Lesson 17: Fitting boxes into boxes

Goals

- Compare and contrast (orally and using other representations) different ways jewellery boxes could be packed inside larger shipping boxes.
- Determine which size shipping box is least expensive, and present (orally and in writing) a justification.
- Make simplifying assumptions and determine what information is needed to solve a problem about shipping costs.

Learning Targets

- I can use multiplication and division of fractions to reason about real-world volume problems.

Lesson Narrative

In this three-part culminating activity, students use what they have learned to determine the most economical way to ship jewellery boxes using the United States Postal Service (USPS) flat-rate options.

In Part 1, students make sense of the task, outline what they will need to know and do to answer the question, and map out their plan. In Part 2, they model the problem, calculate the number of jewellery boxes that will fit into each shipping box, and determine the associated costs. Students experiment with different orientations for the jewellery boxes to optimise space and minimise cost. In Part 3, they present, reflect, and discuss. Students explain their strategies and reasoning and evaluate the decisions about how to fit all 270 jewellery boxes so they ship at the lowest cost. As a class, students reflect on how the orientation of the jewellery boxes and the size of the shipping boxes affected the unit cost for shipping each box of jewellery.

Depending on the instructional choices made, this lesson could take one or more lessons. The time estimates are intentionally left blank because the amount of time needed might vary depending on factors such as:

- If students will research the flat-rate options themselves, or be provided with this information.
- If each group will explore all size options or only one option.
- How much organisational support is given to students.
- How student work is ultimately shared with the class (e.g., not at all, informally, or with formal presentations).

Consider defining the scope of work further and setting a time limit for each part of the activity to focus students' work and optimise class time.

Alignments

Addressing

- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- Interpret and calculate quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. In general, $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$. How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$ cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?

Instructional Routines

- Stronger and Clearer Each Time
- Collect and Display
- Three Reads
- Discussion Supports

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

When compasses are required they are listed as a separate Required Material.

Measuring tapes
Rulers

Required Preparation

Bring in samples of United States Postal Service flat-rate boxes, or have images of these boxes available.

Student Learning Goals

Let's use what we learned about fractions to find shipping costs.

17.1 Determining Shipping Costs (Part 1)

In this first part of the project, students make sense of the task at hand and determine what they need to know and do to find the most economical shipping box combination. Students should recognise that they will need to:

- Find out the measurements of the jewellery boxes and shipping boxes, as well as the costs for mailing a shipping box of each size. Asking students to find out this information themselves will increase the modelling demand.
- Decide on an orientation for the jewellery boxes inside each shipping box and calculate how many jewellery boxes will fit with that particular orientation.
- Test out different orientations and how they affect the number of jewellery boxes to be fitted and the cost.

Students may want to think of a strategy for considering different configurations efficiently, rather than testing all of them, which would be time consuming and repetitive. As students make plans to try out different jewellery box orientations and associated calculations, encourage them to work systematically to minimise omissions and errors. Urge students to create drawings or models of the boxes to show the calculations they will need to make.

Consider supporting students by discussing the different orientations of jewellery boxes in a shipping box (Which orientations are possible? How much empty space would result?) and possible ways to use drawings or diagrams to show the arrangements of jewellery boxes.

Instructional Routines

- Three Reads

Launch

Give students 1–2 minutes to read the task statement individually and to ask any clarifying questions. Demonstrate the idea of the task by putting a small box inside a larger box in different orientations. Consider displaying USPS flat-rate boxes, or an image of each of the boxes.

Arrange students in groups of 4. Give students 5 minutes of quiet think time to brainstorm about what information is needed to solve this task and share to it with their group. Give another 5 minutes to plan in groups, followed by time to measure boxes or research box options and dimensions for themselves. Provide access to measuring tools.

USPS flat-rate information:

- Small box: $5\frac{3}{8}$ inches by $8\frac{5}{8}$ inches by $1\frac{5}{8}$ inches. Cost: £6.80.

- Medium box 1: 11 inches by $8\frac{1}{2}$ inches by $5\frac{1}{2}$ inches. Cost: £13.45.
- Medium box 2: $11\frac{7}{8}$ inches by $3\frac{3}{8}$ inches by $13\frac{5}{8}$ inches. Cost: £13.45.
- Large box: 12 inches by 12 inches by $5\frac{1}{2}$ inches. Cost: £18.75.

Reading, Speaking, Representing: Three Reads. Use this routine to orient students to the context of the problem. In the first read, students read the problem with the goal of comprehending the situation (an artist is packing jewellery boxes to ship to a store.). Clarify any unknown language, such as a "flat-rate" box or shipping rates, as needed. For the second read, ask students to identify the quantities and mathematical relationships (number of necklaces ordered, the dimensions of the jewellery box). After the final read, ask students to brainstorm possible strategies they may use to solve the problem.
Design Principle(s): Support sense-making

Student Task Statement

An artist makes necklaces. She packs each necklace in a small jewellery box that is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch.

A department store ordered 270 necklaces. The artist plans to ship the necklaces to the department store using flat-rate shipping boxes from the post office.

