Lesson 16: Solving problems with systems of equations

Goals

- Calculate the solution to a system of equations in context, and present (using words and other representations) the solution method.
- Create a system of equations to solve a problem in context.
- Critique (orally) peer solutions to a system of equations.

Learning Targets

- I can use a system of equations to represent a real-world situation and answer questions about the situation.

Lesson Narrative

In this final lesson on systems of equations, students work in groups as they apply what they have learned to solve three problems with different structures and then create a new problem similar in structure to one of the ones they solved. Groups trade problems, prepare well-explained solutions, and take turns sharing their solutions with the class. While groups share, ask other students to interpret particular aspects of the presentation, such as the slope of a graph, the coefficient of a variable, or the solution to a system, in terms of the context of the problem.

Building On

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyse the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Addressing

- Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Instructional Routines

- Group Presentations
- Discussion Supports

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Provide access to tools for making a visual display to each group of 2.

Student Learning Goals

Let's solve some gnarly problems.

16.1 Are We There Yet?

Warm Up: 5 minutes

The purpose of this warm-up is to get students to reason about representing a context about distance as an expression. For students who use the equation $d = rt$ to choose their answer, encourage them to explain how each part of the expression matches the context.

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

A car is driving towards home at 0.5 miles per minute. If the car is 4 miles from home at $t = 0$, which of the following can represent the distance that the car has left to drive?

- $0.5t$
- $4 + 0.5t$
- $4 - 0.5t$
- $4 \times (0.5t)$

Student Response

The expression $4 - 0.5t$ represents the distance the car has left to drive towards home. Since $0.5t$ represents the distance travelled toward home, subtracting that from the 4 miles the car is from home will give us the distance left to drive.

Activity Synthesis

Poll students on which expression they chose. Ask students who chose any expression, right or wrong, to explain their reasoning. After each explanation ask the rest of the class if they agree or disagree and how the context is represented in the expression.

16.2 Cycling, Fundraising, Working, and ___?

35 minutes

In this activity, students reason about situations involving two different relationships between the same two quantities. Then they invent their own problem of the same type. While students are encouraged by the language of the activity to use a system of equations to solve the problems, they may elect to use a different representation to explain their thinking.

As students work through the first three problems, notice the ways students reason about the problems with and without systems of equations. Identify some groups with particularly compelling or clear reasoning to share later.

Instructional Routines

- Group Presentations
- Discussion Supports

Launch

Arrange students in groups of 2. Provide tools for creating a visual display.

Once students have completed the first three problems, select previously identified groups to share their solutions. Bring out why these solutions are particularly good (i.e., clarity, efficiency), and discuss the connections between them, particularly the connections between groups that did and did not use systems of equations where possible. Next, have students begin the second part of the activity and write their own problem to trade with another group.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2–3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far, as well as any questions they have, before continuing.
Supports accessibility for: Organisation; Attention Representing, Conversing: Discussion Supports. Use this routine to help students consider audience when preparing a visual display of their work. When students create their displays for the problems they invent, invite them to consider how to display their strategies so that another student can interpret them. For example, students may wish to add notes or details to their equations to help communicate their thinking. When students receive the problem created by another group, provide 2–3 minutes of quiet think time for students to read and interpret the display before they begin their small-group discussion. During whole-group discussion, ask students to share the kinds of details that were most helpful when interpreting another group's display.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Anticipated Misconceptions

If students struggle to write a system of equations, ask them to identify any unknown quantities in the problem and assign variables to them. Then ask them if there are ways to describe the relationships between the variables. If students still struggle to think about the relationships, ask them about some possible values for each of the variables including some that make sense (such as 20 grapefruits) and some that do not (such as 1 000 grapefruits). Have students explain why some values are not possible to help them understand the relationships between variables.

Student Task Statement

Solve each problem. Explain or show your reasoning.

1. Two friends live 7 miles apart. One Saturday, the two friends set out on their bikes at 8 am and started riding towards each other. One rides at 0.2 miles per minute, and the other rides at 0.15 miles per minute. At what time will the two friends meet?

2. Students are selling grapefruits and nuts for a fundraiser. The grapefruits cost £1 each and a bag of nuts cost £10 each. They sold 100 items and made £307. How many grapefruits did they sell?

3. Jada earns £7 per hour mowing her neighbours' lawns. Andre gets paid £5 per hour for the first hour of babysitting and £8 per hour for any additional hours he babysits. What is the number of hours they both can work so that they get paid the same amount?

4. Pause here so your teacher can review your work. Then, invent another problem that is like one of these, but with different numbers. Solve your problem.

5. Create a visual display that includes:

   - The new problem you wrote, without the solution.
   - Enough work space for someone to show a solution.

6. Trade your display with another group, and solve each other's new problem. Make sure that you explain your solution carefully. Be prepared to share this solution with the class.

7. When the group that got the problem you invented shares their solution, check that their answer is correct.

Student Response

1. The friends will meet at 8:20. Students may write and solve this system: $\begin{cases} y = 0.2x \\ y = -0.15x + 7 \end{cases}$

2. They sold 77 grapefruit. Students may write and solve this system: $\begin{cases} x + 10y = 307 \\ x + y = 100 \end{cases}$

3. Jada and Andre will both earn £21 for working 3 hours. Students may write and solve this system: $\begin{cases} y = 7x \\ y = 5 + 8(x - 1) \end{cases}$

4. Answers vary.

5. Answers vary.

6. Answers vary.

7. Answers vary.

Are You Ready for More?

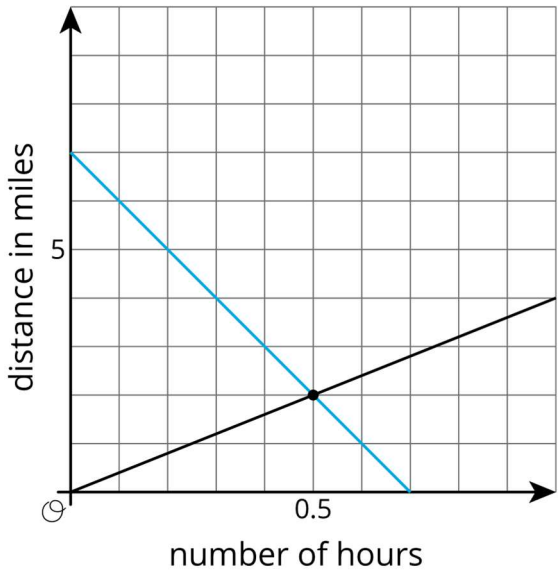
On a different Saturday, two friends set out on bikes at 8:00 am and met up at 8:30 am. (The same two friends who live 7 miles apart.) If one was riding at 10 miles per hour, how fast was the other riding?

Student Response

The other friend is riding 4 miles per hour.

Sample response: 10 miles an hour is equivalent to 5 miles in a half hour. The first friend will be at mile 2 in a half hour because $2 = -10(0.5) + 7$. The second friend will arrive at mile 2 in a half hour which is equivalent to 4 miles an hour. This situation can be represented by the system: $\begin{cases} y = -10x + 7 \\ y = 4x \end{cases}$

Students may use a graph:



Activity Synthesis

Most of the discussions happen within and between groups, but the last question requires a whole-class discussion. Have each group share the peer-generated question they were assigned and the solution. Though the group that wrote the question will be responsible for confirming the answer, encourage all to listen to the reasoning each group used.

Alternatively, after groups have checked the work of the group that solved their problem, have students complete a gallery walk to see all the created problems. Ask students to look for situations similar to theirs and to identify the most common solution methods used. After the gallery, select a few groups to share a problem and how they solved it.