Lesson 6: Estimating areas

Goals

- Estimate the area of a complex, real-world region, e.g., a country or county, by approximating it with an irregular polygon, and indicate that it is an approximation when expressing the answer (orally and in writing).
- Explain (orally and in writing) how to calculate the area of an irregular polygon by decomposing it.
- Interpret floor plans and maps in order to identify the information needed to calculate area.

Learning Targets

- I can calculate the area of a complicated shape by breaking it into shapes whose area I know how to calculate.

Lesson Narrative

The purpose of this lesson is for students to practise composing and decomposing irregular regions to calculate their area, in preparation for estimating the area of circles in the next lesson. In the first activity, the region is polygonal and students can calculate an exact answer for the area of the floorplan. In the second activity, students must approximate the area of the state.

Students use polygons to model regions on a map or floorplan. To complete each task, students need to identify relevant information, choose an appropriate strategy, and make simplifying assumptions. Students also have an opportunity to think about what factors affect the estimates. Students see that the nature of the information we have or could obtain, the assumptions we make, and the estimation methods we use affect how close our estimates are to the actual area measures. Adjust the length of lesson and the expectations on the depth of students' investigations depending on the time available.

Building On

- Write and interpret numerical expressions.

Addressing

- Solve problems involving scale drawings of geometric shapes, including calculating actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions
- Compare and Connect
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Required Preparation

Provide access to geometry toolkits.

Student Learning Goals

Let's estimate the areas of unusual shapes.

6.1 Mental Calculations

Warm Up: 5 minutes

This warm-up encourages using different strategies to perform arithmetic operations mentally. One of these is the idea of compensation. These methods for performing mental maths are arithmetic analogues of the composition and decomposition techniques students use in this lesson to calculate areas of shapes.

Instructional Routines

- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy.

Student Task Statement

Find a strategy to make each calculation mentally:

$599 + 87$

$254 - 88$

99×75

Student Response

- Taking one away from 87 and adding it to 599 turns this into $600 + 86$, which is 686.
- Instead of subtracting 88, it is easier to subtract 90. Since this is subtracting 2 more, 254 needs to be increased by 2: $256 - 90 = 166$.
- Since 99 is 1 short of 100, this is the same as 100 times 75 minus 75, or 7 425.

Activity Synthesis

Ask students to share their ideas for how to perform these calculations mentally.

One key idea to bring out, for all three calculations, is the idea of compensation: identifying numbers close to the given ones for which the calculation can be done more efficiently. For 599 + 87, since 599 is only one away from 600 (a nice round number), it is natural to change 599 to 600. Adding 1 to 599 means that we need to *subtract* one from 87 to keep the sum the same. So the answer is 600 + 86, or 686. For 254 - 88, students may identify 90 or 100 as a nice number near 88 which is simpler to subtract. Subtracting 100 would be subtracting 12 more than 88, so we need to add 12 to 254. So the answer is 266 - 100, or 166. Finally for 99 × 75, 99 is 1 short of 100, so 99 × 75 is 75 short of 7 500 or 7 425.

Tell students that in this lesson they are going to use these kinds of strategies in a geometric context to find areas efficiently.

Speaking:

Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

6.2 House Floorplan

15 minutes

In this activity students calculate the area of an irregular shape presented in a scale drawing. In this case, students can calculate the area exactly by composing and decomposing triangles and parallelograms.

Choosing an appropriate way to compose and decompose the floor plan of the house in order to make calculations efficient is analogous to choosing how to rewrite numbers in order to make finding their sum, difference, or product as efficient as possible.

Monitor for students who focus on decomposing the floor plan and for students who compose the floor plan with additional shapes to make a rectangle. For students who focus on decomposing the floor plan, the biggest challenge will be the right side of the house.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions

Launch

Given students 4–5 minutes of quiet work time followed by whole-class discussion.