1. Consider the problem: Which of the flat-rate boxes should she use to minimise her shipping cost?

   What other information would you need to be able to solve the problem?

2. Discuss this information with your group. Make a plan for using this information to find the most inexpensive way to ship the jewellery boxes. Once you have agreed on a plan, write down the main steps.

Student Response

Answers vary. Possible response:

1. We need to know the shipping cost and the size (i.e., the dimensions) for each flat-rate box.
2. Possible plan:
   - Decide which group member or members will work with which shipping box.
   - Find out how many jewellery boxes fit into each shipping box.
   - Find out how many and what combination of shipping boxes we need.
   - Compute the total cost for shipping the jewellery boxes.

Activity Synthesis

Reconvene as a class before continuing with the next part. Ask each group to share a couple of specific steps they have taken toward answering the question and a couple of steps they plan on taking to move forward. Highlight any ideas students might have about making the problem-solving process more efficient and systematic. If not already mentioned by students, suggest that each group divide up the calculations to be done so each person is responsible for one shipping box.

17.2 Determining Shipping Costs (Part 2)

After planning and gathering information in Part 1, students now calculate the cost of shipping jewellery boxes in each of the USPS flat-rate boxes. Each member of the group will select one of the 4 flat-rate shipping box sizes, decide on the best orientation for the jewellery boxes inside the shipping box, and calculate the cost to ship 270 boxes.

Notice groups using different strategies for division with fractions. Ask students to think about the different ways they have used fractions in calculations. If they are stuck, remind students that drawing the boxes, or making them from paper, might help them to visualise what calculations would be most helpful in finding a solution for the task at hand.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Keep students in the same groups of 4. Ask each group member to select a different size of shipping box so that all boxes are represented in each group. Provide access to geometry toolkits and rulers (or tape measures).

Once shipping boxes are assigned, give students quiet time to work. After most students have attempted a few box orientations and made their first calculations, ask the class to pause their work. Use Stronger and Clearer Each Time: Students successively share their orientations and reasoning for their particular orientation, and get feedback on both language and orientation choices from their partners before the post-write.

Conversing: This activity recommends using *Stronger and Clearer Each Time* to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners to share and get feedback on their work. Display prompts for feedback that students can use to help their partner strengthen and clarify their ideas. For example, "Your explanation tells me . . .", "Can you say more about why you . . . ?", and "A detail (or word) you could add is _____, because . . . ." Give students with 3–4 minutes to revise their initial draft based on feedback from their peers.
Design Principle(s): Optimise output (for explanation) Engagement: Internalise Self-Regulation. Provide a project checklist that chunks the various steps of this activity into a set of manageable tasks.
Supports accessibility for: Organisation; Attention

Student Task Statement

Work with your group to find the best plan for shipping the boxes of necklaces. Each member of your group should select a different type of flat-rate shipping box and answer the following questions. Recall that each jewellery box is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch, and that there are 270 jewellery boxes to be shipped.

For each type of flat-rate shipping box:

1. Find how many jewellery boxes can fit into the box. Explain or show how the jewellery boxes can be packed in the shipping box. Draw a sketch to show your thinking, if needed.

2. Calculate the total cost of shipping all 270 jewellery boxes in shipping boxes of that type. Show your reasoning and organise your work so it can be followed by others.

Student Response

1. Answers vary. Students should decide how they want to orient the jewellery boxes inside the mailing boxes. Then they can compute how many boxes will fit into the mailing box with that particular orientation. Often there will be a gap left where additional jewellery boxes will fit. Each mailing box allows for several different orientations.

   Small box: The jewellery box has dimension $\frac{7}{4} \times \frac{9}{4} \times \frac{3}{4}$ or $\frac{14}{8} \times \frac{18}{8} \times \frac{6}{8}$ and the small shipping box has dimension $\frac{43}{8} \times \frac{69}{8} \times \frac{13}{8}$. At most 18 (3 by 3 by 2) jewellery boxes can fit into the shipping box (other orientations would lead to fewer boxes).

   Medium box 1: Different orientations and results are possible. If the jewellery box is oriented with the $\frac{3}{4}$ inch side as the height and lining up the $1\frac{3}{4}$ inch side along the 11 inch side of the mailing box, then we can fit $6 \times 3 \times 7 = 126$ boxes. This will leave a gap in the back of the box where we can fit an additional $1 \times 3 \times 7 = 21$ boxes for a total of 147 boxes.

   Medium box 2: Different orientations and results are possible. If the jewellery box is oriented with the $\frac{3}{4}$ inch side as the height and lining up the $2\frac{1}{4}$ inch side along the $13\frac{5}{8}$ inch side of the mailing box, then we can fit $6 \times 6 \times 4 = 144$ boxes. This will leave a gap in the back of the box where we can fit an additional $1 \times 1 \times 7 = 7$ boxes for a total of 151 boxes.

