

Lesson 13: Median

Goals

- Comprehend that the “median” is another measure of centre, or average, which uses the middle of all the values in an ordered list to summarise the data.
- Identify and interpret the median of a data set given in a table or on a dot plot.
- Informally estimate the centre of a data set and then compare (orally and in writing) the mean and median with this estimate.

Learning Targets

- I can find the median for a set of data.
- I can say what the median represents and what it tells us in a given context.

Lesson Narrative

In this lesson, students consider another measure of centre, the **median**, which divides the data into two groups with half of the data greater and half of the data less than the median. To find the median, they learn that the data are to be arranged in order, from least to greatest. They make use of the structure of the data set to see that the median partitions the data into two halves: one half of the values in the data set has that value or smaller values, and the other half has that value or larger. Students learn how to find the median for data sets with both even and odd number of values.

Students find the median of a numerical data set and interpret it in context. They begin to see that, just like the mean, the median can be used to describe what is typical in a distribution, but that it is interpreted differently than the mean.

Addressing

- Summarise and describe distributions.
- Giving quantitative measures of centre (median and/or mean) and variability (interquartile range and/or range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Building Towards

- Relating the choice of measures of centre and variability to the shape of the data distribution and the context in which the data were gathered.

Instructional Routines

- Stronger and Clearer Each Time
 - Clarify, Critique, Correct
 - Think Pair Share
-

Required Materials

Index cards

Required Preparation

For the Finding the Middle activity, each student will need an index card.

Student Learning Goals

Let's explore the median of a data set and what it tells us.

13.1 The Plot of the Story

Warm Up: 5 minutes

This warm-up introduces/reinforces the **range** as a measure of the spread of data. The range is the difference between the highest and lowest items of data. In the given scenarios, the number of people attending the two events and their mean age are the same, but the ranges are different. In the first question, students interpret these measures in the context of the situations. In the second, they draw a dot plot that could represent an age distribution with the same mean and yet another range.

As students work and discuss, identify several students who drew dot plots that correctly meet the criteria in the second question. Ask students with different dot plots to share during whole-class discussion.

Students may need more time to make sense of how to generate their own dot plot for the second question. If it is not possible to give students additional time, consider presenting the second question at a different time.

Instructional Routines

- Think Pair Share

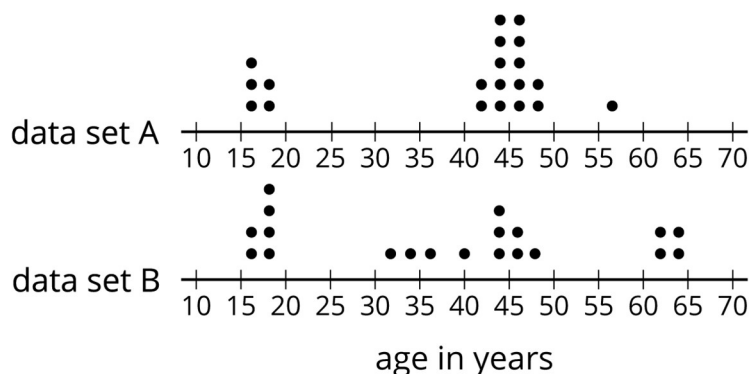
Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time for the first question, and then 2–3 minutes to work on the second question with a partner. Display the following questions for all to see. Ask students to think about and discuss them before drawing their dot plots:

- “How many data points should be on the dot plot?”
- “How would the mean help us place the data points?”
- “How would the range help us place the data points?”
- “How should our dot plot compare to the dot plots of data sets A and B?”

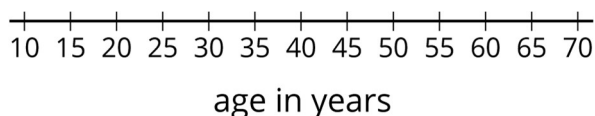
Student Task Statement

1. Here are two dot plots and two stories. Match each story with a dot plot that could represent it. Be prepared to explain your reasoning.



