

Lesson Plan: Estimating Pi with Archery - The Monte Carlo Method

Objective: Students will use a simulation of shooting arrows at a target to estimate the value of pi, applying the Monte Carlo method. This activity will help students understand the application of probability, the law of large numbers, and statistical estimation in mathematics.

Grade Level: IB Diploma Programme Mathematics Applications and Interpretation SL/HL and Analysis and Approaches SL/HL.

Duration: 60 minutes

Warm-up Activity (5 minutes)

- Inquiry Question: How can random experiments help us understand deterministic quantities?
- Activity: Discuss the concept of using probability to estimate fixed values, introducing the idea of the Monte Carlo method as a bridge between randomness and determinism.

Introduction to the Scenario (10 minutes)

- Presentation: Introduce the scenario of the Archery Analytics Adventure in the kingdom of Numeria, where the goal is to estimate pi through archery.
- Objective: Explain that students will simulate this process using a cognitive activator applet, linking the activity to the mathematical concept of pi, areas, and probabilities.

Simulating the Shots (15 minutes)

- Group Activity: Students will use the applet to simulate shooting 100 and then 1000 arrows at a target, recording the number of hits and misses.
- Data Collection: Emphasize the importance of accurate record-keeping and how this simulation mirrors the process of gathering empirical data.

Understanding the Math (10 minutes)

- Exploration: Dive into the mathematical foundation of the simulation, explaining how the target represents a quarter circle within a square and how the ratio of hits to total shots relates to the area of the circle and thus to pi.
- Discussion: Facilitate a conversation on how this process uses random sampling to estimate a precise mathematical constant.

Estimating Pi (10 minutes)

- Calculation: Guide students through using their data to estimate pi, highlighting the role of the law of large numbers in refining their estimates as the number of arrows increases.

- Analysis: Discuss the implications of their findings and the potential for variance in their results.

Sharing Findings and Reflecting on the Process (5 minutes)

- Presentation: Allow groups to share their methods, results, and any observations about the distribution of their arrows.
- Reflection: Engage in a discussion about the importance of π , the utility of statistical methods in understanding mathematical constants, and the real-world applications of such techniques.

Questions for Investigation and Homework (5 minutes)

- Homework Assignment: Assign questions for further investigation, such as exploring the impact of changing simulation conditions or the reliability of the Monte Carlo method compared to traditional geometric methods.
- Extension Activity: Encourage students to experiment with variations of the simulation to explore concepts like the law of large numbers and statistical variance further.

Materials Needed:

- Computers with internet access for the simulation applet
- Calculators for analysis
- Notebooks for data collection and calculations

Assessment:

- Formative Assessment: Participation in simulation activities and group discussions.
- Summative Assessment: Analysis and presentation of findings, as well as responses to the assigned homework questions.