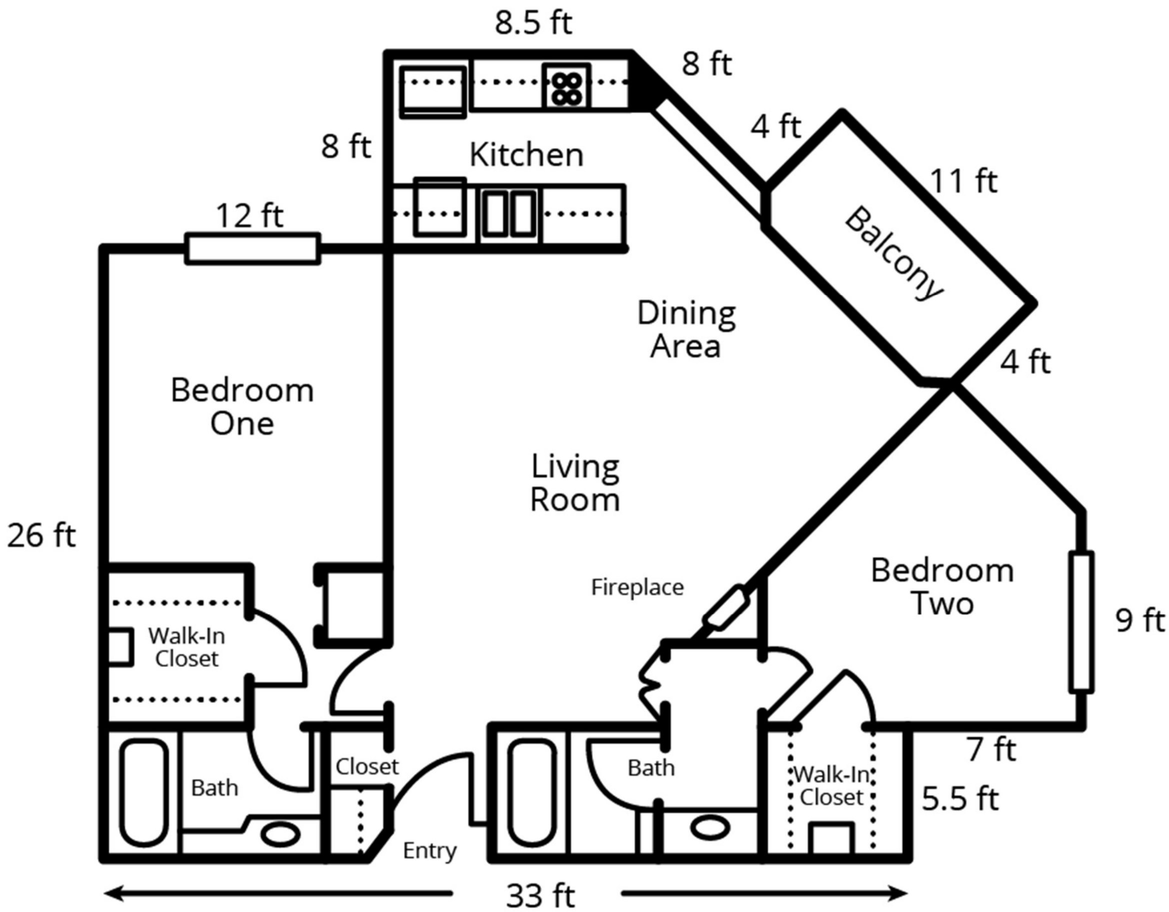
Action and Expression: Internalise Executive Functions. Provide students with a graphic organiser to organise the information provided in the problem and to structure their problem-solving strategy. The graphic organiser should include the prompts: "What do I need to find out?", "What do I need to do?", "How I solved the problem.", and "How I know my answer is correct."
Supports accessibility for: Language; Organisation Conversing: Co-Craft Questions. Display the diagram of the floor plan without revealing the task statement to students. Ask pairs of students to write a list of possible mathematical questions about the situation. Then, invite pairs to share their questions with the class. This will provide students with an opportunity to orient themselves to the context, ensuring that students understand the components of a floor plan, and also to produce the language of mathematical questions related to finding the area of irregular shapes.
Design Principle(s): Cultivate conversation; Support sense-making

Anticipated Misconceptions

If students struggle getting started finding the area of the floor plan, consider suggesting that they use composition and decomposition to break the floor plan up into familiar shapes whose area can be calculated.