- Twenty people—secondary school students, teachers, and invited guests—attended a rehearsal for a school musical. The mean age was 38.5 years and the range was 48 years.
 - Secondary school football team practice is usually watched by supporters of the players. One evening, twenty people watched the team practice. The mean age was 38.5 years and the range was 41 years.
2. Another evening, twenty people watched the football team practice. The mean age was similar to that from the first evening, but the range was greater (about 60 years).

Make a dot plot that could illustrate the distribution of ages in this story.



Student Response

1. Data set A goes with the secondary-school football practice story. Data set B goes with the musical performance story. Sample explanation: The data points in data set B are more spread out, so the range for that data set would be larger.
2. Answers vary, but should show a centre of between 38 and 40 and a wider spread than data set B. Sample dot plot:



Activity Synthesis

Poll students on their response to the first question. Ask a student to explain how they matched one context to its dot plot and another student to explain the second matching context and dot plot. Record and display their responses for all to see. If possible, record their responses directly on the dot plots.

Ask selected students to share their dot plots for the second question and their reasoning. To involve more students in the conversation, consider asking some of the following questions:

- “What was the first piece of information you used to draw your dot plot? Why?”
- “How did you decide where to place your dots?”
- “How is your dot plot the same or different than the first evening of football practice?”
- “Do you agree or disagree with this representation of the context? Why?”
- “Do you have any questions to ask the student who drew the dot plot?”

13.2 Siblings in the House

15 minutes

The aim of this activity is to expose the limits of the mean in summarising a data set that has gaps and values far from the centre, and to motivate a need to have another measure of centre. Students first use a table of values and a dot plot to estimate a “typical” value for a data set. Then, they calculate the mean and notice that it does not match their estimate of a typical value. A closer look helps them see that when a data set contains values that are far away from the bulk of the data, or when there are gaps in the data set, the mean can be a little or a lot higher or lower than what we would consider typical for the data.

In the next activity, the median will be introduced as another measure of centre of a data set.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 7–8 minutes of quiet work time and then 3–4 minutes to discuss their responses with a partner. If there are disagreements, ask them to discuss them until they reach agreement. Follow with a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge about determining means of data sets. Allow students to use calculators to ensure inclusive

participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Since previous lessons have used the mean as the best way to find a typical value, some students may go directly to that method from the beginning. Although this is valid at this stage, encourage them to look at the dot plot and think about what a typical value *should* be.

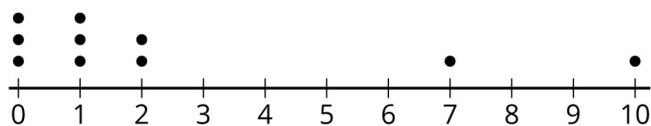
Student Task Statement

Here is data that shows the numbers of siblings of ten students in Tyler’s class.

1 0 2 1 7 0 2 0 1 10

1. Represent the data shown with a dot plot.
2. Without making any calculations, estimate the centre of the data based on your dot plot. What is a typical number of siblings for these year 7 students? Mark the location of that number on your dot plot.
3. Find the mean. Show your reasoning.
 - a. How does the mean compare to the value that you marked on the dot plot as a typical number of siblings? (Is it a little larger, a lot larger, exactly the same, a little smaller, or a lot smaller than your estimate?)
 - b. Do you think the mean summarises the data set well? Explain your reasoning.

Student Response



1. number of siblings
2. Answers vary. Sample response: I’d estimate the centre to be between 1 and 2 siblings.
3. The mean is 2.4 siblings. $\frac{1+0+2+1+7+0+2+0+1+10}{10} = \frac{24}{10} = 2.4$.
 - a. Answers vary. Sample response: The mean is a little larger than my estimate.
 - b. Answers vary. Sample response: I don’t think the mean summarises the data very well. Eight out of 10 of the data points are below the mean, and more than

half of the students have either no siblings or only 1 sibling, so to say that 2.4 is a typical number of siblings is not accurate.

