Lesson 3: Enlargements with no grid

Goals

- Create an enlargement of a shape given a scale factor and centre of enlargement.
- Explain (orally) the effect of the scale factor on the size of the image of a polygon and its distance from the centre of enlargement.
- Identify the centre, scale factor, and image of an enlargement without a circular grid.

Learning Targets

- I can apply an enlargement to a polygon using a ruler.

Lesson Narrative

In the previous lesson, students applied enlargements on a circular grid. The circular grid provides two levels of scaffolding:

- The radial lines give rays from the centre of the grid which help find the enlarged image of points on those rays.
- The circles provide a way to measure the distance of points from the centre of enlargement.

In this lesson, students apply enlargements to points with no grid. In order to perform an enlargement, three pieces of information are still needed: a centre of enlargement, a scale factor, and a point which is enlarged. Students practise identifying centres, scale factors, and images of enlargement. They also use enlargements to make perspective drawings.

Addressing

- Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- Compare and Connect
- Discussion Supports

Required Materials

Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Required Preparation

Ensure that rulers, index cards, and coloured pencils are available in the geometry toolkits.

Student Learning Goals

Let's enlarge shapes not on grids.

3.1 Points on a Ray

Warm Up: 5 minutes

Students apply an enlargement to points on a ray. The scaffold of the circular grid has been removed but the structure of enlargements is the same.

Without the grid, students will need to come up with a way to measure in order to find the point twice as far from A as B and half as far from A as B. They can use a ruler or the edge of an index card.

Monitor for these methods:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

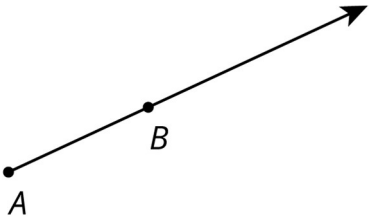
Select students who use these methods and invite them to present.

Launch

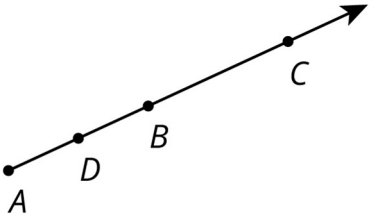
Provide access to geometry toolkits.

Student Task Statement

1. Find and label a point C on the ray whose distance from A is twice the distance from B to A.
2. Find and label a point D on the ray whose distance from A is half the distance from B to A.



Student Response



Activity Synthesis

Invite selected students to present their methods for finding the points which may include:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

Point out how this is similar to work with enlargements on a circular grid (the points lie on the same ray at different distances) and how it is different (there are no marked distances). For the next activity, it is important for students to understand that C is the enlargement of B with centre A and scale factor 2. And D is the enlargement of B with centre A and scale factor $\frac{1}{2}$.

3.2 Enlargement Obstacle Course

10 minutes (there is a digital version of this activity)

This activity investigates enlargements with no grid. Students have seen these for the first time in the warm-up, which had a ray drawn between two points. That scaffold has been removed here so the teacher may need to provide guidance by suggesting that students draw appropriate rays.

Encourage students to measure distances carefully at first, since the problem statement does not state that the image of each point, after the enlargement indicated, is one of the labelled points. After doing a few of the problems, the students should notice that the enlarged point is always one of the labelled points and then use this observation to expedite the work. Also monitor for students who see the relationship between the scale factors used to send G to E and E to G (both with centre H).

Instructional Routines

- Discussion Supports

Launch

Ask students to work on the first question and then pause. Demonstrate, or have a student demonstrate, drawing a ray from point A through point B (the ray goes through points H and I). Show that the length of AI is five times as long as the length of AB, either with a ruler or by marking intervals on the edge of a blank piece of paper. Or, if using the digital activity, use the measuring tool.(Click two points to measure the distance between them.)

If using the digital activity, it may be easiest for students to work with a partner, with one device used to manipulate the applet and the other device used to display the questions.

