Lesson 5: More graphs of functions

Goals

- Describe (orally and in writing) a graph of a function as “increasing” or “decreasing” over an interval, and explain (orally) the reasoning.
- Interpret (orally and in writing) a graph of temperature as a function of time, using language such as “input” and “output”.

Learning Targets

- I can explain the story told by the graph of a function.

Lesson Narrative

In this lesson, students begin to analyse graphs of functions and use them to answer questions about a context. Students also look at what happens over intervals of input values and learn that graphs can be viewed as dynamic objects that tell stories.

In the temperature activity, students connect specific features of the graph, such as the highest point, with specific features of the contextual situation, i.e., the highest temperature of the day and when it was attained. In the activity about garbage production, students investigate what happens over ranges of input values. The graph tells us how much garbage was produced at certain times and we can also determine if the amount of garbage was increasing or decreasing over time.

As students learn to interpret graphs in terms of a context and use them to answer questions, they learn an important skill in mathematical modelling.

Addressing

- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in KS3.
- Describe qualitatively the functional relationship between two quantities by analysing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Building Towards

- Use functions to model relationships between quantities.
- Describe qualitatively the functional relationship between two quantities by analysing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Student Learning Goals

Let's interpret graphs of functions.

5.1 Which One Doesn't Belong: Graphs

Warm Up: 10 minutes

The purpose of this warm-up is for students to notice and describe the features of graphs using their own language. Students will encounter a variety of graphs over the next several lessons and throughout these lessons they will gradually develop more precise language around graphs as the needs of activities dictate.

Instructional Routines

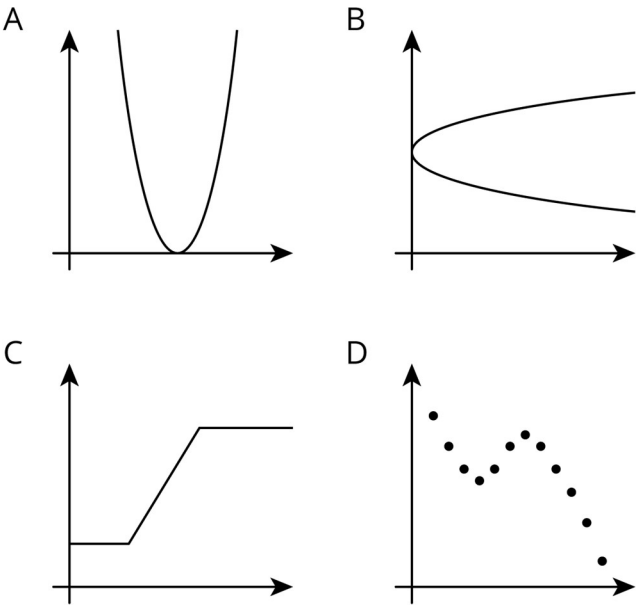
- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the image of the four graphs for all to see. Ask students to indicate when they have noticed one graph that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. Follow with a whole-class discussion.

Student Task Statement

Which graph doesn't belong?



Student Response

Answers vary. Sample response:

A doesn't belong because it is the only graph that touches the horizontal axis.

B doesn't belong because it is the only one that is not a function.

C doesn't belong because it is the only one made of straight line segments or because it is the only graph with no interval where it is decreasing.

D doesn't belong because it is the only one made of discrete points or because it is the only graph with two distinct intervals where it is decreasing.

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular graph might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which graph does not belong, attend to students' explanations and ensure the reasons given are correct.

Try to highlight the two facts that there are points with the same first coordinate and different second coordinates in graph B (which means it is not a function), the straight segments of C vs. curves of the others, and the discrete nature of D during this discussion, using whatever language students bring to it. Avoid introducing the traditional x and y names for the axes into the discussion unless students use them first. More formal vocabulary will be developed in later activities, lessons, and much of the motivation of this

added vocabulary is to improve upon the somewhat clunky language we are led to use without it.

