Mathematics Education Innovation

Lesson 11: Representing small numbers on the number line

Goals

- Coordinate (orally and in writing) decimals and multiples of powers of 10 representing the same small number.
- Use number lines to represent (orally and in writing) small numbers as multiples of powers of 10 with negative exponents.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

Lesson Narrative

Previously, students used the number line and positive exponents to explore very large numbers. In this lesson, they use the number line and negative exponents to explore very small numbers. Students create viable arguments and critique the reasoning of others when discussing how to represent powers of 10 with negative exponents on a number line. They attend to precision when deciding how to label the powers of 10 on the number line and how to plot numbers correctly.

Addressing

- Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

Student Learning Goals

Let's visualise small numbers on the number line using powers of 10.

11.1 Small Numbers on a Number Line

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about expressions with negative exponents on a number line. Students explore a common misunderstanding about negative exponents that is helpful to address before standard form is used to describe very small numbers.

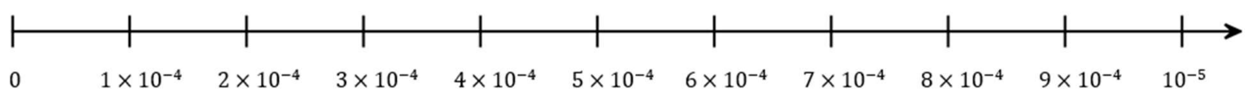
For students' reference, consider displaying a number line from a previous lesson that shows powers of 10 on a number line.

Launch

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Kiran drew this number line.



Andre said, "That doesn't look right to me."

Explain why Kiran is correct or explain how he can fix the number line.

Student Response

Answers vary. Sample response: Change each instance of 10^{-4} into 10^{-6} or change 10^{-5} to 10^{-3}.

Activity Synthesis

The important idea to highlight during the discussion is that the larger the size (or absolute value) of a negative exponent, the closer the value of the expression is to zero. This is because the negative exponent indicates the number of factors that are $\frac{1}{10}$. For example, 10^{-5} represents 5 factors that are $\frac{1}{10}$ and 10^{-6} represents 6 factors that are $\frac{1}{10}$, so 10^{-6} is 10 times smaller than 10^{-5}.

Ask one or more students to explain whether they think the number line is correct and ask for their reasoning. Record and display their reasoning for all to see, preferably on the number line. If possible, show at least two correct ways the number line can be fixed.

11.2 Comparing Small Numbers on a Number Line

10 minutes

This task is analogous to a previous activity with positive exponents. The number line strongly encourages students to think about how to change expressions so they all take the form $b \times 10^k$, where b is between 1 and 10, as in the case of standard form. The number line also is a useful representation to show that, for example, 29×10^{-7} is about half as much as 6×10^{-6}. The last two questions take such a comparison a step further, asking students to estimate relative sizes using numbers expressed with powers of 10.

As students work, look for different strategies they use to compare expressions that are not written as a product of a number and 10^{-6}. Also look for students who can explain how they estimated in the last two problems. Select them to share their strategies later.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time, followed by partner discussion and whole-class discussion. During partner discussion, ask students to share their responses for the first two questions and reach an agreement about where the numbers should be placed on the number line.

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the visual display of rules for exponents.
Supports accessibility for: Memory; Language Speaking: Discussion Supports. Display sentence frames to support students as they describe how they compared and estimated the difference in magnitude between the pairs of values 29×10^{-7} and 6×10^{-6} or 7×10^{-8} and 3×10^{-9}. For example, "First I ____ , then I ____ ." or "_____ is (bigger/smaller) because ______ ."
Design Principle(s): Optimise output (for explanation)

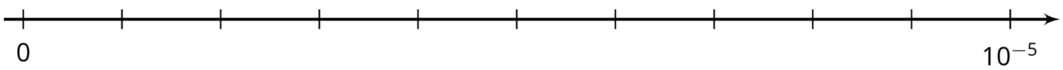
Anticipated Misconceptions

Students may have trouble comparing negative powers of 10. Remind these students that, for example, 10^{-5} is 5 factors that are $\frac{1}{10}$ and 10^{-6} is 6 factors that are $\frac{1}{10}$, so 10^{-5} is 10 times larger than 10^{-6}.