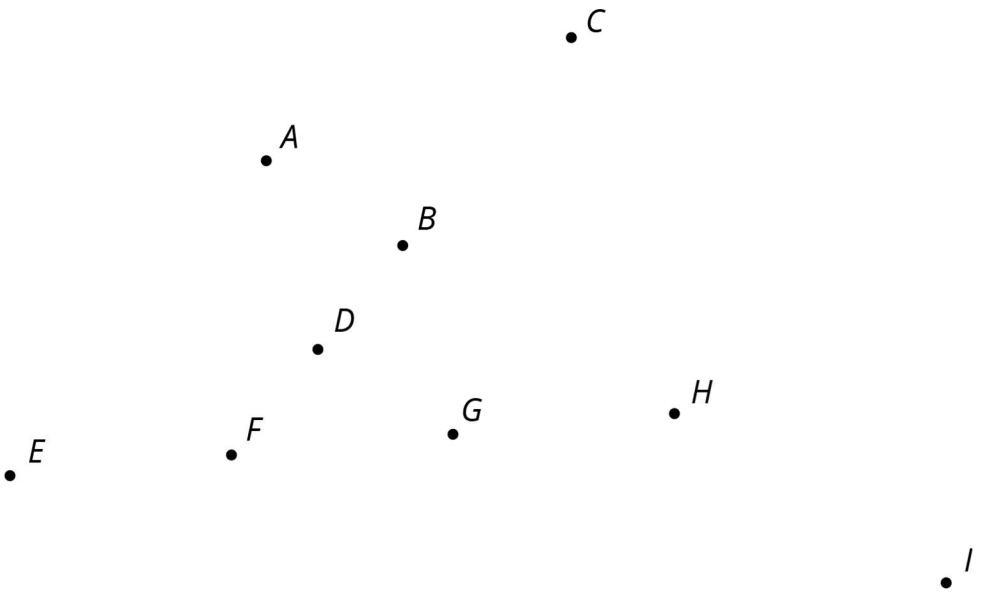
Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisational skills in problem solving. For example, allow time for students to work on the first question, then pause for a whole-class think aloud and discussion. Follow by presenting one question at a time and monitor students to ensure they are making progress throughout the activity. *Supports accessibility for: Organisation; Attention*

Anticipated Misconceptions

Students might need to be reminded that the image of a point under enlargement must lie on the same line as the point being enlarged and the centre of enlargement. Students might think that for a point to be an enlargement of itself, the scale factor is 0. If this happens, ask them to consider multiplying the distance of the point by 0. (If they want the distance to be the same, they actually need to multiply it by 1 instead.)

Student Task Statement

Here is a diagram that shows nine points.



1. Enlarge B using a scale factor of 5 and A as the centre of enlargement. Which point is its image?
2. Using H as the centre of enlargement, enlarge G so that its image is E. What scale factor did you use?
3. Using H as the centre of enlargement, enlarge E so that its image is G. What scale factor did you use?
4. To enlarge F so that its image is B, what point on the diagram can you use as a centre?
5. Enlarge H using A as the centre and a scale factor of $\frac{1}{3}$. Which point is its image?

6. Describe an enlargement that uses a labelled point as its centre and that would take F to H.

7. Using B as the centre of enlargement, enlarge H so that its image is itself. What scale factor did you use?

Student Response

1. I is on the same line as B and A and line segment AI is 5 times the length of line segment AB.

2. 3. Line segment EH is 3 times as long as line segment GH.

3. $\frac{1}{3}$. Line segment GH is $\frac{1}{3}$ as long as line segment EH.

4. C. It needs to be on the same line as F and B, but can't be between them (if restricted to positive scale factors).

5. B. It needs to be on the same line as A and H, but closer to A than H is.

6. Use E as the centre and a scale factor of 3. The centre must be on the same line as F and H and not between them, so the centre is E. Line segment EH is 3 times as long as line segment EF, so the scale factor should be 3.

