Lesson 21: Combining like terms (Part 2)

Goals

- Critique (in writing) methods for generating equivalent expressions with fewer terms.
- Generate expressions that are not equivalent, but differ only in the placement of brackets, and justify (orally) that they are not equivalent.
- Write expressions with fewer terms that are equivalent to a given expression that includes negative coefficients and brackets.

Learning Targets

- I am aware of some common pitfalls when writing equivalent expressions, and I can avoid them.
- When possible, I can write an equivalent expression that has fewer terms.

Lesson Narrative

In this lesson, students are still working toward gaining fluency in writing equivalent expressions. The goal of this lesson is to highlight a particular common error: mishandling the subtraction in an expression like $8 - 3(4 + 9x)$. To this end, students first analyse and explain the error in several incorrect ways of rewriting this expression. Then, they consider the effect of inserting brackets in different places in an expression with four terms.

Building On

- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no brackets to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Addressing

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Building Towards

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports
- Think Pair Share
- True or False

Required Materials

Index cards

Required Preparation

Access to index cards is suggested for students who need help isolating one expression at a time. They can use the index card to cover up nearby expressions.

Student Learning Goals

Let's see how to use properties correctly to write equivalent expressions.

21.1 True or False?

Warm Up: 10 minutes

In this warm-up, students consider some correct and incorrect numerical examples of expanding brackets when the expression is being subtracted. A main goal of this lesson is to help students to understand how to write equivalent expressions that contain variables. By first looking at numbers only, students have a way to tell if the expressions are equivalent by evaluating each side.

Look for students who evaluate each side and students who reason about operations and properties.

Instructional Routines

- Think Pair Share
- True or False

Launch

Tell students that their job is to consider four equations and decide which of them are true.

The amount of numbers and symbols presented relatively close together might present a challenge. It is important that students think of how the different equations compare to each other, but they also need to consider them one at a time. Provide access to index cards, so that, for example, students can cover up questions 2, 3, and 4 while considering question 1.

Arrange students in groups of 2. Give them 3 minutes of quiet work time and time to share their thoughts with a partner, followed by whole-class discussion.

Student Task Statement

Select **all** the statements that are true. Be prepared to explain your reasoning.

1. $4 - 2(3 + 7) = 4 - 2 \times 3 - 2 \times 7$
2. $4 - 2(3 + 7) = 4 + \text{-}2 \times 3 + \text{-}2 \times 7$
3. $4 - 2(3 + 7) = 4 - 2 \times 3 + 2 \times 7$
4. $4 - 2(3 + 7) = 4 - (2 \times 3 + 2 \times 7)$

Student Response

Sample reasoning:

1. true, because the 2 must be multiplied by both 3 and 7 because of the distributive property, and each product must be subtracted since the product of 2 and $(3 + 7)$ is subtracted
2. true, because subtracting 2 is the same as adding -2, and then the distributive property is applied
3. false, because 2×7 needs to be subtracted
4. true, because the 2 is distributed but the brackets indicate that the result of $2(3 + 7)$ is subtracted

Activity Synthesis

Ask students to explain why the true statements are true. Select students who reason by evaluating each side, and also students who reason using properties. For example, statement 2 is true because subtracting 2 is the same as adding negative 2, and then the distributive property is applied.

Then, spend some time on why statement 3 is false. First, we can tell it's false because when each side is evaluated, we get $\text{-}16 = 12$. The order of operations is just a convention, but we need to all follow one convention so that we can communicate mathematically. When the order of operations is followed on the left side, the result of $2(3 + 7)$ is subtracted from 4. However on the right side of statement 3, only the 2×3 is being subtracted, and the 2×7 is being added.

21.2 Seeing it Differently

10 minutes

In this activity, students encounter typical errors with directed numbers, operations, and properties. They are tasked with identifying which strategies are correct and for those that are not, describing the error that was made.

