Mathematics Education Innovation

Lesson 16: When Is the same size not the same size?

Goals

- Apply ratios and Pythagoras' theorem to solve a problem involving the aspect ratio of screens or photos, and explain (orally) the reasoning.
- Describe (in writing and using other representations) characteristics of rectangles with the same aspect ratio or with different aspect ratios.

Learning Targets

- I can apply what I have learned about Pythagoras' theorem to solve a more complicated problem.
- I can decide what information I need to know to be able to solve a real-world problem using Pythagoras' theorem.

Lesson Narrative

Before 2017, smartphones by major manufacturers all had screens with a 16: 9 aspect ratio. In 2017, two major brands released phones with screens in an 18.5: 9 aspect ratio. However they still reported their screen size using the same diagonal length of the earlier phones, 5.8 inches. How did the change in aspect ratio affect the screen size, if at all?

There is an element of mathematical modelling in the last activity, because in order to quantify the screens' sizes to compare them, students need to refine the question that is asked.

Building On

- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2: 1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

Addressing

- Apply Pythagoras' theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Instructional Routines

- Three Reads

Required Materials
Scientific calculators

Student Learning Goals
- Let’s figure out how aspect ratio affects screen area.

16.1 Three Figures

Warm Up: 5 minutes

The purpose of this activity is to notice that rectangles can have the same diagonal length but different areas. The concept of aspect ratio is also introduced in the activity synthesis.

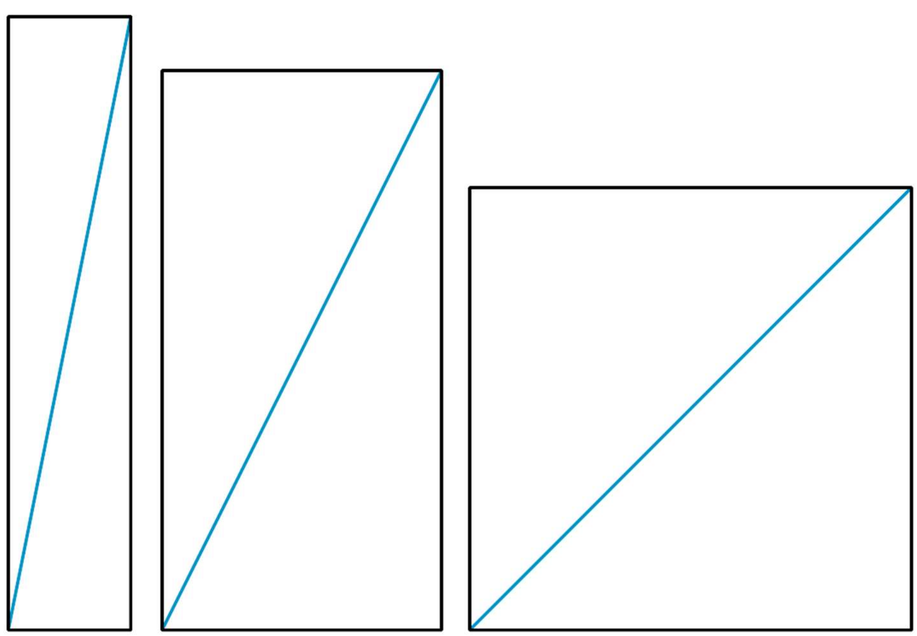
Monitor for students checking whether the diagonals are all the same length by using a ruler, compass, or edge of a piece of paper to compare their lengths. Also monitor for students mentioning that the rectangles have different areas.

Launch

Provide access to rulers, or suggest students use the edge of a blank piece of paper if they would like to compare lengths.

Student Task Statement

How are these shapes the same? How are they different?



Student Response

They are the same because they are all rectangles. Also, their diagonals are all the same length. They are different because they have different areas and aspect ratios. (If students mention the aspect ratio, they are likely to use informal language.)

Activity Synthesis

Ask selected students to share their observations. Ensure that these ideas are mentioned:

- The diagonals of the rectangles are all the same length.
- The rectangles have different areas. The leftmost rectangle has a relatively small area compared to the others, and the rightmost rectangle has a relatively large area.
- Each rectangle has a different ratio of height to base. Students might mention the slope of the diagonals, which is related to this idea.

Tell students that in photography, film, and some consumer electronics with a screen, the ratio of the two sides of a rectangle is often called its *aspect ratio*. In the rectangles in this activity, the aspect ratios are $5:1$, $2:1$, and $1:1$.

