Lesson 14: Evaluating expressions with exponents

Goals

- Evaluate numerical expressions that have an exponent and one other operation, and justify (orally) the process.
- Explain (orally and in writing) that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols.
- Interpret expressions with exponents that represent the surface area or volume of a cube.

Learning Targets

- I know how to evaluate expressions that have both an exponent and addition or subtraction.
- I know how to evaluate expressions that have both an exponent and multiplication or division.

Lesson Narrative

The focus of this lesson is evaluating expressions that have an exponent and one other operation by carrying out operations in the conventional order. This is accomplished through an example with surface area, where the context provides a clear reason for evaluating the exponential expression before performing the multiplication. Students practice evaluating numeric expressions that include exponents.

Alignments

Addressing

- Write and evaluate numerical expressions involving whole-number exponents.
- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no brackets to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

Instructional Routines

- Clarify, Critique, Correct
- Three Reads

Student Learning Goals

Let's find the values of expressions with exponents.

14.1 Revisiting the Cube

Warm Up: 10 minutes (there is a digital version of this activity)

The purpose of this warm-up is for students to recall previous understandings of area, volume, and surface area of cubes, and how to record these measurements as expressions using exponents. Students might respond with either verbal or numerical descriptions, saying, for example, "We can find the area of the square," or "The area of the square is 9 square units."

After students share their responses, display the following table for all to see and give students time to discuss the information with a partner. The table is used to encourage students to think about the expressions with exponents in addition to the numeric responses.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3			
as an expression using an exponent	3			

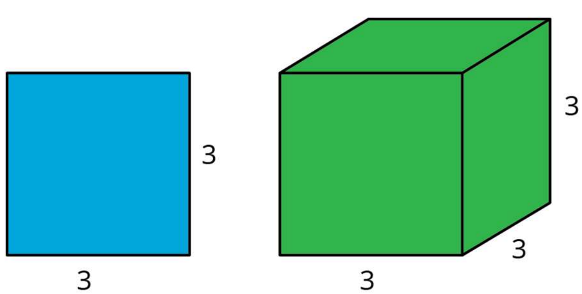
Launch

Arrange students in groups of 2. Display the table and give students 1 minute of quiet work time to complete as much of the table as they can. Then complete the table and discuss as a group.

If students have access to digital activities they can explore the applet and generate dimensions that can be determined. After sharing, they can complete the table.

Student Task Statement

Based on the given information, what other measurements of the square and cube could we find?



Student Response

Answers vary. Sample responses:

- We can find the area of the square.
- The area of the square is 9 square units.
- The perimeter is 12 units.
- We can find the volume of the cube.
- The volume of the cube is 27 cubic units.
- The surface area is 54 square units.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3	9	27	54
as an expression using an exponent	3	3^2	3^3	$6(3^2)$

Activity Synthesis

Ask students to share their responses for the cells in the table. Poll the students on whether they agree or disagree with each response. Record and display the responses for all to see. As students share responses, ask the following questions to help clarify their answers:

- First row: What calculation did you do to arrive at that answer? Where are those measurements in the image?
- Second row: How did you decide on the exponent for your answer? Where are those measurements in the image?

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3	9	27	54
as an expression using an exponent	3	3^2	3^3	$6(3^2)$

In the next activity, students will analyse calculations of the surface area of a cube.
Take time now to discuss why $6(3^2)$ expresses the surface area of this cube. Ask students to think about how they computed surface area, and then analyse this expression. Where did the 3^2 come from? (It's the area of one face of the cube.) Why are we multiplying by 6? (We want to add up 6 3^2 s, and that is the same as multiplying 3^2 by 6.)

14.2 Calculating Surface Area

10 minutes

In this activity, students use surface area as a context to extend the order of operations to expressions with exponents. The context provides a reason to evaluate the exponent before performing the multiplication.

Instructional Routines

- Three Reads

Launch

Give students 10 minutes of quiet work time, followed by a class discussion.

Reading: Three Reads. Use this routine to support students' comprehension of the situation. In the first read, students read the text with the goal of comprehending the situation (e.g., Jada and Noah have different solutions for the surface area of the same cube). In the second read, ask students to identify important quantities that can be counted or measured (e.g., the side length of the cube; the number of faces of a cube; the area of each face of the cube). In the third read, reveal the question, "Do you agree with either of them? Explain your reasoning." Ask students to brainstorm possible strategies to answer the question (e.g., Find the area of each face of the cube. Multiply the area by 6 to find the surface area of the cube). This will help students concentrate on making sense of the situation before rushing to a solution or method.
Design Principle(s): Support sense-making

Student Task Statement

A cube has side length 10 cm. Jada says the surface area of the cube is 600 cm^2, and Noah says the surface area of the cube is 3,600 cm^2. Here is how each of them reasoned:

Jada's Method:

6×10^2
6×100
600

Noah's Method:

6×10^2
60^2
$3\,600$

Do you agree with either of them? Explain your reasoning.