Student Task Statement

Here is a floor plan of a house. Approximate lengths of the walls are given. What is the approximate area of the home, including the balcony? Explain or show your reasoning.



Student Response

About 1 080 ft². Sample reasoning: Enclose the house plan with a rectangle. One side of the rectangle is 40 ft because $33 + 7 = 33$. The other side is 34 ft because $26 + 8 = 34$. So the area of the enclosing rectangle is 1 360 ft², since $40 \times 34 = 1\,360$. Next, find the areas of the two smaller rectangles outside the house (at upper left and lower right), then subtract those areas from 1 360 ft². The two rectangles are 96 ft² and 38.5 ft² because $12 \times 8 = 96$ and $(5.5) \times 7 = 38.5$. Next, find the area of the right-angled triangle at upper right, and then subtract it: the triangle includes part of the balcony which will be added back in later. The area of the triangle is about 190 ft², because $\frac{1}{2} \times (19.5) \times (19.5) = 190.125$ The balcony is 44 sq ft, since $4 \times 11 = 44$. $1360 - 96 - 38.5 - 190 + 44 = 1\,079.5$. Therefore, the area of the house is about 1 080 ft².

Activity Synthesis

Select students to share their reasoning in this sequence:

- decomposing the floorplan into various rectangles and triangles

- composing the floorplan with other shapes to create a large rectangle

Composing to make a bigger shape and then taking away the excess area is very much like finding 99×75 in the warm-up by first calculating the larger product 100×75 and then removing 75. In both cases, something is being added to facilitate the calculation and then an adjustment is made at the end to take away the excess that was added.

6.3 Area of Nevada

15 minutes

In this activity students first identify the information needed to estimate the area of the state of Nevada from a map. Next, they use strategies developed in earlier work to make an estimate. The area can only be estimated as the shape is more complex and not a polygon. Like in the previous activity, monitor for these two strategies:

- Enclosing the image of the state in a rectangle, finding the area of the rectangle, and subtracting the area of a right-angled triangle
- Decomposing the image of the state into a rectangle and a right-angled triangle, finding the area of each, and combining the two

These two methods work equally well for this shape. Also, some students may notice and account for the missing area in the southeast corner of the state and others may not. As students work, notice the approaches they use and select one or two students who use each strategy to share during the discussion.

Instructional Routines

- Compare and Connect
- Notice and Wonder
- Think Pair Share

Launch

Arrange students in groups of 2. Have students close their books or devices. Display this map of Nevada that does not have a scale, and invite students to share what they notice and wonder.

Some things students might notice:

- The shape of the state looks like a rectangle with a corner cut off and a bite taken out.
- The shape of the state could be decomposed into a rectangle and a triangle.

Some things students might wonder:

- How long are the side lengths of the state?
- What is the area of Nevada?


Denio
McDermitt
Santa Rosa Range
Black Rock Desert
Independence Mts
Humboldt R
Winnemucca
Wells
Elko
Battle Mountain
80
Pyramid Lake
Trinity Range
Ruby Mts
Lovelock
Stillwater Range
G R E A T
Carson Sink
Shoshone Mts
Reno
Sparks
Fallon
Austin
Toiyabe Range
Eureka
Egan Range
Schell Creek Range
Virginia City
Carson City
Ruth
Ely
Snake Ra
Yerington
Monitor Range
Hot Creek Range
Wheeler Pk 13063
Gardnerville
Walker Lake
Round Mountain
Mt Grant 11239
Hawthorne
Grant Range
B A S I N
Tonopah
Pioche
Boundary Pk 13140
Panaca
Caliente
Pahute Mesa
Beatty
Mesquite
15
Sheep Range
Indian Springs
Charleston Pk 11918
Lake Mead
North Las Vegas
Las Vegas
Henderson
Boulder City
Colorado River
Lake Mohave
Laughlin
0 50 100mi

Ask students what information they would need to know to calculate the area of Nevada. If desired, show students the scale and ask them to estimate the needed measurements. Alternatively, instruct students to open their books or devices. Give students 5 minutes of quiet work time followed by partner discussion.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students as they discuss and explain their strategies. For example, “We are trying to….”, “How did you get…?”, and “First, I _____ because….”
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

If students decompose the image of the state into a rectangle and triangle, they may use the 270 miles for a side length of the rectangle instead of finding the difference of the 490 miles on the opposite side and 270 miles. Ask them to check their answer with a partner and re-evaluate their calculations.

