Lesson 9: Solutions of inequalities

Goals

- Draw and label a number line diagram to represent the solutions to an inequality.
- Recognise and explain (orally and in writing) that an inequality may have infinitely many solutions.
- Use substitution to justify (orally) whether a given value is a "solution" to a given inequality.

Learning Targets

- I can determine if a particular number is a solution to an inequality.
- I can explain what it means for a number to be a solution to an inequality.
- I can graph the solutions to an inequality on a number line.

Lesson Narrative

In this lesson, students consider situations where there might be more than one condition. Students have already learned "solution to an equation" to mean a value of the variable that makes the equation true. Here, they learn a similar definition about inequalities: a **solution to an inequality** is a value of the variable that makes the inequality true. But while the equations students solved in the last unit generally had one solution, the inequalities they solve in this unit have many, sometimes infinitely many, solutions.

Constraints in real-world situations reduce the range of possible solutions. Students reason abstractly by using inequalities or graphs of inequalities to represent those situations and interpreting the solutions. Students think carefully about whether to include boundary values as solutions of inequalities in various contexts.

Addressing

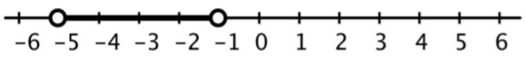
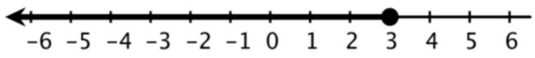
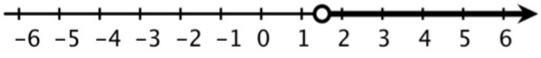
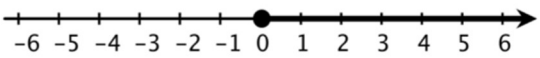
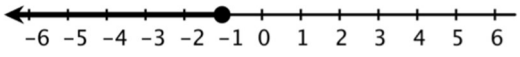
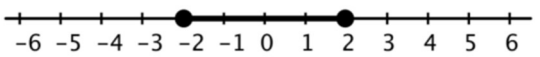
- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognise that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Discussion Supports
- Think Pair Share

Required Materials

Coloured pencils
Pre-printed slips, cut from copies of the blackline master

What Number Am I? -5	What Number Am I? -4	What Number Am I? -1	What Number Am I? 0
What Number Am I? 1	What Number Am I? 2	What Number Am I? 4	What Number Am I? 5
What Number Am I? $x > 1$	What Number Am I? $x < 1$	What Number Am I? $x < -3$	What Number Am I? $x < -1$
What Number Am I? $x > -6$	What Number Am I? $x < 6$	What Number Am I? $x > -3$	What Number Am I? $x > -1$
What Number Am I? 		What Number Am I? 	
What Number Am I? 		What Number Am I? 	
What Number Am I? 		What Number Am I? 	

Required Preparation

The included blackline master is for the *optional* activity, "What Number Am I?" Print and cut up slips from the blackline master. Prepare 1 set of inequalities and 1 set of

numbers for each group of 4 students. Coloured pencils are only needed for an "Are You Ready for More" problem.

Student Learning Goals

Let's think about the solutions to inequalities.

9.1 Unknowns on a Number Line

Warm Up: 10 minutes

The purpose of this warm-up is for students to compare and name values on a number line based on their relative position to one another and 0. Students also review completing inequality statements based on their comparisons.

Since there are many ways to make each inequality true, it may not be possible for students to share all of the possible ways due to time. Consider sharing 2 possibilities for each before moving on to the next question.

Instructional Routines

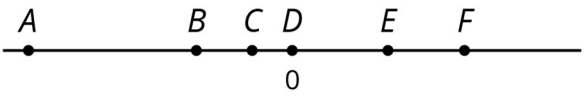
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time. Tell students there are many possible answers for the questions. Give students 1 minute to compare their responses to their partners and decide if they are both correct, even if they are different. Follow with whole-class discussion.

Student Task Statement

The number line shows several points, each labelled with a letter.



1. Fill in each blank with a letter so that the inequality statements are true.

   a. _____ > _____

   b. _____ < _____

2. Jada says that she found three different ways to complete the first question correctly. Do you think this is possible? Explain your reasoning.

3. List a possible value for each letter on the number line based on its location.

Student Response

1.

   a. Answers vary. The first point should be to the right of the second point. Sample response: $B > A$

   b. Answers vary. The first point should be to the left of the second point. Sample response: $A < B$

2. Jada is correct, there are many ways to answer question 1 correctly. Here are 3 possible answers, though there are more: $B > A, C > B, C > A$.