Instructional Routines

- Clarify, Critique, Correct

Launch

Ensure students understand the task: first they decide whether they agree with each person's strategy, but they also need to describe the errors that were made. Give 5 minutes quiet work time followed by a whole-class discussion.

Representation: Internalise Comprehension. Begin by providing students with rules for adding, subtracting, and multiplying directed numbers. Invite students to share their prior knowledge and ideas using simple examples.
Supports accessibility for: Conceptual processing Speaking, Writing: Clarify, Critique, Correct. This activity provides students with the opportunity to improve upon the written work of another by correcting errors and clarifying meaning. Ask students to select one of the errors they notice, and to produce a written explanation, intended for the student who made the error (Noah, Lin, Jada, or Andre), that describes the error that was made, and how to fix it. Give students 3–5 minutes to complete a first draft before they read their writing to a partner. Provide students with prompts they can use to give each other feedback such as “Can you say that another way?” or “Can you try to explain this using an example?” This will provide students with an additional opportunity to produce language related to writing equivalent expressions.
Design Principle(s): Optimise output (for explanation)

Student Task Statement

Some students are trying to write an expression with fewer terms that is equivalent to $8 - 3(4 - 9x)$.

Noah says, “I worked the problem from left to right and ended up with $20 - 45x$.”

$8 - 3(4 - 9x)$

$5(4 - 9x)$

$20 - 45x$

Lin says, “I started inside the brackets and ended up with $23x$.”

$8 - 3(4 - 9x)$

$8 - 3(\text{-}5x)$

$8 + 15x$

$23x$

Jada says, "I used the distributive property and ended up with $27x - 4$."

$8 - 3(4 - 9x)$

$8 - (12 - 27x)$

$8 - 12 - (\text{-}27x)$

$27x - 4$

Andre says, "I also used the distributive property, but I ended up with -4 $- 27x$."

$8 - 3(4 - 9x)$

$8 - 12 - 27x$

-4 $- 27x$

1. Do you agree with any of them? Explain your reasoning.

2. For each strategy that you disagree with, find and describe the errors.

Student Response

1. Answers vary. Sample response: I agree with Jada because I tried some values of x and Jada's expression always evaluates to the same number as the original expression.

2. Answers vary. Sample response: Noah subtracted before multiplying. Lin combined 4 and $9x$. Andre multiplied -3 and -9 and got -27.

Are You Ready for More?

1. Jada's neighbour said, "My age is the difference between twice my age in 4 years and twice my age 4 years ago." How old is Jada's neighbour?

2. Another neighbour said, "My age is the difference between twice my age in 5 years and twice my age 5 years ago." How old is this neighbour?

3. A third neighbour had the same claim for 17 years from now and 17 years ago, and a fourth for 21 years. Determine those neighbours' ages.

Student Response

1. 16. An expression for the neighbour's age is $2(a + 4) - 2(a - 4)$ or $2a + 8 - 2a + 8$ which is 16.

2. 20. An expression for this neighbour's age is $2(a + 5) - 2(a - 5)$ or $2a + 10 - 2a + 10 = 20$.

3. 68 and 84. The expression is always twice the number of years + twice the number of years, or $4y$ where y is the number of years: $4(17) = 68$, $4(21)$ = 84.

Activity Synthesis

Ask students, "Which way do you see it?" Earlier in KS3, students learned that equivalent expressions meant two expressions had to be equal for any value of the variable. Since each student's work contains at least two steps, the steps can be used to identify where the error occurs—where the expressions are no longer equal for a value of the variable.

A handy approach is to rewrite subtraction operations as adding the opposite. If desired, demonstrate such an approach for this expression along with a box to organise the work of multiplying -3$(4 - 9x)$:

	4	-9x
-3		

	4	-9x
-3	-3×4	-3×-9x

	4	-9x
-3	-12	27x

$8 - 3(4 - 9x)$
$8 +$ (-3)$(4 +$ (-9x))
$8 +$ (-3)(4) $+$ (-3)(-9x)
$8 +$ (-12) $+ 27x$
-4 $+ 27x$

21.3 Grouping Differently

15 minutes

In this activity students continue the work of generating equivalent expressions as they decide where to place a set of brackets and explore how that placement affects the expressions.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Tell students to first complete both questions independently. Then, trade one of their expressions with their partner. The partner's job is to decide whether the new expression is equivalent to the original or not, and explain how they know.

