

Problemas – Tema 6

Problemas resueltos - 18 - propiedades de la matriz inversa

1. Sea $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$. Calcular A^{-1} y $(2025A)^{-1}$.

Aplicamos Gauss-Jordan.

$$\left\langle \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array} \right\rangle \rightarrow F'_3 = F_3 + F_1 \rightarrow \left\langle \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 1 \end{array} \right\rangle \rightarrow F'_3 = F_3 - 2F_2 \rightarrow$$

$$\left\langle \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array} \right\rangle \rightarrow F'_2 = 2F_2 - F_3 \rightarrow \left\langle \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 4 & -1 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array} \right\rangle$$

$$\rightarrow F'_1 = 2F_1 - 3F_3 \rightarrow \left\langle \begin{array}{ccc|ccc} 2 & 4 & 0 & -1 & 6 & -3 \\ 0 & 2 & 0 & -1 & 4 & -1 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array} \right\rangle \rightarrow F'_1 = F_1 - 2F_2 \rightarrow$$

$$\left\langle \begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -2 & -1 \\ 0 & 2 & 0 & -1 & 4 & -1 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array} \right\rangle \rightarrow \text{Normalizamos cada fila, dividiendo entre 2 cada fila} \rightarrow$$

$$\left\langle \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & -1/2 \\ 0 & 1 & 0 & -1/2 & 2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1 & 1/2 \end{array} \right\rangle$$

Quedando la matriz inversa:

$$A^{-1} = \begin{pmatrix} 1/2 & -1 & -1/2 \\ -1/2 & 2 & -1/2 \\ 1/2 & -1 & 1/2 \end{pmatrix} \rightarrow \text{Recordamos la propiedad } (k \cdot A)^{-1} = \frac{1}{k} \cdot A^{-1} \text{ . Por lo tanto.}$$

$$(2025A)^{-1} = \frac{1}{2025} \begin{pmatrix} 1/2 & -1 & -1/2 \\ -1/2 & 2 & -1/2 \\ 1/2 & -1 & 1/2 \end{pmatrix} \rightarrow (2025A)^{-1} = \begin{pmatrix} 1/4050 & -1/2025 & -1/4050 \\ -1/2025 & 2/2025 & -1/4050 \\ 1/4050 & -1/2025 & 1/4050 \end{pmatrix}$$