Student Task Statement

Estimate the area of Nevada in square miles. Explain or show your reasoning.

Student Response

Answers vary. Sample response: About 110 000 square miles. Possible strategies:

- Enclose Nevada with a 320 mi by 490 mi rectangle, and subtract the area of the right-angled triangle in the lower left corner that has side lengths of 320 mi and 270 mi: $(320 \times 490) - (\frac{1}{2} \times 320 \times 270) = 113\,600$. The area is about 110 000 square miles.

- Decompose Nevada into a rectangle and a right-angled triangle: $(320 \times 220) + (\frac{1}{2} \times 320 \times 270) = 113\,600$. The area is about 110 000 square miles.

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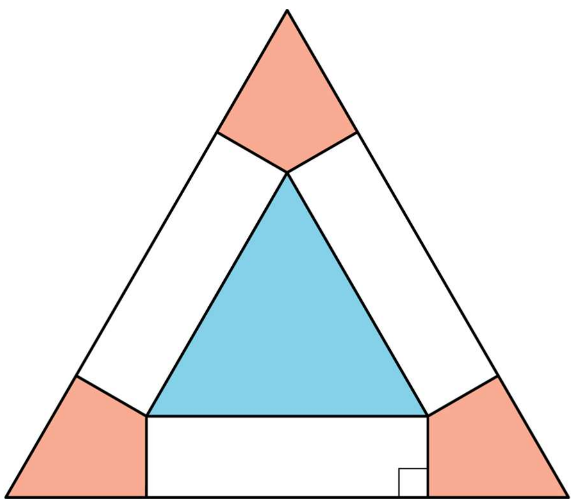
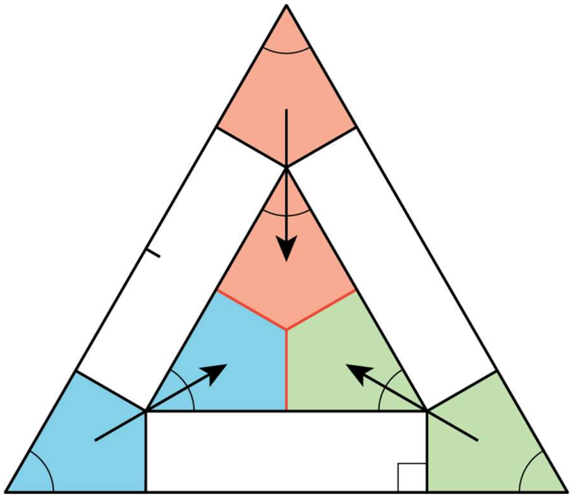
Are You Ready for More?

The two triangles are equilateral, and the three pink regions are identical. The blue equilateral triangle has the same area as the three pink regions taken together. What is the ratio of the sides of the two equilateral triangles?



Student Response

$2:1$ or $1:2$. Since both triangles are equilateral, the three pink regions are identical, and the pink regions have the same area as the blue region, they must fit inside as shown.



Activity Synthesis

The goal of this discussion is for students to understand the distinction between calculating the areas of geometric objects and estimating areas of regions on maps.

First, display these questions for students to discuss with their partners:

- How did you make your estimate?
- If your estimates are not the same, are they close? What accounted for the difference?

Ask students how finding the area of Nevada in this activity was the same as finding the area of the floorplan in the previous activity and how it was different.

- One way it was the same was that it was still helpful to decompose the region into rectangles and triangles. Strategies involving addition and strategies involving subtraction were both possible.
- An important difference is that the state is not a polygon. Some of the boundaries are not straight and the overall land is not completely flat. Assuming the state is flat and approximating the boundaries with line segments both lead to some error in the estimate.

Consider telling students that the actual area of Nevada is about 110 560 square miles.