7. 1. To enlarge H to itself using B as the centre, the distance from B to H must stay the same. That means the scale factor must be exactly 1.

Activity Synthesis

Discuss any strategies used to solve the problems. Ask selected students who noticed that the answers to all of the questions were labelled points to share their observation and how it helped them answer the questions. Next ask selected students to share their observation about the scale factors for enlarging G to E and enlarging E back to G. One way to reverse or "undo" an enlargement is to use the same centre and reciprocal scale factor.

Other important ideas to bring out include:

- The centre of enlargement, the point being enlarged, and the image of the point after enlargement must all lie on the same line.
- A scale factor of 1 does not move any points. If the scale factor is not 1, only one point does not move (the centre of enlargement).

Speaking, Listening: Discussion Supports. As students share their strategies with the class, encourage students to use full sentences and press for details in their explanations. Ask questions such as, "How do you know this is the image of the point?", "How do you know this is the scale factor?", "How do you know this is the centre of enlargement?", and "Why can't that point be located somewhere else?". If necessary, rephrase "collinear" and explain that collinear means on the same line. Point out the Latin root "co," which means "together." For example, "collaborate" means to work together, and therefore "collinear"

means on the same line together. This will support a rich and inclusive discussion about how to determine the scale factor, centre, and image of an enlargement.
Design Principle(s): Support sense-making

3.3 Getting Perspective

15 minutes (there is a digital version of this activity)

In this activity, students continue to apply enlargements without a grid. Unlike in the previous activity, the enlarged images of the points are not plotted. So rather than identifying the correct point, they will need to find an appropriate way to take measurements, most likely with the aid of a ruler or the edge of an index card. Different students will work with different scale factors and will produce perspective drawings of a box.

Watch for students who pick a point close to one vertex of the given rectangle. If the point is too close, it will be more difficult to visualise the box. Suggest that they move the point further away. Monitor for students who produce accurate drawings with different scale factors and invite them to share during the discussion.

Instructional Routines

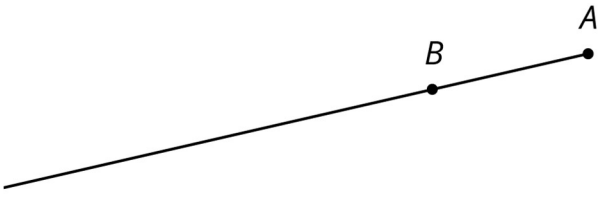
- Compare and Connect

Launch

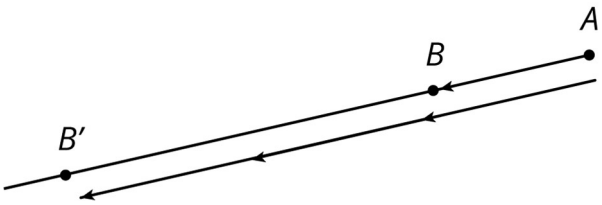
Provide access to geometry toolkits.

First, a demonstration about enlarging a point on a plane with no grid.

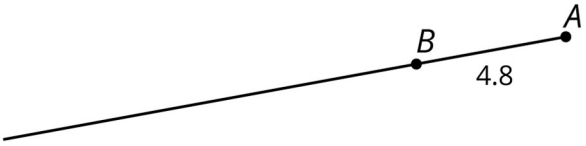
We want to enlarge point B using A as the centre of enlargement and a scale factor of 3.



Use a straightedge to draw ray AB.

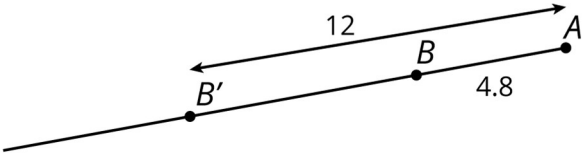


Measure the distance from B to A. Multiply the distance by 3. Draw B' so that it is 3 times as far away from A. For scale factors that are integers, an unmarked edge of an index card or a compass can also be used to transfer the distance along the ray.