3. Answers vary. Sample response: $A = -11, B = -5, C = -2, D = 0, E = 5, F = 9$

Activity Synthesis

Ask students to share their responses and explanations for each question. Record and display their responses for all to see. If possible, as students share, record their reasoning directly on the displayed number line and reference the points' locations.

If there is time after students share the possible values of each point in the last question, ask students how they could complete the inequality: __+__>__. Ask students to share their responses and explanations for how they know the inequality is true based on their assigned values for each point.

9.2 Amusement Park Rides

25 minutes

The purpose of this activity is for students to represent situations with inequalities and investigate whether values are solutions to multiple inequalities at the same time. Students are formally introduced to the term **solution to an inequality** and are given the opportunity to use it precisely during discussion. A solution to an inequality is a value of the variable that makes the inequality true. Students explore these ideas using given height restrictions for a variety of amusement park rides. Students represent the height restrictions as inequality statements and graph those inequalities on the number line. Students reason abstractly when determining whether a value is a solution to one or more of the inequalities and what that means in context.

Question 3 will likely lead to a discussion of whether or not the endpoints of the inequality are included. As students work, take note that students' inequalities and graphs should match their reasoning on the inclusion or exclusion of the endpoints 55 and 72. Monitor for one student who included 55 and 72 as a solution and one student who did not.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time

Launch

Prior to beginning this activity, remind students that in their previous work with inequalities, they considered which values made an inequality true and which values did not. Introduce the more formal definition of solution here by using students' previous work with solutions to equations as a starting point. Just as a solution to an equation was a value of the variable that made the equation true, a **solution to an inequality** is a value of the variable that makes the inequality true. But while the equations students solved in the last unit generally had one solution, inequalities have many, sometimes infinitely many, solutions.

Arrange students in groups of 2. Give students 5 minutes of quiet think time for questions 1 and 2. Pause after question 2 to tell students to work with their partner for questions 3 through 6. Tell students that if there is disagreement, work to reach agreement. Give students 5 minutes of work time. Follow with a whole-class discussion.

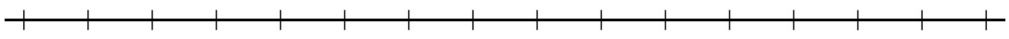
Student Task Statement

Priya finds these height requirements for some of the rides at an amusement park.

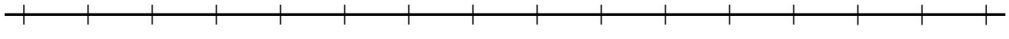
To ride the . . .	you must be . . .
High Bounce	between 55 and 72 inches tall
Climb-A-Thon	under 60 inches tall
Twirl-O-Coaster	58 inches minimum

1. Write an inequality for each of the three height requirements. Use h for the unknown height. Then, represent each height requirement on a number line.

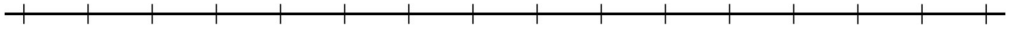
   - High Bounce

   - Climb-A-Thon

   - Twirl-O-Coaster

   Pause here for additional instructions from your teacher.

2. Han's cousin is 55 inches tall. Han doesn't think she is tall enough to ride the High Bounce, but Kiran believes that she is tall enough. Do you agree with Han or Kiran? Be prepared to explain your reasoning.

3. Priya can ride the Climb-A-Thon, but she cannot ride the High Bounce or the Twirl-O-Coaster. Which, if any, of the following could be Priya's height? Be prepared to explain your reasoning.

- 59 inches
- 53 inches
- 56 inches

4. Jada is 56 inches tall. Which rides can she go on?

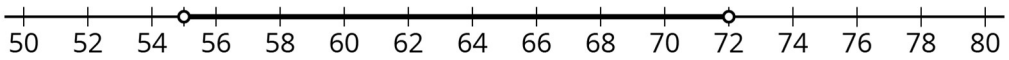
5. Kiran is 60 inches tall. Which rides can he go on?

6. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, of another ride. Write three values that are **solutions** to both of these inequalities.