5.2 Time and Temperature

15 minutes

The purpose of this activity is for students to begin using a graph of a functional relationship between two quantities to make quantitative observations about their relationship. For some questions students must identify specific input-output pairs while in others they can use the shape of the graph. For example, when asked for which time the temperature was warmer, students need only compare the relative height of the graph at the two different times. Similarly, students can identify another time the temperature was the same as 4:00 p.m. without actually knowing the temperature at 4:00 p.m.

Identify students who reason about the graph without identifying specific values to share during the discussion. For example, a student can identify that the temperature was highest at about 5:45 p.m. by finding the highest point on the graph without stating that that highest temperature was approximately 59°F.

Instructional Routines

- Discussion Supports
- Think Pair Share

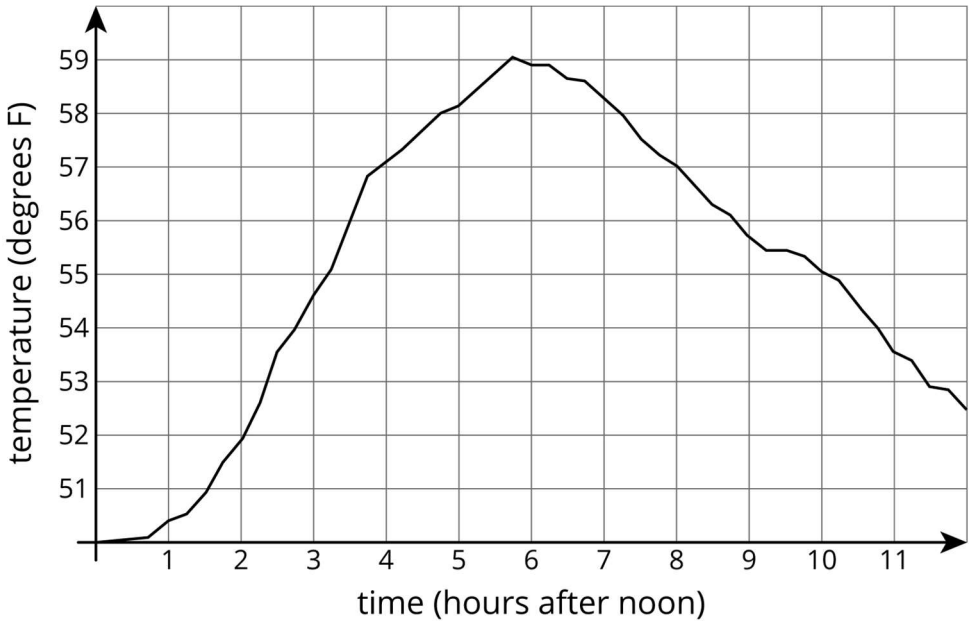
Launch

Arrange students in groups of 2. Give students 4–6 minutes of quiet work time and then time to share their responses with their partner. Follow with a whole-class discussion.

Representation: Internalise Comprehension. Represent the same information as the graph through different modalities by using tables and sentences. If students are unsure where to begin, suggest they create a table to represent the time and temperature.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Student Task Statement

The graph shows the temperature between noon and midnight in one day in a certain city.



1. Was it warmer at 3:00 p.m. or 9:00 p.m.?
2. Approximately when was the temperature highest?
3. Find another time that the temperature was the same as it was at 4:00 p.m.
4. Did the temperature change more between 1:00 p.m. and 3:00 p.m. or between 3:00 p.m. and 5:00 p.m.?
5. Does this graph show that temperature is a function of time, or time is a function of temperature?
6. When the input for the function is 8, what is the output? What does that tell you about the time and temperature?