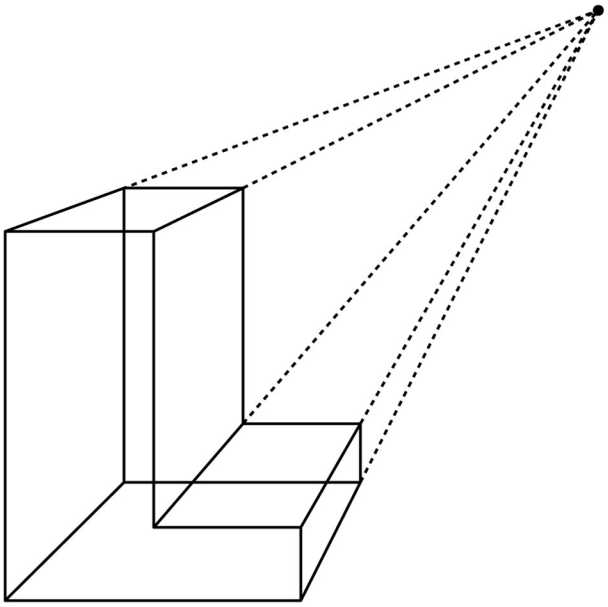
If we wanted a scale factor that is not an integer the procedure is the same. Measure the distance from A to B, multiply by the scale factor, and place B' at that new distance from A.

Let's say the distance from A to B is 4.8 cm.



If we wanted to use a scale factor of 2.5, the distance from A to the enlarged point B' would be 12 cm, because $(4.8) \times (2.5) = 12$.

A perspective drawing is an optical illusion that makes an image printed on paper have a three-dimensional look. Display at least one example of a perspective drawing:



Students will practise some simple enlargements of points, and then they will create a perspective drawing. Tell students to complete the first part of the activity enlarging points P and Q. After you review their work, assign each student a scale factor to use for the second part. Appropriate scale factors include $\frac{1}{3}$, $\frac{1}{2}$, $1\frac{1}{2}$, and 2. It will work best if the centre of the enlargement is not too close to the rectangle the students are enlarging.

Representation: Provide Access for Perception. Display or provide students with a physical copy of the Launch demonstration about enlarging a point on a plane with no grid. Check for understanding by inviting students to rephrase directions for creating an enlargement of points in their own words. Consider keeping the display of directions visible throughout the activity.
Supports accessibility for: Language; Memory

Anticipated Misconceptions

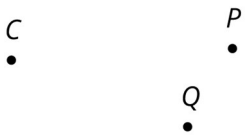
Students may all try to make their drawing match any example drawings shown in the launch. For example, if the centre of enlargement in an example is above and to the right, everyone might place their centre of enlargement above and to the right of the rectangle. Any point is fine as an enlargement point, but the effect on what the picture looks like may vary.

Students may not recall that to enlarge a polygon, they can first enlarge the vertices and then connect them in the proper order. It may be necessary to show students how to enlarge one of the vertices and allow them to perform the enlargement on the other three vertices.

Student Task Statement

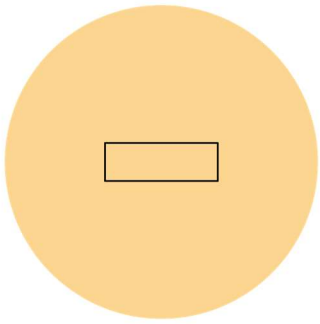
1. Using one coloured pencil, draw the images of points P and Q using C as the centre of enlargement and a scale factor of 4. Label the new points P' and Q'.

2. Using a different colour, draw the images of points P and Q using C as the centre of enlargement and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q''.



Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Now you'll make a perspective drawing. Here is a rectangle.



a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the centre of enlargement. Label it C.

