the number of pages that he read each day for five days. The dot plot shows his data.




a. Is 30 pages a good estimate of the mean number of pages that Han read each day? Explain your reasoning.

b. Find the mean number of pages that Han read during the five days. Draw a triangle to mark the mean on the dot plot.

c. Use the dot plot and the mean to complete the table.

| number of pages | distance from mean | left or right of mean |
|---|---|---|
| 25 | | left |
| 28 | | |
| 32 | | |
| 36 | | |
| 42 | | |

d. Calculate the mean absolute deviation (MAD) of the data. Explain or show your reasoning.

**Solution**

a. The value of 30 is not a good estimate of the mean as it would not balance the other values.

b. The mean is 32.6 pages.

c.

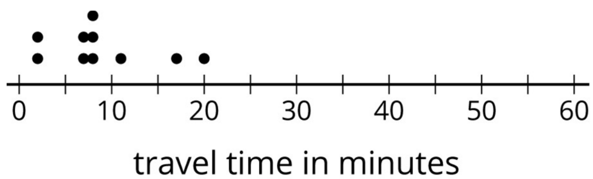
| number of pages | distance from mean | left of mean or right of mean |
|---|---|---|
| 25 | 7.6 | left |
| 28 | 4.6 | left |
| 32 | 0.6 | left |
| 36 | 3.4 | right |

| | | |
|---|---|---|
| 42 | 9.4 | right |

d. The MAD is 5.12 pages: $\frac{7.6+4.6+0.6+3.4+9.4}{5} = 5.12$

**Problem 2 Statement**

Ten year 7 students recorded the amounts of time each took to travel to school. The dot plot shows their travel times.




The mean travel time for these students is approximately 9 minutes. The MAD is approximately 4.2 minutes.

a. Which number of minutes—9 or 4.2—is a typical amount of time for the ten year 7 students to travel to school? Explain your reasoning.

b. Based on the mean and MAD, Jada believes that travel times between 5 and 13 minutes are common for this group. Do you agree? Explain your reasoning.

c. A different group of ten year 7 students also recorded their travel times to school. Their mean travel time was also 9 minutes, but the MAD was about 7 minutes. What could the dot plot of this second data set be? Describe or draw how it might look.

**Solution**

a. 9 minutes. Explanations vary. Sample reasoning: On the dot plot, the centre or balance point of the data set is located at or near 9, so that value is a good description of a typical travel time.

b. Agree. Explanations vary. Sample reasoning: The MAD tells us that, on average, the travel times of the students in this group are 4.2 minutes below the mean (about 5 minutes) or 4.2 minutes above the mean (about 13 minutes), so travel times between 5 and 13 minutes are common.

c. Answers vary. Sample response: The data points on the second dot plot would be more spread out, with more points further from 9, because the MAD is larger. Travel times between 2 minutes and 16 minutes would be typical for this group.

**Problem 3 Statement**

In an archery competition, scores for each round are calculated by averaging the distance of 3 arrows from the centre of the target.

An archer has a mean distance of 1.6 inches and a MAD distance of 1.3 inches in the first round. In the second round, the archer's arrows are further from the centre but are more consistent. What values for the mean and MAD would fit this description for the second round? Explain your reasoning.

**Solution**

Answers vary. Correct responses should have a mean greater than 1.6 inches and a MAD less than 1.3 inches.