Are You Ready for More?

Invent a data set with a mean that is significantly lower than what you would consider a typical value for the data set.

Student Response

Answers vary. Sample response: The data could have most values close to or equal to 8 and a small number of much lower values: 0, 0, 1, 7, 8, 8, 8, 9, 9, 10. The mean of this data set is 6, while we might say that 8 is typical.

Activity Synthesis

Select a few students to share their estimate for a typical number of siblings. Consider asking students:

- “When you looked at the table of values, what was your sense of a typical number of siblings for the ten students in Tyler’s class?”
- “When you looked at the dot plot, did your estimate change?”

Then, discuss how the calculated mean compared to their estimates. Draw students’ attention to the idea that the mean may not always represent a typical value for a data set. Discuss:

- “We have learned that the mean is a way to measure the centre of a distribution. How did your calculated mean compare to your estimate of what was typical for the data set?”
- “Why do you think the mean was higher than your estimate?” (Only two of the points are above the mean of 2.4 and both are quite far above it, and seven points are below 2.4, so the mean might not paint an accurate picture of what is typical in this situation.)
- “If the mean does not always reflect what is typical in a data set, should we always rely on it as the best way to describe the centre? If not, in what other ways might we measure the centre of a data set?”

Explain to students that in the next activity we will look at a different measure of centre.

Writing, Conversing: Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to refine their response to “Do you think the mean summarises the data set well?” Give students time to meet with 2–3 partners, to share and get feedback on their responses. Provide listeners with prompts for feedback that will help their partner strengthen their ideas and clarify their language. For example, “Can you explain what a typical value should be?” and “How can you expand on the using the mean to find a typical value?” Give students 2–3 minutes to revise their initial draft based on feedback from their

peers. This helps students clarify their reasoning about how the mean may or may not summarise the data well.

Design Principle(s): Optimise output (for generalisation); Cultivate conversation

13.3 Finding the Middle

15 minutes

This activity introduces students to the term **median**. They learn that the median describes the middle value in an ordered list of data, and that it can capture what we consider typical for the data in some cases.

Students learn about the median through a kinaesthetic activity. They line up in order of the number of letters in their name. Then, those at both ends of the line count off and sit down simultaneously until one or two people in the middle remain standing. If one person remains standing, that person has the median number of letters. If two people remain standing, the median is the mean or the average of their two values.

Students then practise identifying the median of other data sets, by analysing both tables of values and dot plots.

Instructional Routines

- Clarify, Critique, Correct

Launch

Explain to students that, instead of using the mean, sometimes we use the “middle” value in an ordered list of data set as a measure of centre. We call this the **median**. Guide students through the activity:

- Give each student an index card. Ask them to write their first and last names on the card and record the total number of letters in their name. Display an example for all to see.
- Ask students to stand up, holding their index cards in front of them, and arrange themselves in order based on the number of letters in their name. (Consider asking students to do so without speaking at all.) Look for the student whose name has the fewest letters and ask him or her to be the left end of the line. Ask the student with the longest full name to be the right end of the line. Students who have the same number of letters should stand side-by-side.
- Tell students that, to find the median or the middle number, we will count off from both ends at the same time. Ask the students at the two ends of the line say “1” at the same time and sit on the floor, and the students next to them to say “2” and then sit down, and so on. Have students count off in this fashion until only one or two students are standing.
- If the class has an odd number of students, one student will remain standing. Tell the class that this student’s number is the median. Give this student a sign that says

“median” If the class has an even number of students, two students will remain standing. The median will be the mean or average of their numbers. Ask both students to hold the sign that says “median.” Explain that the median is also called the “50th percentile,” because half of the data values are the same size or less than it and fall to the left of it on the number line, and half are the same size or greater than it and fall to the right.