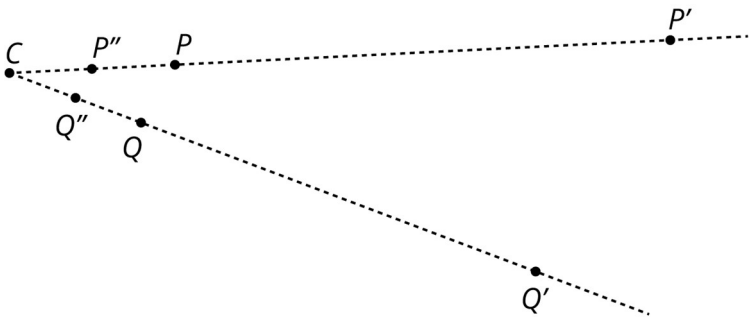
b. Using your centre C and the scale factor you were given, draw the image under the enlargement of each vertex of the rectangle, one at a time. Connect the enlarged vertices to create the enlarged rectangle.

c. Draw a line segment that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If there's time, you can shade the sides of the box to make it look more realistic.

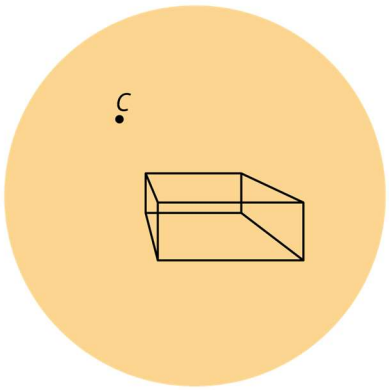
d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your enlarged rectangle closer to C than to the original rectangle, or farther away? How is that decided?

Student Response

1,2

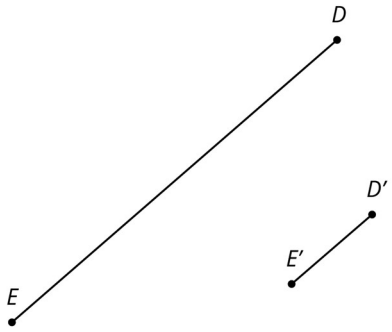


1. Answers vary. Possible response (scale factor 1.5):



Are You Ready for More?

Here is line segment DE and its image $D'E'$ under an enlargement.



1. Use a ruler to find and draw the centre of enlargement. Label it F.
2. What is the scale factor of the enlargement?

Student Response

1. Draw ray DD' and ray EE'. F is their intersection.
2. $\frac{1}{4}$

Activity Synthesis

Display the work of several students selected based on the different scale factors. Then ask students:

- "What are the effects of using a scale factor greater than 1?" (The image is larger than the original *and* farther away from the centre of enlargement than the original.)
- "What are the effects of using a scale factor less than 1?" (The image is smaller than the original *and* closer to the centre of enlargement than the original.)
- "What effect does the location of C, the centre of enlargement, have?" (It impacts the size and location of the enlarged rectangle: if the scale factor is less than 1 then the enlarged rectangle is closer to C than the original and if the scale factor is larger than 1 then the enlarged rectangle is further away from C than the original.)

Time permitting, consider showing several student drawings with the same scale factor but a different location for the point C. How are they the same? How are they different? Two faces of these boxes (the original rectangle and the scaled copy) are congruent but the point of view or perspective on them is different.

Speaking, Listening: Compare and Connect. As students prepare their perspective drawings, identify the drawings with scale factors greater than 1 or less than 1. As students investigate each other's work, ask them to share what is similar about the drawings with scale factor greater than 1 (or less than 1). Listen for and amplify statements such as "a scale factor greater than 1 results in an image larger than the original" and "a scale factor less than 1 results in an image smaller than the original." Then encourage students to make connections between the value of the scale factor and the effect on the image. Listen for and amplify language students use to describe how the scale factor affects the size of the image and its distance from the centre of enlargement. This will foster students' meta-awareness and support constructive conversations as they compare perspective drawings and make connections between the value of the scale factor and the image of the original polygon. *Design Principles(s): Cultivate conversation; Maximise meta-awareness*