Anticipated Misconceptions

Students may not realise that they can break up a term and place brackets, for example, between the 8 and x in the term $8x$. Clarify that they may place the brackets anywhere in the expression.

Student Task Statement

Diego was taking a maths quiz. There was a question on the quiz that had the expression $8x - 9 - 12x + 5$. Diego's teacher told the class there was a typo and the expression was supposed to have one set of brackets in it.

1. Where could you put brackets in $8x - 9 - 12x + 5$ to make a new expression that is still equivalent to the original expression? How do you know that your new expression is equivalent?

2. Where could you put brackets in $8x - 9 - 12x + 5$ to make a new expression that is not equivalent to the original expression? List as many different answers as you can.

Student Response

Answers vary. Sample responses:

1. $(8x - 9 - 12x + 5)$, $(8x - 9) - 12x + 5$

2. $8x - 9 - (12x + 5) = -4x - 14$ $\quad$ $8x - (9 - 12x) + 5 = 20x - 4$
   $8x - (9 - 12x + 5) = 20x - 14$ $\quad$ $8(x - 9 - 12x + 5) = -88x - 32$
   $8(x - 9 - 12x) + 5 = -88x - 67$ $\quad$ $8(x - 9) - 12x + 5 = -4x - 67$

Activity Synthesis

Questions for discussion:

- "How can you write the original expression in different ways with fewer terms?"
  ($-4x - 4$, $-4(x + 1)$, $4(-x - 1)$)

- "Where did you place the brackets to create an equivalent expression?"

- "Share other ways you placed brackets and the resulting expression with fewer terms."

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: parenthesis. The display should include examples of how to create equivalent expressions using brackets by breaking up the components. Be sure to emphasise how the placement of the brackets affects the expression.
Supports accessibility for: Memory; Language Speaking, Writing: Discussion Supports. When students interpret the new expression they receive, ask students to press their partner for more details in their explanations by challenging an idea, or asking for an example. Listen for, and amplify language students use to describe how the brackets influence the resulting expression with fewer terms. This will help call students' attention to the types of details and language to look for to determine if an expression is equivalent to another or not.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

Display the expression $5 - 2(3x - x)$. Ask students to think of a mistake someone would be likely to make when trying to write an expression that is equivalent to this one. Select a student to share, and then ask if anyone can think of a different likely mistake. Continue until each of these common errors arises:

- Subtracting 2 from 5 first, resulting in $3(3x - x)$
- Distributing positive 2, resulting in $5 - 6x - 2x$
- Thinking that $3x - x$ is 3, resulting in $5 - 2(3)$

Then, ask students for strategies for preventing these errors. Reliable properties to use are: rewriting subtraction as adding the opposite, the commutative property of multiplication and addition, and the distributive property. Suggest this way of rewriting this example:

$5 - 2(3x - x)$

$5 + \text{-}2(3x + \text{-}x)$

$5 + \text{-}2 \times 3x + \text{-}2 \times \text{-}x$

$5 + \text{-}6x + 2x$

$5 + x(\text{-}6 + 2)$

$5 + \text{-}4x$ or $5 - 4x$

21.4 How Many Are Equivalent?

Cool Down: 5 minutes

Student Task Statement

Select **all** the expressions that are equivalent to $16x - 12 - 24x + 4$. Explain or show your reasoning.