Student Response

1. The temperature in the city was warmer at 9:00 p.m.
2. The temperature in the city was highest at approximately 5:45 p.m.
3. At 8:00 p.m. the temperature in the city was the same as at 4:00 p.m.
4. The temperature changed approximately 4 degrees between 1:00 p.m. and 3:00 p.m. but only approximately 3.5 degrees between 3:00 p.m. and 5:00 p.m.
5. Temperature is a function of time.
6. 57. At 8:00 p.m., the temperature is 57°F.

Activity Synthesis

Display the graph for all to see during the discussion. Select a few previously identified students per problem to model how they found their answers on the displayed graph for the first five questions. If not mentioned by students, demonstrate how to find the solution to the fourth problem by either identifying the temperature values at each time and subtracting or by measuring the vertical change for each time interval.

For the final question, ask students to plot the point on their graphs if they did not do so already. Invite students to describe what the point means in the context.

If time allows, give 1 minute quite think time for groups to come up with their own question that someone else could answer using the graph. Invite groups to share their question and ask a different group to give the answer.

Speaking: Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language to restate and/or revoice the response presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language based on what they can interpret from a graph.
Design Principle(s): Support sense-making; Maximise meta-awareness

5.3 Garbage

15 minutes

The purpose of this activity is for students to identify where a function is increasing or decreasing from a graphical representation. In the previous activity students focused more on single points. In this activity they focus on collections of points within time intervals and what the overall shape of the graph says about the relationship between the two quantities.

As students work, monitor for strategies for identifying increasing or decreasing intervals. Some strategies might be:

- finding the amount of garbage that corresponds to different years and comparing their values
- drawing line segments between discrete points and observing whether the line segment slants up or down as you move from left to right on the graph
- using a finger to show that, as you move from left to right on the graph, the function trends upward or downward

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Display the first graph for all to see. if students are not familiar with time plots, explain that each point represents the value for one year starting with the point for 1991. Ask students "Does the graph show the amount of garbage produced as a function of time, or the time as a function of the amount of garbage produced?" (Amount of garbage produced as a function of time.)

The unit for the amount of garbage is ton. This is the American ton (equivalent to 2000 lbs) as the graph refers to the amount of garbage produced in the US each year and not the British tonne (equivalent to 1000 kg).

Give groups 1 minute to decide on a question that the information in the graph can answer. For example, "About how much garbage was produced in 2010?" (About 250 000 thousand or 250 billion tons.) Invite groups to share their question and ask a different group to give the answer.

Tell students to reopen their books or devices and read the first problem. Ask:

- "What do the words increase and decrease mean?" (Increase means a value is going up and decrease means a value is going down.)
- "From 1999 to 2000, did the amount of garbage produced *increase* or *decrease*?" (It increased.)
- "How can you tell it increased?"

One way to answer this last question is to find the amount of garbage (about 235 000 thousand tons in 1999 and about 245 000 in 2000) and compare the values, but there are easier ways. Call upon students to articulate other methods, such as tracing from left to right with your finger and noting that your finger is travelling upwards.

Give students 3–5 minutes work time for the remaining problems. Encourage students to discuss the last question pertaining to the second graph. If partners do not agree have them work together until they come to agreement. Follow with a whole-class discussion.