Lesson Synthesis

Ask students to think about how they would explain the steps for enlarging a point, and either write them down or share them with a partner. Ask a few students to share their steps. Ensure that all of the important aspects are mentioned:

- "You need to know which point you want to enlarge, which point is the centre of enlargement, and what scale factor to use."
- "Use a straightedge to draw a ray from the centre of enlargement through the point you want to enlarge."
- "Measure the distance from the centre of enlargement through the point. Multiply this distance by the scale factor. Place the new point at this distance from the centre of enlargement and also on the ray you drew."
- "If the scale factor is greater than 1, the new point will be farther from the centre than the original point. If the scale factor is less than 1, the new point will be closer to the centre than the original point."

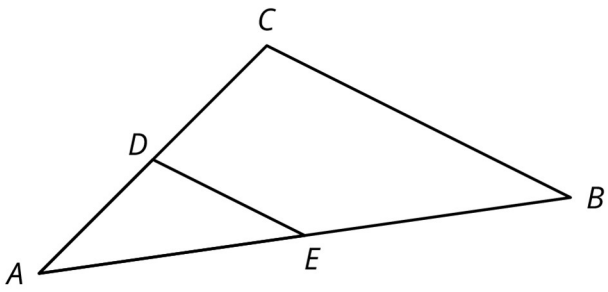
3.4 A Single Enlargement of a Triangle

Cool Down: 5 minutes

Students identify the centre of an enlargement given a shape, its enlargement, and the scale factor.

Student Task Statement

Lin drew a triangle and an enlargement of the triangle with scale factor $\frac{1}{2}$:



1. What is the centre of the enlargement? Explain how you know.
2. Which triangle is the original and which triangle is the enlargement? Explain how you know.

Student Response

1. The centre of enlargement is A. Lines emanate from A and points lie along those lines.
2. Triangle ABC is the original and triangle AED is the enlargement. Since the scale factor is less than 1, the enlargement is smaller than the original shape.

Student Lesson Summary

If A is the centre of enlargement, how can we find which point is the enlargement of B with scale factor 2?

Since the scale factor is larger than 1, the point must be farther away from A than B is, which makes C the point we are looking for. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B.

An enlargement with scale factor less than 1 brings points closer. The point D is the enlargement of B with centre A and scale factor $\frac{1}{3}$.

Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Line segment AB measures 3 cm. Point O is the centre of enlargement. How long is the image of AB after an enlargement with . . .

   a. Scale factor 5?

   b. Scale factor 3.7?

   c. Scale factor $\frac{1}{5}$?

   d. Scale factor s?

   Solution

   a. 15 cm

   b. 11.1 cm

   c. $\frac{3}{5}$ cm

   d. $3s$ cm

2. **Problem 2 Statement**

   Here are points A and B. Plot the points for each enlargement described.

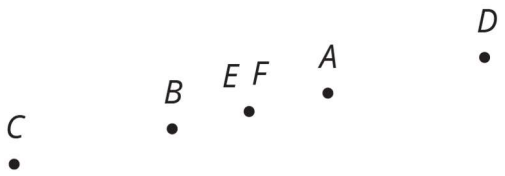
   a. C is the image of B using A as the centre of enlargement and a scale factor of 2.

   b. D is the image of A using B as the centre of enlargement and a scale factor of 2.

   c. E is the image of B using A as the centre of enlargement and a scale factor of $\frac{1}{2}$.

   d. F is the image of A using B as the centre of enlargement and a scale factor of $\frac{1}{2}$.

   Solution



3. **Problem 3 Statement**

Make a perspective drawing. Include in your work the centre of enlargement, the shape you enlarge, and the scale factor you use.

Solution

Answers vary.