Demonstrate how the length of one side is a multiple of the other, on each rectangle. Students may be familiar with selecting an aspect ratio when taking or editing photos. Some common aspect ratios for photos are $1:1$, $4:3$, and $16:9$. Also, from ordering school pictures, 5 by 7 and 8 by 10 may be common sizes they've heard of.

16.2 A $4:3$ Rectangle

20 minutes

The purpose of this activity is to really understand what an aspect ratio means when one side is not a multiple of the other, and to think about how you can figure out the side lengths if you know the rectangle's aspect ratio and some other information. This problem is a simpler version of the type of work needed for the more complicated activity that follows.

Launch

Provide access to calculators that can take the square root of a number.

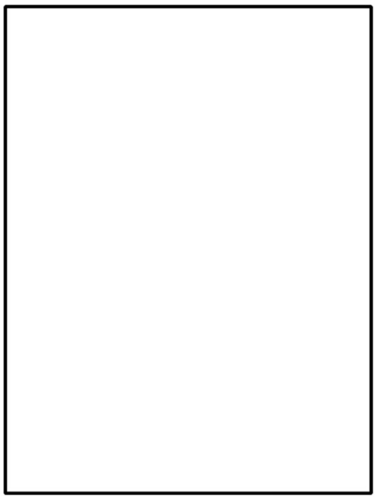
Make sure that students understand what it means for the rectangle to have a $4:3$ aspect ratio before they set to work on figuring out the side lengths. Give them time to productively struggle before showing any strategies. It may be necessary to clarify that the rectangle's diagonal refers to the segment that connects opposite corners (which is not drawn).

Representation: Internalise Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other

text-based content.
Supports accessibility for: Language; Conceptual processing

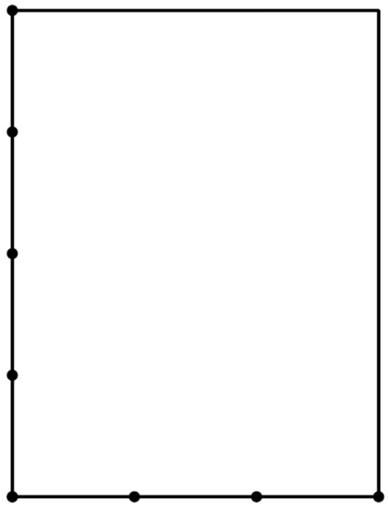
Student Task Statement

A typical aspect ratio for photos is 4: 3. Here's a rectangle with a 4: 3 aspect ratio.



1. What does it mean that the aspect ratio is 4: 3? Mark up the diagram to show what that means.
2. If the shorter side of the rectangle measures 15 inches:
    a. What is the length of the longer side?
    b. What is the length of the rectangle's diagonal?
3. If the diagonal of the 4: 3 rectangle measures 10 inches, how long are its sides?
4. If the diagonal of the 4: 3 rectangle measures 6 inches, how long are its sides?

Student Response



1. See diagram.
2. With a shorter side of 15 inches, each equal "piece" along the two subdivided sides is 5 inches.
    a. 20 inches
    b. 25 inches, by solving $15^2 + 20^2 = c^2$, or scaling up a 3-4-5 right triangle.
3. 6 inches and 8 inches. Students may be able to figure this out if they know that 6-8-10 is a Pythagorean triple, recognising that 6 and 8 are in the desired ratio.
4. 3.6 inches and 4.8 inches. Possible strategies:
    a. Solve the equation $(3x)^2 + (4x)^2 = 6^2$, where x represents the length of one of the little subdivisions of sides in the diagram. x is $\frac{6}{5}$, which can be multiplied by 3 and 4 to find the side lengths.
    b. Scale the 6 and 8 found previously by a factor of $\frac{6}{10}$, since this triangle would necessarily be similar.

Activity Synthesis

Invite students to share their solutions. If any students solved an equation such as $(3x)^2 + (4x)^2 = 6^2$ for the last question, ensure they have an opportunity to demonstrate their approach.

16.3 The Screen Is the Same Size . . . Or Is It?

20 minutes

The purpose of this activity is to give students an opportunity to solve a relatively complicated application problem that requires an understanding of aspect ratio, Pythagoras' theorem, and realising that a good way to compare the sizes of two screens is to compare their areas. The previous activities in this lesson are meant to prepare students to understand the situation and suggest some strategies for tackling the problem.