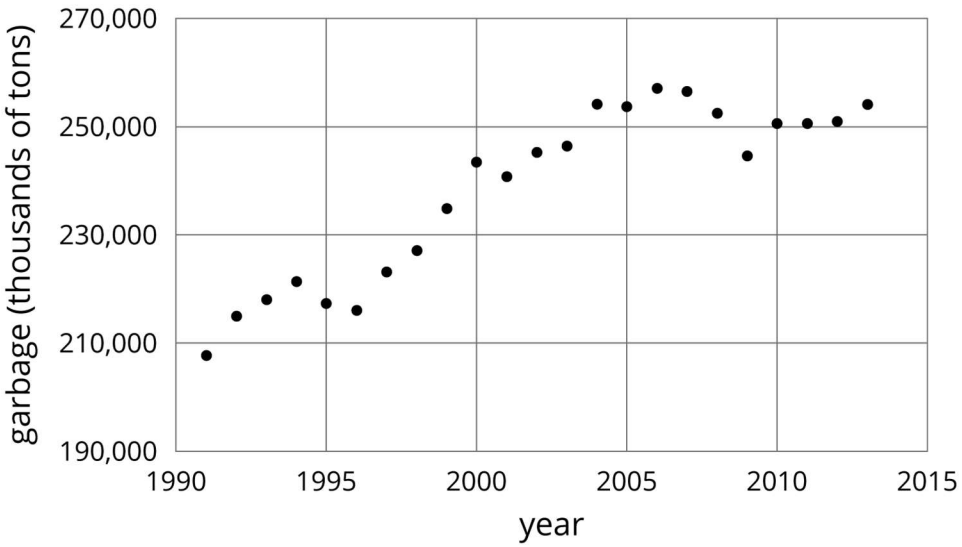
Representation: Develop Language and Symbols. Create a display of important terms and vocabulary, and maintain the display for reference throughout the unit. Invite students to suggest language or diagrams to include that will support their understanding of: increase, decrease, steep, independent, dependent, function, and variable.
Supports accessibility for: Memory; Language

Anticipated Misconceptions

Students may not answer with a range of dates, they might instead list each year it increased. A list is acceptable, but be sure students see the connection between, for example, the list 1996, 1997, and 1998 and the same years stated as "from 1996 to 1998."

Student Task Statement

1. The graph shows the amount of garbage produced in the US each year between 1991 and 2013.

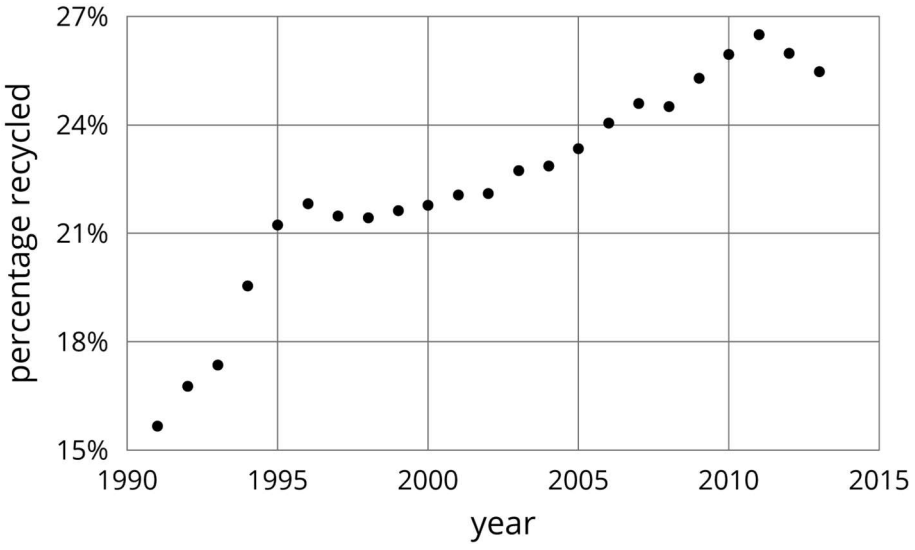


   a. Did the amount of garbage increase or decrease between 1999 and 2000?

   b. Did the amount of garbage increase or decrease between 2005 and 2009?

   c. Between 1991 and 1995, the garbage increased for three years, and then it decreased in the fourth year. Describe how the amount of garbage changed in the years between 1995 and 2000.

2. The graph shows the percentage of garbage that was recycled between 1991 and 2013.



a. When was it increasing?

b. When was it decreasing?

c. Tell the story of the change in the percentage of garbage recycled in the US over this time period.