Students may also have trouble estimating how many times one larger expression is than another. Offer these students an example to illustrate how representing numbers as a single digit times a power of 10 is useful for making rough estimations. We have that 9×10^{-12} is roughly 50 times as much as 2×10^{-13} because 10^{-12} is 10 times as much as

10^{-13} and 9 is roughly 5 times as much as 2. In other words, $\frac{9\times10^{-12}}{2\times10^{-13}} \approx \frac{10\times10^{-12}}{2\times10^{-13}} = 5 \times 10^{-12-(-13)} = 5 \times 10^{1} = 50$

Student Task Statement



1. Label the tick marks on the number line.
2. Plot the following numbers on the number line:

   A. 6×10^{-6}

   B. 6×10^{-7}

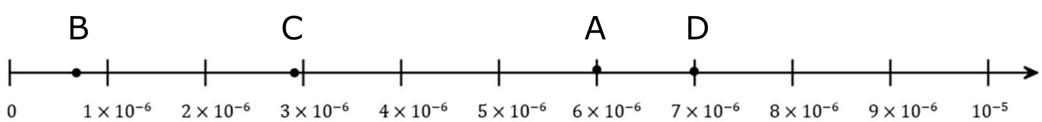
   C. 29×10^{-7}

   D. $(0.7) \times 10^{-5}$

3. Which is larger, 29×10^{-7} or 6×10^{-6}? Estimate how many times larger.
4. Which is larger, 7×10^{-8} or 3×10^{-9}? Estimate how many times larger.

Student Response

1. The tick marks should be labelled as increasing multiples of 10^{-6}.
2.



3. 6×10^{-6} is about twice as large as 29×10^{-7} because $29 \times 10^{-7} = (2.9) \times 10^{-6}$, which is roughly 3×10^{-6}.
4. 7×10^{-8} is roughly 20 times as large as 3×10^{-9} because 7 is roughly twice as much as 3, and 10^{-8} is 10 times as much as 10^{-9}.

Activity Synthesis

Select previously identified students to explain how they used the number line and powers of 10 to compare the numbers in the last two problems.

One important concept is that it's always possible to change an expression that is a multiple of a power of 10 so that the leading factor is between 1 and 10. For example, we can think of 29×10^{-7} as $(2.9) \times 10 \times 10^{-7}$ or $(2.9) \times 10^{-6}$. Another important concept is that powers of 10 can be used to make rough estimates. Make sure these ideas are uncovered during discussion.

11.3 Atomic Scale

20 minutes

Students convert a decimal to a multiple of a power of 10 and plot it on a number line. The first problem leads to a product of an integer and a power of 10, and the second leads to a product of a decimal and a power of 10. It is difficult to fit the numbers on the number line without using standard form. Again, students build experience with standard form before the term is formally introduced.

As students work, notice those who connect the number of decimal places to negative powers of 10. For example, they might notice that counting decimal places to the right of the decimal point corresponds to multiplying by $\frac{1}{10}$ a certain number of times.

Instructional Routines

- Clarify, Critique, Correct

Launch

This is the first time students convert small decimals into a multiple of a power of 10 with a negative exponent. Before students begin the activity, review the idea that a decimal can be thought of as a product of a number and $\frac{1}{10}$. Explain, for example, that 0.3 is $3 \times \frac{1}{10}$ (3 tenths), 0.03 is $3 \times \frac{1}{10} \times \frac{1}{10}$ (three hundredths). Similarly, 0.0003 is 3 multiplied by $\frac{1}{10}$ 4 times (3 ten-thousandths). So $(0.0003) = 3 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 3 \times 10^{-4}$.