Student Response

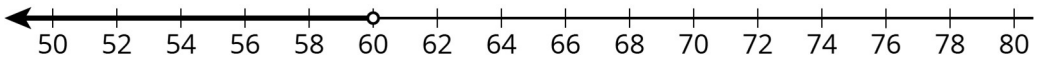
1. High Bounce: $h > 55$ and $h < 72$. Students may also interpret the height description to include $h = 55$ and $h = 72$. This is a point to discuss. Optional notation for that interpretation: $h \geq 55$ and $h \leq 72$. Additionally, students may write: $55 \leq h \leq 72$ though this is not required notation. Climb-A-Thon: $h < 60$ Twirl-O-Coaster: $h > 58$ or $h = 58$.

   High Bounce:

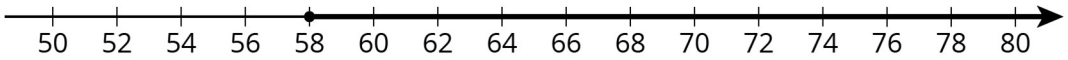
   

   (Some students may opt to shade in 55 and 72.)

   Climb-A-Thon:

   Twirl-O-Coaster:

2. Answers vary due to ambiguity of the stated restriction. If one interprets the restriction to include the endpoints, then Han's cousin will be able to go on the High Bounce. If one believes the endpoints are not included, then Han's cousin cannot go on this ride.

3. 53 inches. Explanations vary. Sample response: The lowest restriction is for the High Bounce, which Priya cannot ride. Therefore, Priya's height is less than 55 inches. The only choice that meets this criteria is 53.

4. High Bounce and Climb-A-Thon. Explanations vary. Sample response: Jada can go on the High Bounce because $56 > 55$ and $56 < 72$ are both true. Jada can go on the Climb-A-Thon because $56 < 60$ is true. Jada cannot go on the Twirl-O-Coaster because $56 > 58$ is false.

5. High Bounce and Twirl-O-Coaster. Kiran can go on the High Bounce because $60 > 55$ and $60 < 72$ are both true. Kiran can also go on the Twirl-O-Coaster because $60 > 58$ is true. Kiran cannot go on the Climb-A-Thon because $60 < 60$ is false.

6. Answers vary. Three sample responses: 65.5, 65, and 74 inches. Any heights between 64 and 75 inches, not including 64 and 75, are possible responses.

Are You Ready for More?

1. Represent the height restrictions for all three rides on a single number line, using a different colour for each ride.

2. Which part of the number line is shaded with all 3 colours?

3. Name one possible height a person could be in order to go on all three rides.

Student Response

1. High Bounce: Number line with open circles at 55 and 72. Shaded in between these circles. (Note to teacher: students may opt to shade in 55 and 72.)

   Climb-A-Thon: Number line with open circle at 60 and shaded to the left. Bold left arrow.

   Twirl-O-Coaster: Number line with closed circle at 58 and shaded to the right. Bold right arrow.

2. The space between 58 and 60, not including 58 and 60.

3. Answers vary. One possible response is: 59 inches. This is the only whole number response possible, but other numbers, such as $59\frac{1}{4}$ are possible.

Activity Synthesis

The discussion should include these topics:

1. What does it mean to be a solution to an inequality? (A value of the variable that makes the statement true.)

2. What are some ways to find solutions to an inequality? (A substitution can be used to check if values make the inequality true, or the inequality can be graphed on a number line and the value checked against the shaded part.)

3. How many solutions are there to the inequality for the high bounce? In other words, how many possible heights are there that are allowed? (There are infinitely many solutions because 58.1, 58.01, 58.001, 58.0001, etc. are all allowable heights in inches.)

4. Continue the discussion started in the last lesson on the meaning of an open circle vs. a closed circle on the number line. This will come up when students are writing

inequalities for the High Bounce ride and then trying to determine if Han's cousin can go on this ride. There is ambiguity in the language "between x and y," so both interpretations should be included in the discussion. Take time to make sure students' inequalities and number lines match their reasoning. Select previously identified students to share their graphs of the height restrictions for the High Bounce ride.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: inequality, solution to an inequality.
Supports accessibility for: Conceptual processing; Language; Memory Writing, Speaking, Listening: Stronger and Clearer. To help students strengthen their understanding of solutions to inequalities, ask students to write a response to the prompt "What does it mean to be a solution to an inequality?" and ask them to provide an example of an inequality and a solution to it. Have students read their writing to a partner. The partner can ask clarifying questions as well as confirm that their partner has written an inequality and has a correct solution. Have students share with two partners, reminding students to capture new ideas and language after each time they share. For students needing extra support, provide a sentence frame such as "If a number is a solution to an inequality, it means that _____."
Design Principle(s): Optimise output (for explanation); Cultivate conversation

9.3 What Number Am I?

Optional: 15 minutes

This activity is optional due to time considerations. The purpose of this activity is for students to reason about whether given values make an inequality true and justify their answers using inequality statements and graphs. Students explored this concept in the previous activity, so they should work to articulate how they check if a number is a solution to an inequality statement and an inequality on a number line.