Compare and Connect. Use this routine when students present their strategy for finding the area of Nevada. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the language used to describe the different ways the area can be calculated (decompose, rearrange, enclose, area, etc.). These exchanges can strengthen students' mathematical language use and reasoning to make sense of strategies used to calculate the area of irregular shapes.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

Point out that students estimated areas of both large and small things in the world by approximating them with polygons. Go over the different strategies students used to estimate the area in this lesson and emphasise we can find the area of any polygon by decomposing it into triangles and rectangles and using formulae we know to find the area. In practice, it is important to be strategic when composing and decomposing, taking advantage of measurements that are known and avoiding measurements that are unknown or difficult to calculate.

Ask students to reflect on their recent work in finding area and discuss the following questions:

- "What things are important to think about when asked to find the area of a shape?"
- "What things do we know help us find area of any shape?"

It is important to consider the shape of the region, how polygons are helpful, and the ways polygons can be decomposed, rearranged or enclosed to find the area of the region.

6.4 The Area of Alberta

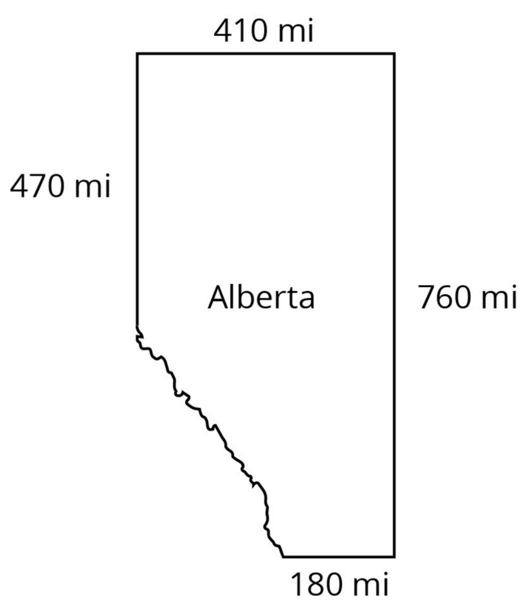
Cool Down: 5 minutes

Launch

Consider telling students that Alberta is a province in Canada.

Student Task Statement

Estimate the area of Alberta in square miles. Show your reasoning.



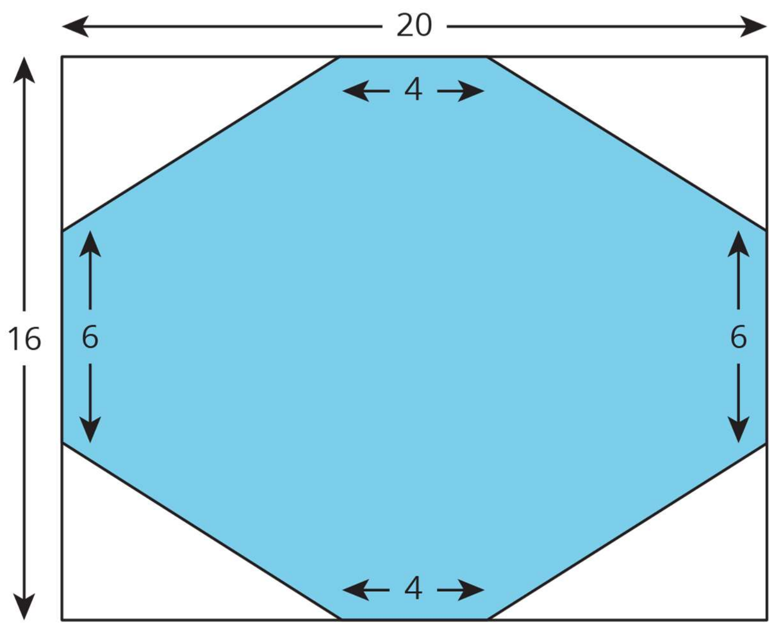
Student Response

About 250 000 square miles. Sample reasoning: Alberta can be surrounded with a 410-mile-by-760-mile rectangle with a 290-mile-by-230-mile triangle removed in the lower left corner. The answer has been rounded because the part missing in the lower left is not exactly a triangle.