Student Response

a. Increase. Based on the graph, from 1999 to 2000 the amount of garbage produced increased from about 235 000 to 245 000 thousand tons.

b. Decrease. Based on the graph, from 2005 to 2009 the amount of garbage produced decreased from about 255 000 to 245 000 thousand tons.

c. The amount of garbage decreased for one year, then increased for four years in a row.

a. The graph is increasing from 1991 until 1996, and from 1998 to 2011.

b. The graph is decreasing from 1996 to 1998, and 2011 to 2013.

c. Answers vary. Sample response: The percentage of garbage recycled generally increased from 1991 until a peak at 2011, and then began to decrease. There were brief periods of decrease from 1996 to 1998 and 2007 to 2008.

Are You Ready for More?

Refer to the graph in the first part of the activity.

1. Find a year where the amount of garbage produced increased from the previous year, but not by as much it increased the following year.

2. Find a year where the amount of garbage produced increased from the previous year, and then increased by a smaller amount the following year.

3. Find a year where the amount of garbage produced decreased from the previous year, but not by as much it decreased the following year.

4. Find a year where the amount of garbage produced decreased from the previous year, and then decreased by a smaller amount the following year.

Student Response

1. 2003

2. 1997

3. 2008

4. 1995

Activity Synthesis

While discussing each graph, display for all to see. For the first graph, ask previously selected students to share their responses and their strategies for finding years where the

amount of garbage increased or decreased. Sequence student responses in the order listed in the Activity Narrative.

Ask students to share their responses for the second graph. Close the discussion by asking "How might you describe this graph in general to someone who couldn't see it?" Invite student to share their descriptions. For example, the percentage of garbage that was recycled increased overall from 1990 to 2011, but began decreasing from 2011 to 2013.

Writing, Speaking, Listening: Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners, to share their response to, "Tell the story of the change in the percentage of garbage produced in the US over this time period." Provide prompts for feedback that will help students strengthen their ideas and clarify their language. For example, "What happens first in your story?", "How do the points in your story match the graph?", "What happened in 2011?", etc. Give students 1–2 minutes to revise their writing based on the feedback they received. *Design Principle(s): Optimise output (for explanation)*

Lesson Synthesis

The graph of a function tells us a story about the context it represents. Specific points on the graph connect to specific features of the situation.

Consider asking some of the following questions about the graphs from the activities to reinforce these ideas:

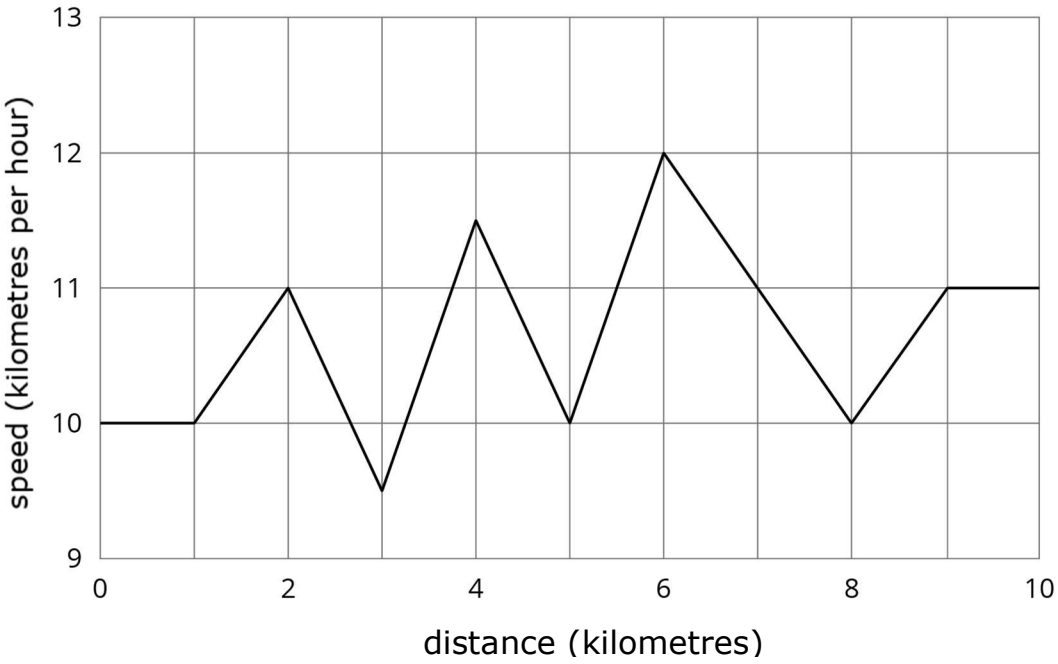
- "On the temperature graph, how do we find the time when it was the coolest?" (By finding the point on the graph that is the lowest.)
- "On the temperature graph, how do we find the difference between the hottest and the coolest temperatures?" (We can find the lowest and the highest points on the graph and count the difference between them using the grid or we can find the hottest temperature and subtract from it the coolest temperature. Either way the difference is about 9 degrees.)
- "Looking at the garbage production graph, how does the production before 2005 compare with the production since 2005?" (From 1990 to 2005, production was increasing. After 2005 production levelled off and seemed to decrease slightly.)