Arrange students in groups of 2. Give students 10 minutes to work, followed by whole-class discussion. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

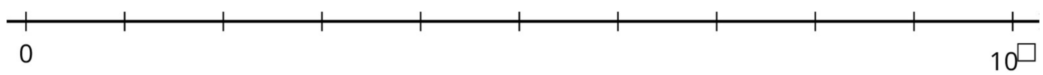
Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support use of structure. For example, check in with students within the first 2-3 minutes of work time. Ask students to share how they decide what power of 10 to put on the right side of this number line.
Supports accessibility for: Visual-spatial processing; Organisation Writing: Clarify, Critique, Correct. Display the incomplete statement: "I just count how many places and write the number in the exponent." Prompt discussion by asking, "What is unclear?" or "What do you think the author is trying to say?" Then, ask students to write a more precise version to explain the strategy of converting. Improved statements should include reference to the relationship between the number of decimal places to the right of the decimal point and repeated multiplication of $\frac{1}{10}$. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Optimise output (for explanation); Support sense-making

Anticipated Misconceptions

For the mass of the proton, students might find that it is equal to 17×10^{-25}, but 17 does not fit on the number line because there are not 17 tick marks. Ask students whether 1.7 would fit on the number line
(it does). Follow up by asking how to replace 17 with 1.7 times something $(17 = (1.7) \times 10)$. So $17 \times 10^{-25} = (1.7) \times 10 \times 10^{-25} = (1.7) \times 10^{-24}$.

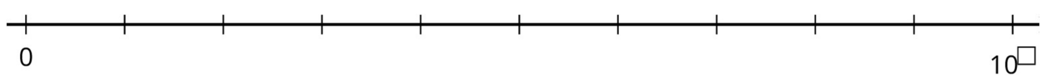
Student Task Statement

1. The radius of an electron is about 0.0000000000003 cm.
   a. Write this number as a multiple of a power of 10.
   b. Decide what power of 10 to put on the right side of this number line and label it.
   c. Label each tick mark as a multiple of a power of 10.



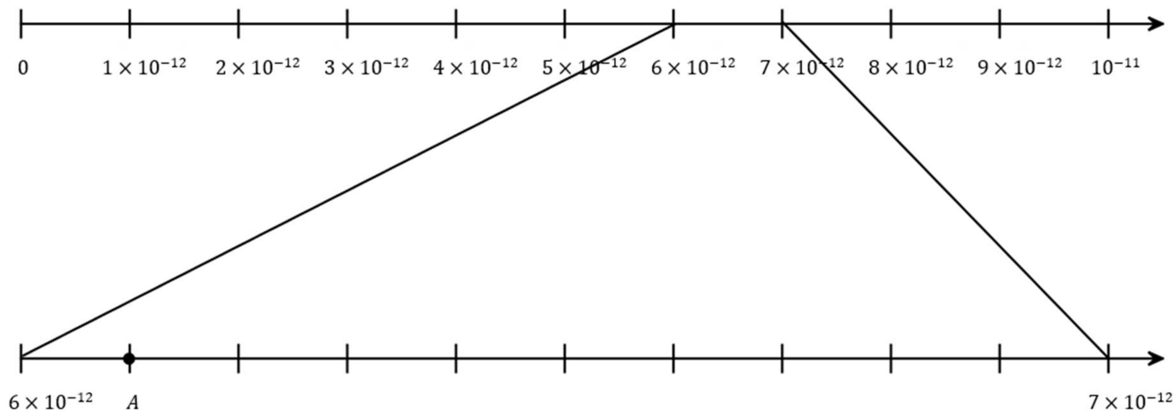
   d. Plot the radius of the electron in centimetres on the number line.

2. The mass of a proton is about 0.0000000000000000000000017 grams.
   a. Write this number as a multiple of a power of 10.
   b. Decide what power of 10 to put on the right side of this number line and label it.
   c. Label each tick mark as a multiple of a power of 10.



   d. Plot the mass of the proton in grams on the number line.

3. Point A on the zoomed-in number line describes the wavelength of a certain X-ray in metres.



a. Write the wavelength of the X-ray as a multiple of a power of 10.

b. Write the wavelength of the X-ray as a decimal.