4. **Problem 4 Statement**

Triangle ABC is a scaled copy of triangle DEF. Side AB measures 12 cm and is the longest side of ABC. Side DE measures 8 cm and is the longest side of DEF.

a. Triangle ABC is a scaled copy of triangle DEF with what scale factor?

b. Triangle DEF is a scaled copy of triangle ABC with what scale factor?

Solution

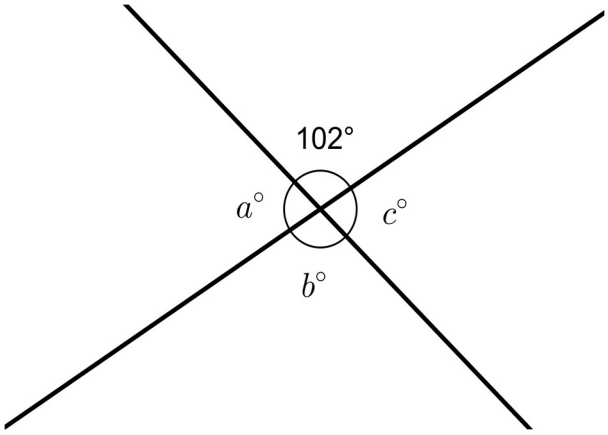
a. $\frac{3}{2}$

b. $\frac{2}{3}$

5. **Problem 5 Statement**

The diagram shows two intersecting lines.

Find the missing angles.



Solution

$a = 78$

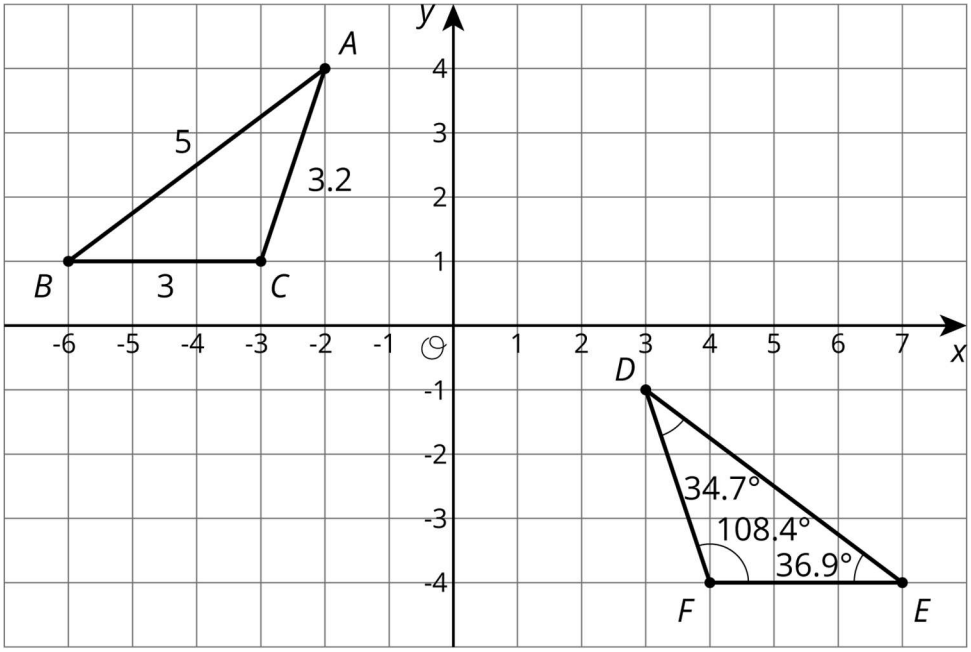
$b = 102$

$c = 78$

6. **Problem 6 Statement**

   a. Show that the two triangles are congruent.

   b. Find the side lengths of DEF and the angles of ABC.



Solution

   a. Reflect in the y-axis and translate until A meets D.

   b. Angle ABC is 36.9 degrees. Angle BCA is 108.4 degrees. Angle CAB is 34.7 degrees. $DE = 5$. $EF = 3$. $FD = 3.2$.