5.4 Diego's 10K Race

Cool Down: 5 minutes

Student Task Statement

Diego runs a 10 kilometre race and keeps track of his speed.



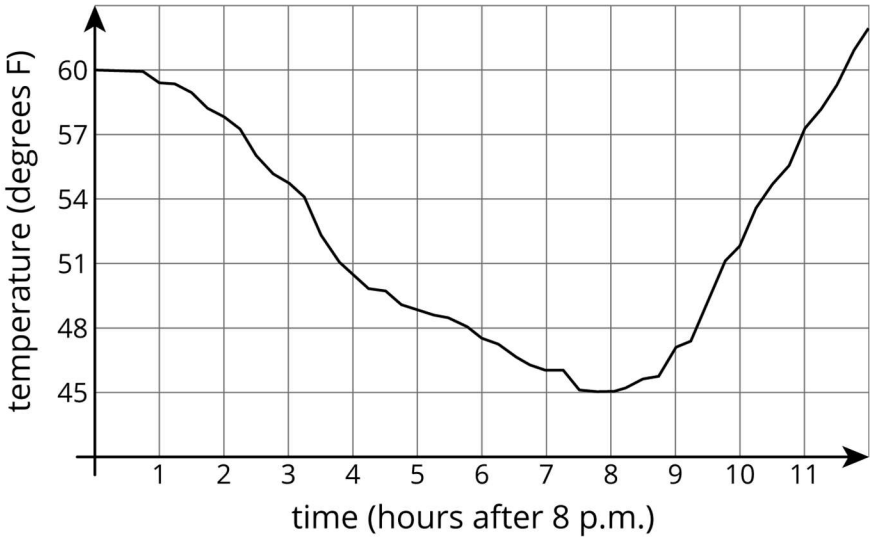
1. What was Diego's speed at the 5 kilometre mark in the race?
2. According to the graph, where was Diego when he was going the slowest during the race?
3. Describe what happened to Diego's speed in the second half of the race (from 5 km to 10 km).

Student Response

1. 10 kilometres per hour. This is the second coordinate of the point with first coordinate representing 5 km.
2. 3 kilometres into the race. This is the first coordinate of the lowest point on the graph, representing the slowest speed.
3. From 5 km to 6 km, Diego went faster but slowed down from 6 km to 8 km. He sped up again from 8 km to 9 km and finished the last kilometre at the same speed.

Student Lesson Summary

Here is a graph showing the temperature in a town as a function of time after 8:00 p.m.



The graph of a function tells us what is happening in the context the function represents. In this example, the temperature starts out at 60° F at 8:00 p.m. It decreases during the night, reaching its lowest point at 8 hours after 8:00 p.m., or 4:00 a.m. Then it starts to increase again.