Students will play a game to practice this skill. The game's goal is for one student to guess a mystery number using as few inequalities as possible. To create a class competition, keep track of how many inequalities each group uses for each number and display scores after each round. The winning group is the group with the lowest score.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 4. As students read the game instructions with their group, give each group 1 set of inequalities and 1 set of numbers, pre-cut from the blackline master. Review the game instructions as a whole class. Students will play 1 round and then pause to reflect and plan strategies for the next rounds. Allow students 10 minutes of game time followed by a whole-class discussion.

Once student groups have completed one round of game play, pause the game and ask groups to reflect on their strategies. Groups should make a plan before continuing to the second round of the game. Post or ask these questions to guide a short discussion:

- "Clue givers, how did you decide which inequalities would be the most helpful for your detective?"
- "Clue givers, did you work together to decide which 3 clues to give or did you decide independently?"
- "Detective, were some inequalities more helpful than others as you tried to guess the mystery number? If so, what made an inequality more helpful?"

Action and Expression: Internalise Executive Functions. Begin with a small-group or whole-class demonstration of how to play the game. Check for understanding by inviting students to rephrase directions in their own words.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some groups may misunderstand the directions, thinking that each person giving clues is supposed to take a different number card from the stack. Explain to them that there is only one unknown number per round and everyone gives clues about this same number. When a new person becomes the detective, that is when a new number card is drawn.

Student Task Statement

Your teacher will give your group two sets of cards—one set shows inequalities and the other shows numbers. Place the inequality cards face up where everyone can see them. Shuffle the number cards and stack them face down.

To play:

- One person in your group is the detective. The other people will give clues.
- Pick one number card from the stack and show it to everyone except the detective.
- The people giving clues each choose an inequality that will help the detective identify the unknown number.
- The detective studies the inequalities and makes three guesses.
  - If the detective does not guess the right number, each person chooses another inequality to help.
  - When the detective does guess the right number, a new person becomes the detective.
- Repeat the game until everyone has had a turn being the detective.

Student Response

Responses vary depending on what values are randomly chosen.

Activity Synthesis

Once all students have had a turn as the detective, start a whole-class discussion.
Ask groups to share their most successful strategies for choosing helpful clue inequalities as well as strategies for using those clues to guess the mystery number. Emphasise how students can check that a value is or is not a solution to an inequality: both symbolic statements (substitute the value and check that the resulting statement is true) and on the number line (plot the value to test and make sure that it falls within a shaded region).

Speaking: Discussion Supports. To support all students to participate in the discussion, provide students with sentence frames, such as "One strategy I used to choose a clue was _____" or "One way that I checked that a value was a solution to the inequality (number line representation) was to _____." Ask them to complete one of the sentences in writing or share a completed response verbally with a partner.
Design Principle(s): Maximise meta-awareness

Lesson Synthesis

Ask students to think about situations where a quantity can take on a range of values (for example, the ages of students eligible for a certain program, the salary range that applies to a particular tax rate, the speed you can drive on the highway). Ask students to decide on a variable and represent their situation with two inequality statements. Ask whether the maximum and minimum are included in the range of possible values of the variable (for example, can your maximum speed on a highway be equal to 65 miles per hour or does it have to be less than 65?). Then ask them to graph the solutions on one or two number lines. Invite selected students to share their situations, inequalities, and graphs.

9.4 Solutions of Inequalities

Cool Down: 5 minutes

Student Task Statement

a. Select **all** numbers that are solutions to the inequality $w < 1$.

- 5
- -5
- 0
- 0.9
- -1.3

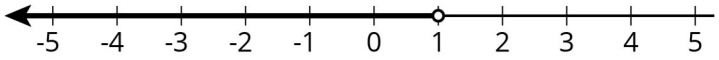
b. Draw a number line to represent this inequality.

c. Write an inequality for which 3, -4, 0, and 2 300 are solutions.

d. How many total solutions are there to your inequality?

Student Response

a. -5, 0, 0.9, -1.3

b. 

c. Answers vary. One possible response $x > -5$.

d. There are infinitely many solutions.