Student Response

Jada’s solution is correct. Explanations vary. Sample response: The cube has 6 faces and each has an area of 10^2 or 100. The area calculation comes before multiplying by 6.

Activity Synthesis

In finding the surface area, there is a clear reason to find 10^2 and then multiply by 6. Tell students that sometimes it is not so clear in which order to evaluate operations. There is an order that we all generally agree on, and when we want something done in a different order, brackets are used to communicate what to do first. When an exponent occurs in the same expression as multiplication or division, we evaluate the exponent first, unless brackets say otherwise. Examples: $(3 \times 4)^2 = 12^2 = 144$, since the brackets tell us to multiply 3×4 first. But $3 \times 4^2 = 3 \times 16 = 48$, because since there are no brackets, we evaluate the exponent before multiplying.

If students bring up BIDMAS or another mnemonic for remembering the order of operations, point out that BIDMAS can be misleading in indicating multiplication before division, and addition before subtraction. Discuss the convention that brackets indicate that something should be evaluated first, followed by exponents, multiplication, or division (evaluated left to right), and last, addition or subtraction (evaluated left to right).

14.3 Row Game: Expression Explosion

15 minutes

In this activity, students use the order of operations to evaluate expressions with exponents. They listen and critique their partner’s reasoning when they do not agree on the answers.

Instructional Routines

- Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Partners work individually on their expression in each row, then check their answers and discuss. Follow with a whole-class discussion.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they work together to find errors. For example, “How did you . . .?”, “First, I _____ because...”, “I agree/disagree because . . .”
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$5^2 + 4$	$2^2 + 25$
$2^4 \times 5$	$2^3 \times 10$
3×4^2	12×2^2
$20 + 2^3$	$1 + 3^3$
9×2^1	3×6^1
$\frac{1}{9} \times \left(\frac{1}{2}\right)^3$	$\frac{1}{8} \times \left(\frac{1}{3}\right)^2$

Student Response

1. 29
2. 80
3. 48
4. 28
5. 18
6. $\frac{1}{72}$

Are You Ready for More?

1. Consider this equation: $\square^2 + \square^2 = \square^2$. An example of 3 different whole numbers that could go in the boxes are 3, 4, and 5, since $3^2 + 4^2 = 5^2$. (That is, $9 + 16 = 25$.)

   Can you find a different set of 3 whole numbers that make the equation true?

2. How many sets of 3 different whole numbers can you find?

3. Can you find a set of 3 different whole numbers that make this equation true? $\square^3 + \square^3 = \square^3$

4. How about this one? $\square^4 + \square^4 = \square^4$

Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (Alas, this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

Student Response

1. Sample responses: {6, 8, 10}, {5, 12, 13}
2. Answers vary. There are an infinite number of these triples.
3. No. (No such triple exists.)
4. No. (No such triple exists.)

Activity Synthesis

The purpose of the discussion is to ensure that students understand and can apply the agreed on rules for order of operations when expressions contain exponents. Consider asking some of the following questions:

- "Were there any expressions that were difficult to evaluate? Why were they difficult?"
- "Did you disagree with your partner about any rows? How did you settle the disagreement?"
- "Did you learn anything new about evaluating expressions with exponents?"

Reading, Writing, Speaking: Clarify, Critique, Correct. Present an incorrect solution based on a common misconception about evaluating expressions with exponents. For example, "The expression $1 + 3^3$ is equal to 64 because $1 + 3$ is 4 and 4^3 is 64." Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who state that exponents must be evaluated before addition, unless brackets say otherwise. Therefore, 3^3 must be evaluated first before adding 1. This routine will engage students in meta-awareness as they critique and correct the language used to evaluate expressions with exponents.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

Ask students to write and evaluate a numerical expression with an exponent and one other operation. Then have students switch with a partner and evaluate the partner's expressions. Invite some students to share their expressions with the class.

14.4 Calculating Volumes

Cool Down: 5 minutes

Student Task Statement

Jada and Noah wanted to find the total volume of a cube and a rectangular prism. They know the prism's volume is 20 cubic units, and they know the cube has side lengths of 10 units. Jada says the total volume is 27,000 cubic units. Noah says it is 1,020 cubic units. Here is how each of them reasoned:

Jada's Method:

$20 + 10^3$
30^3
$27\,000$

Noah's Method:

$20 + 10^3$
$20 + 1\,000$
$1\,020$

Do you agree with either of them? Explain your reasoning.