- Ask students to find the median a couple more times by changing the data set (e.g., asking a few students to leave the line or adding new people who are not members of the class with extremely long or short names). Make sure that students have a chance to work with both odd and even numbers of values.
- Collect the index cards and save them; they will be used again in the lesson on box plots.

Ask students to complete the rest of the questions on the task statement.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: median. Invite students to suggest language or diagrams to include on the display that will support their understanding of this term.

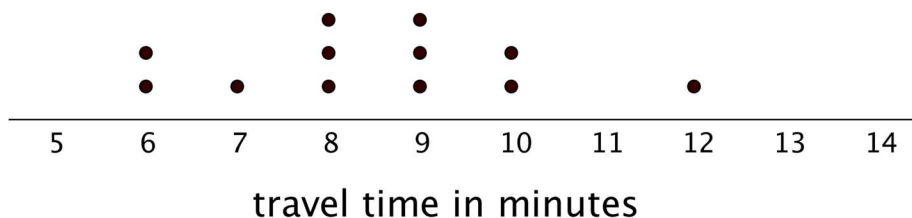
Supports accessibility for: Memory; Language

Anticipated Misconceptions

When determining the median, students might group multiple data points that have the same value and treat it as a single point, instead of counting each one separately. Remind them that when they lined up to find the median number of letters in their names, every student counted off, even if their name had the same number of letters as their neighbours.

Student Task Statement

1. Your teacher will give you an index card. Write your first and last names on the card. Then record the total number of letters in your name. After that, pause for additional instructions from your teacher.
2. Here is the data set on numbers of siblings from an earlier activity.
1 0 2 1 7 0 2 0 1 10
 - a. Sort the data from least to greatest, and then find the **median**.
 - b. In this situation, do you think the median is a good measure of a typical number of siblings for this group? Explain your reasoning.
3. Here is the dot plot showing the travel time, in minutes, of Elena’s bus rides to school.



- Find the median travel time. Be prepared to explain your reasoning.
- What does the median tell us in this context?

Student Response

- No answer required.
 - The numbers of siblings in order: 0, 0, 0, 1, 1, 1, 2, 2, 7, 10 has median 1. The median is the average of the fifth and sixth values.
 - Answers vary. Sample response: Yes, I think the median is a good measure, because 1 sibling is in the middle of where most of the points in the data set are. Almost a third of the students have 1 sibling.
- The median length of travel is 8.5 minutes. It is the average of the sixth and seventh data points, which are 8 and 9 minutes.
 - Answers vary. Sample response: The median tells that half of her trips to school took 8.5 minutes or less and the other half took 8.5 minutes or more.

Activity Synthesis

Select a few students to share their responses to the questions about number of siblings and Elena's travel times. Focus the discussion on the median as another measure of the centre of a data set and whether it captures what students would estimate to be a typical value for each data set.

Emphasise to students that the median is a *value* and not an *individual*. For example, if the last person standing in the class has 5 letters in their first name, the median is the number 5 and not the person standing. If there is another student who had 5 letters in their name, they might have switched places with the last person standing when lining up initially. Although the person standing changed, the median remains the same value of 5.

At this point, it is unnecessary to compare the mean and the median. Students will have many more opportunities to explore the median and think about how it differs from the mean in the upcoming activities.

Writing, Conversing: Clarify, Critique, Correct. Before students share their responses to the questions about number of siblings and Elena's travel times, present an incorrect response. For example, display the statement: "There is no median for this data set because two students remained standing." Demonstrate the process of interpreting the statements to

uncover possible errors by thinking aloud. Voice the questions you ask yourself to call students' attention to the possible types of errors. Invite students to work with a partner to clarify the reasoning, and create an improved statement. Select 2–3 groups to share with the whole class. This helps students evaluate, and improve upon the written mathematical arguments of others.

Design Principle(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

In this lesson, we learn about another measure of centre called the **median**. The discussion should focus on what the median is, how to find it, and why it is useful.