Student Response

1. Radius of electron:

    a. Using powers of 10, the radius of the electron is 3×10^{-13} cm.

    b. The power on the right side should be 10^{-12}, because 3×10^{-13} is greater than 10^{-13} but less than 10^{-12}.

    c. The tick marks should be labelled in multiples of 10^{-13}.

    d. The radius of the electron in cm should be placed at the 3rd tick mark.

2. Mass of proton:

    a. Using powers of 10, the mass of the proton is $(1.7) \times 10^{-24}$ grams.

    b. The power on the right side should be 10^{-23}, because $(1.7) \times 10^{-24}$ is greater than 10^{-24} but less than 10^{-23}.

    c. The tick marks should be labelled in multiples of 10^{-24}.

    d. The mass of the proton in grams should be placed between the 1st and 2nd tick marks, closer to the 2nd than the 1st.

3.

    a. The length of the X-ray's wavelength is $(6.1) \times 10^{-12}$ metres.

    b. The length of the X-ray's wavelength is 0.0000000000061 metres.

Activity Synthesis

One key idea is for students to convert small decimals to standard form in the process of placing them on a number line. Select a student to summarise how they wrote the mass of the proton as a multiple of a power of 10. Poll the class on whether they agree or disagree and why. The discussion should lead to one or more methods to rewrite a decimal as a multiple of a power of 10. For example, students might count the decimal places to the right of the decimal point and recognise that number as the number of factors that are $\frac{1}{10}$. With this method, 0.0000003, for example, would equal 3×10^{-7} because 3 has been multiplied by $\frac{1}{10}$ seven times.

Lesson Synthesis

The purpose of the discussion is to check that students know how to convert between decimal numbers and numbers expressed as multiples of powers of 10, and that they understand the order of numbers with negative exponents on the number line.

Some questions for discussion:

- "As we move to the right on the number line, what happens to the value of the numbers we encounter?" (They get larger.)
- "Would 10^{-5} appear to the left or to the right of 10^{-4} on a number line? Explain." (10^{-5} is smaller than 10^{-4}, so it would be to the left.)
- "How does zooming in on the number line help express numbers between the tick marks?" (Zooming in allows us to subdivide the distance between two tick marks into 10 equal intervals, which allows us to describe a number to an additional decimal place.)
- "Describe how to convert a number such as 0.000278 into a multiple of a power of 10." (The number is equivalent to $278 \times (0.000001)$ or $278 \times \frac{1}{100\,000}$. The fraction $\frac{1}{100\,000}$ is $\frac{1}{10^6}$ or 10^{-6}, so 0.000278 can be written as 278×10^{-6}.)

If time allows, give students other small numbers that are written as decimals and ask them to write them as multiples of powers of 10, and vice versa.

11.4 Describing Very Small Numbers

Cool Down: 5 minutes

Student Task Statement

1. Write 0.00034 as a multiple of a power of 10.
2. Write $(5.64) \times 10^{-7}$ as a decimal.

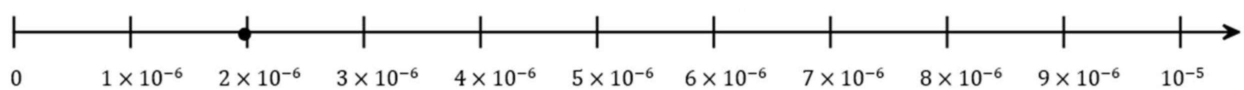
Student Response

1. $(3.4) \times 10^{-4}$, 34×10^{-5}, or equivalent.