Lesson 5 Practice Problems

1. **Problem 1 Statement**

   The solution to a system of equations is (6,-3). Choose two equations that might make up the system.

   a. $y = -3x + 6$

   b. $y = 2x - 9$

   c. $y = -5x + 27$

   d. $y = 2x - 15$

   e. $y = -4x + 27$

   Solution ["C", "D"]

2. **Problem 2 Statement**

   A car is travelling on an A road and is either going 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Letting x represent the amount of time in hours that the car is going 55 miles per hour, and y being the time in hours that the car is going 35 miles per hour, an equation describing the relationship is: $55x + 35y = 200$

   a. If the car spends 2.5 hours going 35 miles per hour on the trip, how long does it spend going 55 miles per hour?

b. If the car spends 3 hours going 55 miles per hour on the trip, how long does it spend going 35 miles per hour?

c. If the car spends no time going 35 miles per hour, how long would the trip take? Explain your reasoning.

Solution

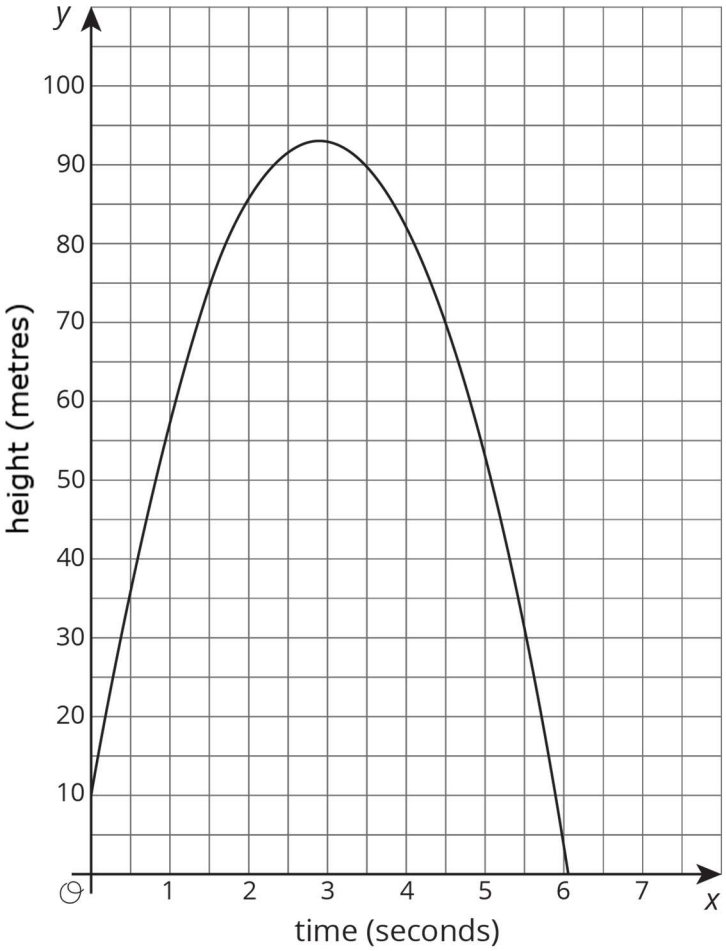
a. About 2.05 hours

b. 1 hour

c. About 3.64 hours. If the car spent the entire trip going 55 mph, the trip would be completed in about 3.64 hours. (3 hours and 38 mins, to the nearest minute)

3. **Problem 3 Statement**

The graph represents an object that is shot upwards from a tower and then falls to the ground. The independent variable is time in seconds and the dependent variable is the object's height above the ground in metres.



a. How tall is the tower from which the object was shot?

b. When did the object hit the ground?

c. Estimate the greatest height the object reached and the time it took to reach that height. Indicate this situation on the graph.

Solution

a. 10 metres

b. 6 seconds after it was shot

c. Approximately 93 metres high at 2.9 seconds. A point should be plotted at (2.9,93).