Student Lesson Summary

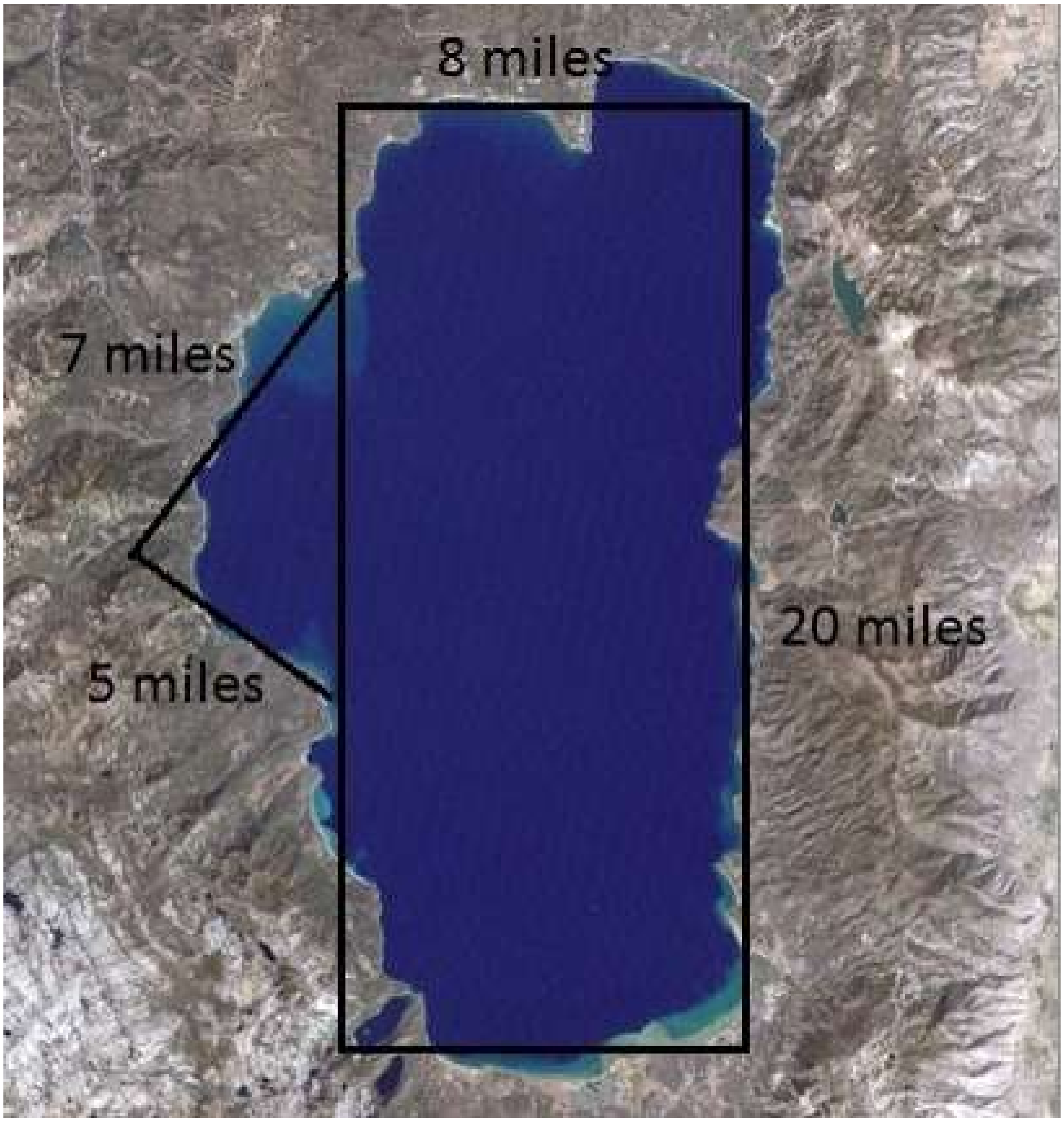
We can find the area of some complex polygons by surrounding them with a simple polygon like a rectangle. For example, this octagon is contained in a rectangle.