2. 0.000000564

Student Lesson Summary

The width of a bacterium cell is about 2×10^{-6} metres. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that 2×10^{-6} is a multiple of 10^{-6}. So our number line will be labelled with multiples of 10^{-6}



Note that the right side is labelled $10 \times 10^{-6} = 10^{-5}$

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

Lesson 11 Practice Problems

1. **Problem 1 Statement**

   Select **all** the expressions that are equal to 4×10^{-3}:

   a. $4 \times \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right)$

   b. $4 \times (-10) \times (-10) \times (-10)$

   c. 4×0.001

   d. 4×0.0001

   e. 0.004

   f. 0.0004

   Solution ["A", "C", "E"]

2. **Problem 2 Statement**

   Write each expression as a multiple of a power of 10:

   a. 0.04

   b. 0.072

   c. 0.0000325

   d. Three thousandths

e. 23 hundredths

f. 729 thousandths

g. 41 millionths

Solution

a. Answers vary. Sample response: 4×10^{-2}

b. Answers vary. Sample response: 7.2×10^{-2}, 72×10^{-3}

c. Answers vary. Sample responses: 3.25×10^{-5}, 325×10^{-7}

d. Answers vary. Sample response: 3×10^{-3}

e. Answers vary. Sample responses: 2.3×10^{-1}, 23×10^{-2}

f. Answers vary. Sample responses: 7.29×10^{-1}, 729×10^{-3}

g. Answers vary. Sample responses: 4.1×10^{-5}, 41×10^{-6}

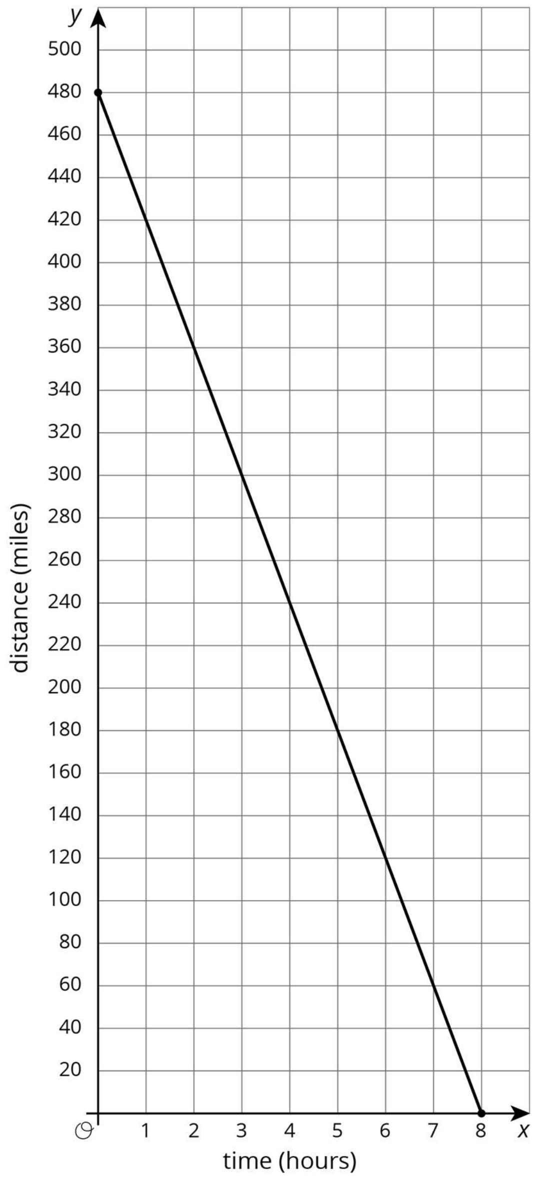
3. **Problem 3 Statement**

A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.

a. How fast are they travelling?

b. Is the gradient positive or negative? Explain how you know and why that fits the situation.

c. How far is the trip and how long did it take? Explain how you know.



Solution

a. 60 miles per hour

b. Negative. Explanations vary. Sample explanation: The gradient is negative because the line moves down toward the right. It shows the change in remaining miles for each hour. There are 60 fewer miles remaining each hour, which means the car is travelling at a steady rate of 60 miles each hour.

c. 480 miles and 8 hours. Explanations vary. Sample explanation: The trip is 480 miles because the remaining distance was 480 miles when they started out (after 0 hours). The trip took 8 hours because after 8 hours, there were 0 miles remaining.