- “What is the median?” (The number in the middle of an ordered list of data.)
- “How can we find it?” (We order the data values from least to greatest and find the value in the middle.)
- “Is the median always one of the values in the data set? If not, when is it not?” (No. When the number of values in a data set is even, there will be two middle values. The median is the number exactly between them which may not be a value in the data set.)
- “What does the median tell you about a data set? Why is it used as a measure of the centre of a distribution?” (It tells us where to divide a data set so that half of the data points have that value or smaller values and the other half have that value or larger.)
- “Why do we need another measure of centre other than the mean?” (Sometimes the mean is not a good indication of what is typical for the data set.)

13.4 Practising the Piano

Cool Down: 5 minutes

Student Task Statement

Jada and Diego are practicing the piano for an upcoming rehearsal. The tables list the number of minutes each of them practiced in the past few weeks.

Jada's practice times:

10 10 20 15 25 25 8 15
20 20 35 25 40

Diego's practice times:

25 10 15 30 15 20 20 25
30 45

1. Find the median of each data set.
2. Explain what the medians tell you about Jada's and Diego's piano practice.

Student Response

1. Jada's times in order: 8, 10, 10, 15, 15, 20, 20, 20, 25, 25, 25, 35, 40. The median is 20 minutes. Diego's times in order: 10, 15, 15, 20, 20, 25, 25, 30, 30, 45. The median is 22.5 minutes.
2. Answers vary. Sample response: One half of Jada's practices were 20 minutes or shorter; the other half of her practices were 20 minutes or longer. Half of Diego's practices were 22.5 minutes or shorter, the other half were 22.5 minutes or longer.

Student Lesson Summary

The **median** is another measure of centre of a distribution. It is the middle value in a data set when values are listed in order. Half of the values in a data set are less than or equal to the median, and half of the values are greater than or equal to the median.

To find the median, we order the data values from least to greatest and find the number in the middle.

Suppose we have 5 dogs whose weights, in pounds, are shown in the table. The median weight for this group of dogs is 32 pounds because three dogs weigh less than or equal to 32 pounds and three dogs weigh greater than or equal to 32 pounds.

20 25 32 40 55

Now suppose we have 6 cats whose weights, in pounds, are as shown in the table. Notice that there are *two* values in the middle: 7 and 8.

4 6 7 8 10 10

The median weight must be between 7 and 8 pounds, because half of the cats weigh less or equal to 7 pounds and half of the cats weigh greater than or equal to 8 pounds.

In general, when we have an even number of values, we take the number exactly in between the two middle values. In this case, the median cat weight is 7.5 pounds because $(7 + 8) \div 2 = 7.5$.

Glossary

- Median

Lesson 13 Practice Problems

Problem 1 Statement

Here is data that shows a student's scores for 10 rounds of a video game.

130 150 120 170 130 120 160 160 190 140

What is the median score?

- a. 125
- b. 145
- c. 147
- d. 150

Solution B

Problem 2 Statement

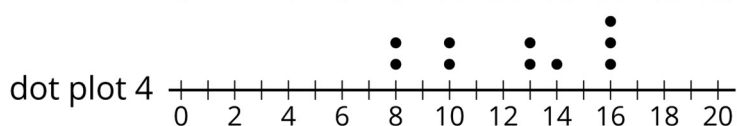
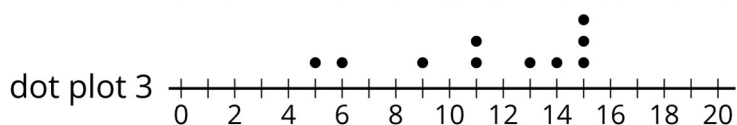
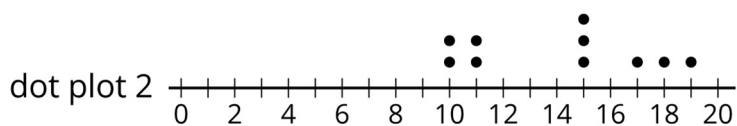
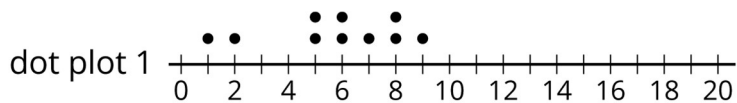
When he sorts the class's scores on the last test, the teacher notices that exactly 12 students scored better than Clare and exactly 12 students scored worse than Clare. Does this mean that Clare's score on the test is the median? Explain your reasoning.