1. $4 + 16x - 12(1 + 2x)$

2. $40x - 16$

3. $16x - 24x - 4 + 12$

4. $-8x - 8$

5. $10(1.6x - 1.2 - 2.4x + 4)$

Student Response

1, 4

Student Lesson Summary

Combining like terms allows us to write expressions more simply with fewer terms. But it can sometimes be tricky with long expressions, brackets, and negatives. It is helpful to think about some common errors that we can be aware of and try to avoid:

- $6x - x$ is not equivalent to 6. While it might be tempting to think that subtracting x makes the x disappear, the expression is really saying take 1 x away from 6 x's, and the distributive property tells us that $6x - x$ is equivalent to $(6 - 1)x$.

- $7 - 2x$ is not equivalent to $5x$. The expression $7 - 2x$ tells us to double an unknown amount and subtract it from 7. This is not always the same as taking 5 copies of the unknown.

- $7 - 4(x + 2)$ is not equivalent to $3(x + 2)$. The expression tells us to subtract 4 copies of an amount from 7, not to take $(7 - 4)$ copies of the amount.

If we think about the meaning and properties of operations when we take steps to rewrite expressions, we can be sure we are getting equivalent expressions and are not changing their value in the process.

Lesson 21 Practice Problems

1. **Problem 1 Statement**

   - Noah says that $9x - 2x + 4x$ is equivalent to $3x$, because the subtraction sign tells us to subtract everything that comes after $9x$.

   - Elena says that $9x - 2x + 4x$ is equivalent to $11x$, because the subtraction only applies to $2x$.

   Do you agree with either of them? Explain your reasoning.

Solution

Elena is correct. Rewriting addition as subtraction gives us $9x + \text{-}2x + 4x$, which shows that the subtraction symbol in front of the $2x$ applies only to the $2x$ and not to the terms that come after it.

2. **Problem 2 Statement**

Identify the error in generating an expression equivalent to $4 + 2x - \frac{1}{2}(10 - 4x)$. Then correct the error.

$4 + 2x + \frac{-1}{2}(10 + (\text{-}4x)) = 4 + 2x + (\text{-}5) + 2x = 4 + 2x - 5 + 2x = \text{-}1$

Solution

The error is in the last step. The second $2x$ was subtracted instead of being added. This would be correct if there were brackets around $5 + 2x$. The last step should be $4x - 1$.

3. **Problem 3 Statement**

Select **all** expressions that are equivalent to $5x - 15 - 20x + 10$.

a. $5x - (15 + 20x) + 10$

b. $5x + \text{-}15 + \text{-}20x + 10$

c. $5(x - 3 - 4x + 2)$

d. $\text{-}5(\text{-}x + 3 + 4x + \text{-}2)$

e. $\text{-}15x - 5$

f. $\text{-}5(3x + 1)$

g. $\text{-}15(x - \frac{1}{3})$

Solution ["A", "B", "C", "D", "E", "F"]

4. **Problem 4 Statement**

The school marching band has a budget of up to £750 to cover 15 new uniforms and competition fees that total £300. How much can they spend for one uniform?

a. Write an inequality to represent this situation.

b. Solve the inequality and describe what it means in the situation.

Solution

a. $15x + 300 \leq 750$

b. $x \leq 30$. They can spend at most £30 on each uniform.

5. **Problem 5 Statement**

Solve the inequality that represents each story. Then interpret what the solution means in the story.

a. For every £9 that Elena earns, she gives x pounds to charity. This happens 7 times this month. Elena wants to be sure she keeps at least £42 from this month's earnings. $7(9 - x) \geq 42$

b. Lin buys a candle that is 9 inches tall and burns down x inches per minute. She wants to let the candle burn for 7 minutes until it is less than 6 inches tall. $9 - 7x < 6$

Solution

a. $x \leq 3$. Elena can give £3 or less to charity for every £9 she earns.

b. $x > \frac{3}{7}$. The candle needs to burn down more than $\frac{3}{7}$ inch each minute.

6. **Problem 6 Statement**

A certain shade of blue paint is made by mixing $1\frac{1}{2}$ litres of blue paint with 5 litres of white paint. If you need a total of 65 litres of this shade of blue paint, how much of each colour should you mix?

Solution

You should mix 15 litres of blue paint with 50 litres of white paint.