The rectangle is 20 units long and 16 units wide, so its area is 320 square units. To get the area of the octagon, we need to subtract the areas of the four right-angled triangles in the corners. These triangles are each 8 units long and 5 units wide, so they each have an area of 20 square units. The area of the octagon is $320 - (4 \times 20)$ or 240 square units.



We can estimate the area of irregular shapes by approximating them with a polygon and finding the area of the polygon. For example, here is a satellite picture of Lake Tahoe with some measurements around the lake.

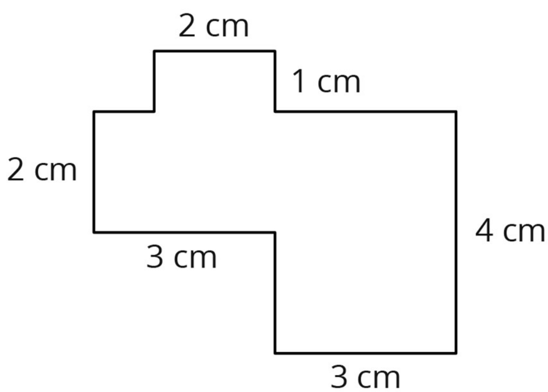
The area of the rectangle is 160 square miles, and the area of the triangle is 17.5 square miles for a total of 177.5 square miles. We recognise that this is an approximation, and not likely the exact area of the lake.


8 miles
7 miles
5 miles
20 miles

Lesson 6 Practice Problems

1. **Problem 1 Statement**

   Find the area of the polygon.



   Solution

   20 cm^2 since the shape can be divided (vertically) into rectangles of area 2, 6, and 12 square centimetres.

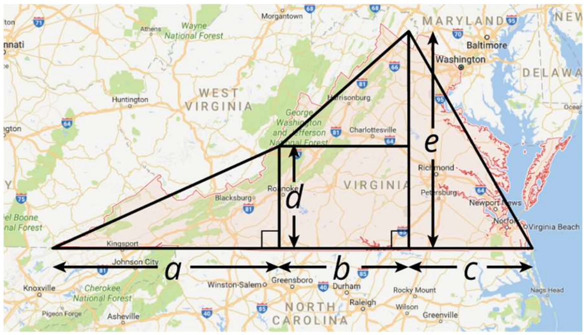
2. **Problem 2 Statement**

   a. Draw polygons on the map that could be used to approximate the area of Virginia.

   b. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)

   Solution

   a. Answers vary. There are many possible ways to draw polygons that would approximate the area of Virginia. One sample response is shown below. Other choices could be made to yield a more or less precise approximation.



b. Answers vary. For rectangles, parallelograms, and triangles, you need both base and height. In the example above, the variables represent measurements needed to find the area of the polygons.

3. **Problem 3 Statement**

Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

Solution

$37 \times 20 \times \pi$ or about 2 325 in.

4. **Problem 4 Statement**

The radius of Earth is approximately 6 400 km. The equator is the circle around Earth dividing it into the northern and southern hemispheres. (The centre of the earth is also the centre of the equator.) What is the length of the equator?

Solution

$6400 \times 2 \times \pi$ is about 40 000 km

5. **Problem 5 Statement**

Here are several recipes for sparkling lemonade. For each recipe, describe how many tablespoons of lemonade mix it takes per cup of sparkling water.

a. 4 tablespoons of lemonade mix and 12 cups of sparkling water

b. 4 tablespoons of lemonade mix and 6 cups of sparkling water

c. 3 tablespoons of lemonade mix and 5 cups of sparkling water

d. $\frac{1}{2}$ tablespoon of lemonade mix and $\frac{3}{4}$ cups of sparkling water

Solution

a. $\frac{4}{12}$ or $\frac{1}{3}$ tablespoons lemonade mix per cup of sparkling water

b. $\frac{4}{6}$ or $\frac{2}{3}$ tablespoons lemonade mix per cup of sparkling water

c. $\frac{3}{5}$ or 0.6 tablespoons of lemonade mix per cup of sparkling water

d. $\frac{1}{2} \div \frac{3}{4}$ or $\frac{2}{3}$ tablespoon of lemonade mix per cup of sparkling water