Solution

Yes. There are 25 students in the class, and Clare's score was exactly in the middle, so her score is the median.

Problem 3 Statement

The medians of the following dot plots are 6, 12, 13, and 15, but not in that order. Match each dot plot with its median.



Solution

- 6: dot plot 1
- 12: dot plot 3
- 13: dot plot 4
- 15: dot plot 2

Problem 4 Statement

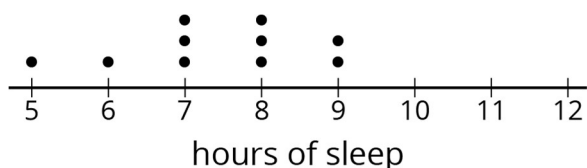
Invent a data set with five numbers that has a mean of 10 and a median of 12.

Solution

Answers vary. Sample response: 1, 11, 12, 13, 13.

Problem 5 Statement

Ten year 7 students reported the hours of sleep they get on nights before a school day. Their responses are recorded in the dot plot.



Looking at the dot plot, Lin estimated the mean number of hours of sleep to be 8.5 hours. Noah's estimate was 7.5 hours. Diego's estimate was 6.5 hours.

Which estimate do you think is best? Explain how you know.

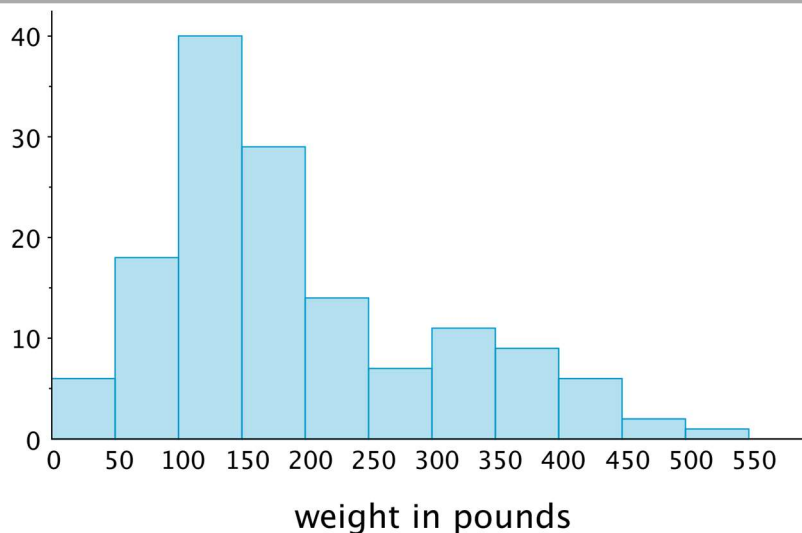
Solution

Noah's estimate of 7.5 hours is best. Explanations vary. Sample explanation: It is a better balance point for the data, balancing the distances to the left and to the right.

Problem 6 Statement

In one study of wild bears, researchers measured the weights, in pounds, of 143 wild bears that ranged in age from new-born to 15 years old. The data was used to make this histogram.

- a. What can you say about the heaviest bear in this group?
- b. What is a typical weight for the bears in this group?
- c. Do more than half of the bears in this group weigh less than 250 pounds?
- d. If weight is related to age, with older bears tending to have greater body weights, would you say that there were more old bears or more young bears in the group? Explain your reasoning.



Solution

- The heaviest bear had a weight between 500 and 550 pounds.
- About 200 pounds
- Yes
- More young bears. There are more bears with smaller weights than large weights, so there are probably more young bears in the group.



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