   Large box: Different orientations should be considered. Orientation 1: If the jewellery box is oriented with the $\frac{3}{4}$ inch side as the height, then we can fit 234 boxes: $5 \times 6 \times 7 = 210$ oriented one way, and an additional $6 \times 2 \times 2 = 24$ oriented differently, squeezed in the gap left by the first set of boxes. Orientation 2: If the jewellery box is oriented with the $1\frac{3}{4}$ inch side as the height, then we can fit 252 boxes:

$16 \times 5 \times 3 = 240$ oriented one way, and an additional $6 \times 1 \times 2 = 12$ oriented differently, squeezed in the gap left by the first set of boxes.

- Small box: We would need 15 small boxes, for a total cost of £102.
- Medium boxes: We would need 2 medium boxes of either type, for a total cost of £26.90.
- Large box: We would need 2 large boxes, for a total cost of £37.50.

Activity Synthesis

Small-group and whole-class reflections will occur in the next activity.

17.3 Determining Shipping Costs (Part 3)

In the final phase of the shipping project, students present, reflect on, and revise their work within their small group. They discuss their decisions, evaluate the accuracy of their calculations, and then revise them as needed. In groups, they discuss which shipping box size, or combination of sizes, will be the most economical for shipping 270 jewellery boxes.

Instructional Routines

- Collect and Display
- Discussion Supports

Launch

Keep students in the same groups of 4. Give them 10–12 minutes to share each member's work in small groups and make revisions as needed. Use *Collect and Display* and refer to select student language while engaging in the whole group discussion. Display questions such as the following. Ask students to use them to guide their discussion.

- How many different ways can the jewellery boxes fit into each shipping box?
- How does the orientation of the jewellery boxes affect how they fit within the shipping boxes?
- Do some shipping boxes have more wasted space than others? Why?
- Can you use diagrams to show and compare the unused spaces in different configurations?
- Are there ways to reduce the amount of wasted space when shipping exactly 270 jewellery boxes?
- How does the orientation of the jewellery boxes affect the cost of shipping with each shipping box?

- Is there a way to increase the number of jewellery boxes that will fit into a shipping box? How?

Once each group member has had a chance to share individual work and before discussing this problem as a whole class, give students 4–5 minutes to decide on the best (least expensive) option for shipping 270 jewellery boxes and write down ideas for explaining their strategies.

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as "____ jewellery boxes can fit into one shipping box because . . . ."
Supports accessibility for: Language; Organisation

Student Task Statement

1. Share and discuss your work with the other members of your group. Your teacher will display questions to guide your discussion. Note the feedback from your group so you can use it to revise your work.

2. Using the feedback from your group, revise your work to improve its correctness, clarity, and accuracy. Correct any errors. You may also want to add notes or diagrams, or remove unnecessary information.

3. Which shipping boxes should the artist use? As a group, decide which boxes you recommend for shipping 270 jewellery boxes. Be prepared to share your reasoning.

Student Response

The cheapest option would be to use one small and one large shipping box. This would fit exactly 270 jewellery boxes for £25.55. However, using two medium-sized boxes would cost only slightly more at £26.90 and allows for roomier packing.

Activity Synthesis

After small groups have reached an agreement on shipping box recommendations, discuss as a class so students can see a variety of strategies for orientation and calculation. Depending on the time available, students could just stand and share, groups could create a gallery walk, or each group could present more formally. Select one group to present their findings about each shipping box. Select an additional 1–2 groups to share their recommended shipping box size, or combination of sizes, needed to ship all 270 jewellery boxes.

To help students tie everything together, consider discussing the following questions:

- "How did the choice of jewellery box orientation affect how many would fit into each shipping box?"

- "How did the quantity of jewellery boxes (270) affect the choice of shipping box size?"

- "How did you calculate how many jewellery boxes would fit in a box? Did you multiply the lengths of the jewellery boxes or divide the lengths of the shipping boxes?"
- "Did the size of fractions affect how you performed division? What methods did you use to divide?"
- "How did you confirm or check your calculations?"
- "If you had a chance to solve a similar problem, what might you do differently to improve the efficiency or accuracy of your work?"

Speaking: Discussion Supports. Use this routine to support whole-class discussion after each group presents their findings. Invite students to restate what they heard the group present using mathematical language. Consider providing students time to restate what they heard to a partner, before selecting one or two students to share with the class. This will provide additional opportunities for all students to speak.
Design Principle(s): Support sense-making; Maximise meta-awareness

Lesson Synthesis

The bulk of students' reflection about the mathematics of the unit should happen in the last task of the lesson. To wrap up this culminating lesson, consider highlighting instances of mathematical modelling in the lesson by asking questions such as:

- "When did you have to make assumptions to make the problem solving possible or more manageable? What assumptions did you make?" (Possible responses: When deciding which shipping boxes to use, I assumed that only one type of shipping box would be used. When deciding how many jewellery boxes would fit into a shipping box, I assumed that no bubble wraps or other packing materials were needed. When deciding how to pack the jewellery boxes, I assumed that using the same orientation within a box was preferable.)
- "Was there any missing information you had to find out before you could proceed?"
- "Were there times when you had to change course or strategy because the approach you had chosen was not productive?"