Monitor for students using different approaches and strategies. Students may benefit from more time to think about this problem than is available during a typical class lesson.

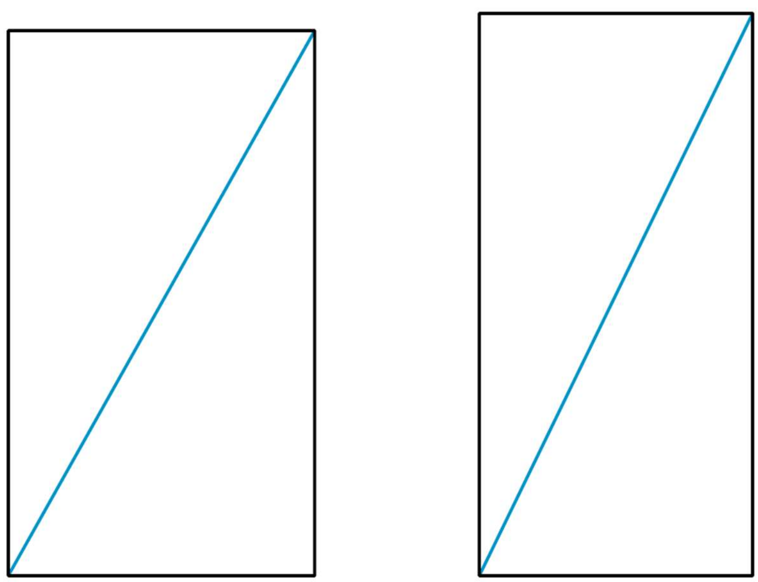
Instructional Routines

- Three Reads

Launch

Provide access to calculators that can find the square root of a number. The task statement is wordy, so consider using the Three Reads protocol to ensure students understand what the problem is saying and what it is asking.

We chose not to provide diagrams drawn to scale in the student materials, since it makes it somewhat obvious that the new phone design has a smaller screen area. However if desired, here is an image to show or provide to students:



Reading, Listening, Conversing: Three Reads. Use this routine to support reading comprehension of this word problem. Use the first read to orient students to the situation. Ask students to describe what the situation is about without using numbers (customers think that newly released smartphones have smaller screens). Use the second read to identify quantities and relationships. Ask students what can be counted or measured without focusing on the values (aspect ratios for phones manufactured before and after 2017). After the third read, invite students to brainstorm possible strategies they can use

determine whether or not the new phones have a smaller screen, and by how much.
Design Principle(s): Support sense-making

Student Task Statement

Before 2017, a smart phone manufacturer's phones had a diagonal length of 5.8 inches and an aspect ratio of $16:9$. In 2017, they released a new phone that also had a 5.8 inch diagonal length, but an aspect ratio of $18.5:9$. Some customers complained that the new phones had a smaller screen. Were they correct? If so, how much smaller was the new screen compared to the old screen?

Student Response

Sample response: For the $16:9$ screen, it's approximately 5.06 inches by 2.84 inches, for an area of approximately 14.4 in^2. This can be found by solving $(16x)^2 + (9x)^2 = 5.8^2$ which gives $x \approx 0.316$. Multiply this value by 16 and 9 to find the lengths of the two sides. Then multiply the lengths of the sides to find the area in square inches.

For the $18.5:9$ screen, it's approximately 5.22 inches by 2.54 inches, for an area of approximately 13.3 in^2. This can be found by solving $(18.5x)^2 + (9x)^2 = 5.8^2$ which gives $x \approx 0.282$. Multiply this value by 18.5 and 9 to find the lengths of the two sides. Then multiply the lengths of the sides to find the area in square inches.

Since $13.3 < 14.4$, the newer phones did, in fact, have a smaller screen when measured in terms of area. The difference was about 1.1 square inches.

This isn't the only possible solution method. For example, you could find that the diagonal of a 16 by 9 rectangle is approximately 18.36, and then scale each of the three measures down by a factor of $\frac{5.8}{18.36}$.

Activity Synthesis

Invite students to share their ideas and progress with the class. If appropriate, students may benefit from an opportunity to clearly present their solution in writing.

Lesson Synthesis

The debrief and presentation of student work provides opportunities to summarise takeaways from this lesson. Aside from opportunities to point out how Pythagoras' theorem can help us tackle difficult problems, this lesson makes explicit some aspects of mathematical modelling. Highlight instances where students had to figure out what additional information they would need to make progress, or restate a question in mathematical terms.