Student Lesson Summary

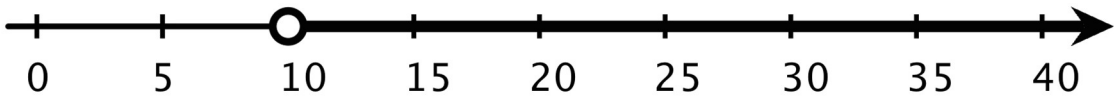
Let's say a cinema ticket costs less than £10. If c represents the cost of a cinema ticket, we can use $c < 10$ to express what we know about the cost of a ticket.

Any value of c that makes the inequality true is called a **solution to the inequality**.

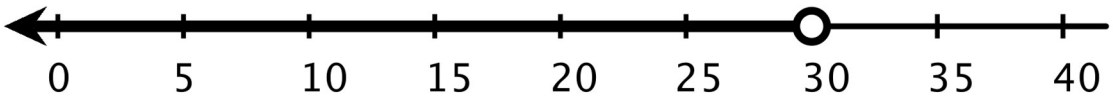
For example, 5 is a solution to the inequality $c < 10$ because $5 < 10$ (or "5 is less than 10") is a true statement, but 12 is not a solution because $12 < 10$ ("12 is less than 10") is *not* a true statement.

If a situation involves more than one boundary or limit, we will need more than one inequality to express it.

For example, if we knew that it rained for *more* than 10 minutes but *less* than 30 minutes, we can describe the number of minutes that it rained (r) with the following inequalities and number lines. $r > 10$

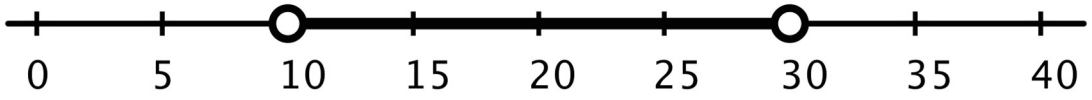


$r < 30$



Any number of minutes greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet the condition of "more than 10 but less than 30," the solutions are limited to the numbers between 10 and 30 minutes, *not* including 10 and 30.

We can show the solutions visually by graphing the two inequalities on one number line.



Glossary

- solution to an inequality

Lesson 9 Practice Problems

1. **Problem 1 Statement**

   a. Select **all** numbers that are solutions to the inequality $k > 5$.

   4

   5

   6

   5.2

   5.01

   0.5

   b. Draw a number line to represent this inequality.

   Solution

   a. 6, 5.2, 5.01

   b. The number line should show an open circle above the number 5 and an arrow pointing to the right.

2. **Problem 2 Statement**

   A sign on the road says: "Speed limit, 60 miles per hour."

   a. Let s be the speed of a car. Write an inequality that matches the information on the sign.

   b. Draw a number line to represent the solutions to the inequality.

   c. Could 60 be a value of s? Explain your reasoning.

   Solution

   a. $s < 60$ or $s = 60$, or equivalent

   b. The number line should show a closed circle at 60 and an arrow pointing to the left.

   c. Yes, 60 is the limit.

3. **Problem 3 Statement**

One day in Boston, MA, the high temperature was 60 degrees Fahrenheit, and the low temperature was 52 degrees.

a. Write one or more inequalities to describe the temperatures T that are between the high and low temperature on that day.

b. Show the possible temperatures on a number line.

Solution

a. $52 < T$ and $T < 60$ or equivalent

b. A graph showing empty circles at 52 and 60 and all of the numbers between.

4. **Problem 4 Statement**

Select **all** the true statements.

a. -5 < |-5|

b. |-6| < -5

c. |-6| < 3

d. 4 < |-7|

e. |-7| < |-8|

Solution ["A", "D", "E"]

5. **Problem 5 Statement**

Match each equation to its solution.

a. $x^4 = 81$

b. $x^2 = 100$

c. $x^3 = 64$

d. $x^5 = 32$

– 2

– 3

– 4

– 10

Solution

a. 3

b. 10

c. 4

d. 2

6. **Problem 6 Statement**

a. The price of a mobile phone is usually £250. Elena's mum buys one of these mobile phones for £150. What percentage of the usual price did she pay?

b. Elena's dad buys another type of mobile phone that also usually sells for £250. He pays 75% of the usual price. How much did he pay?

Solution

a. 60%

b. £187.50

Sample reasoning:

number	percentage
250	100
10	4
150	60
62.5	25
187.5	75