Student Response

Noah's solution is correct. Reasoning varies. Sample reasoning: The cube has a volume of 1 000 cubic units and the additional 20 cubic units from the prism makes the total volume 1 020 cubic units. The exponent calculation comes before addition.

Student Lesson Summary

Exponents give us a new way to describe operations with numbers, so we need to understand how exponents get along with the other operations we know.

When we write 6×4^2, we want to make sure everyone agrees about how to evaluate this. Otherwise some people might multiply first and others compute the exponent first, and different people would get different values for the same expression!

Earlier we saw situations in which 6×4^2 represented the surface area of a cube with side lengths 4 units. When computing the surface area, we evaluate 4^2 first (or find the area of one face of the cube first) and then multiply the result by 6. In many other expressions that use exponents, the part with an exponent is intended to be evaluated first.

To make everyone agree about the value of expressions like 6×4^2, the convention is to *evaluate the part of the expression with the exponent first*. Here are a couple of examples:

6×4^2
6×16
96

$45 \quad +5^2$
$45 \quad +25$
70

If we want to communicate that 6 and 4 should be multiplied first and then squared, then we can use brackets to group parts together:

$(6 \quad \times 4)^2$
24^2
576

$(45 \quad +5)^2$
50^2
$2, \quad 500$

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   Lin says, "I took the number 8, and then multiplied it by the square of 3." Select **all** the expressions that equal Lin's answer.

   a. 8×3^2

   b. $(8 \times 3)^2$

   c. 8×2^3

   d. $3^2 \times 8$

   e. 24^2

   f. 72

   Solution ["A", "D", "F"]

2. **Problem 2 Statement**

   Evaluate each expression.

   a. $7 + 2^3$

   b. 9×3^1

   c. $20 - 2^4$

   d. 2×6^2

   e. $8 \times (\frac{1}{2})^2$

f. $\frac{1}{3} \times 3^3$

g. $(\frac{1}{5} \times 5)^5$

Solution

a. 15

b. 27

c. 4

d. 72

e. 2

f. 9

g. 1

3. **Problem 3 Statement**

Andre says, “I multiplied 4 by 5, then cubed the result.” Select **all** the expressions that equal Andre’s answer.

a. 4×5^3

b. $(4 \times 5)^3$

c. $(4 \times 5)^2$

d. $5^3 \times 4$

e. 20^3

f. 500

g. 8 000

Solution ["B", "E", "G"]

4. **Problem 4 Statement**

Han has 10 cubes, each 5 inches on a side.

a. Find the total volume of Han’s cubes. Express your answer as an expression using an exponent.

b. Find the total surface area of Han’s cubes. Express your answer as an expression using an exponent.

Solution

a. 10×5^3 in^3

b. $10 \times 6 \times 5^2$ in^2 or 60×5^2 in^2

5. **Problem 5 Statement**

Priya says that $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{4}{3}$. Do you agree with Priya? Explain or show your reasoning.

Solution

Answers vary. Sample response: I disagree with Priya. $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$ is really $(\frac{1}{3})^4$, or $\frac{1}{81}$.

6. **Problem 6 Statement**

Answer each question. Show your reasoning.

a. 125% of e is 30. What is e?

b. 35% of f is 14. What is f?

Solution

a. 24. $\frac{125}{100} \times e = 30$, so $e = 30 \div \frac{125}{100}$, so $e = 30 \times \frac{100}{125}$.

b. 40. $(0.35) \times f = 14$, so $f = 14 \div 0.35$.

7. **Problem 7 Statement**

Which expressions are solutions to the equation $2.4y = 13.75$? Select **all** that apply.

a. $13.75 - 1.4$

b. 13.75×2.4

c. $13.75 \div 2.4$

d. $\frac{13.75}{2.4}$

e. $2.4 \div 13.75$

Solution ["C", "D"]

8. **Problem 8 Statement**

Jada explains how she finds 15×23:

"I know that ten 23s is 230, so five 23s will be half of 230, which is 115.
15 is 10 plus 5, so 15 × 23 is 230 plus 115, which is 345."

a. Do you agree with Jada? Explain.

b. Draw a 15 by 23 rectangle. Partition the rectangle into two rectangles and label them to show Jada's reasoning.

Solution

a. Yes, Jada is calculating 15×23 by writing it as $(10 + 5) \times 23$ (using the distributive property). To find 5×23, she thinks of 5 as $\frac{10}{2}$. So Jada needs to multiply 23 by 10 (which gives her 230) and add half of this product (which is 115) to find the value of $(10 + 5) \times 23$.

b.